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www.elsevier.com/locate/physletbThree-Higgs-doublet model with A_4 symmetryA.C.B. Machado ^{*,1}, J.C. Montero ², V. Pleitez ³

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ABSTRACT

We worked out in detail the three-Higgs-doublet extension of the standard model when the A_4 symmetry, which is imposed to solve the flavor problem, is extended to the scalar sector. The three doublets may be related to the fermion mass generation and, in particular, they may be the unique responsible for generating the neutrino masses. If this is the case, the respective VEVs have to be quite smaller than the electroweak scale. The usual hierarchy in the Yukawa couplings is now moved to a hierarchy in the VEVs which may be justified on dynamical grounds. We consider here the mass spectra in the scalar sector in several situations. When the three doublets are those related with neutrino masses some light scalar arise. However, the later fields are safe, from the phenomenological point of view, since they couple mainly to neutrinos.

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1. Introduction

In the standard model of particle interactions (SM) with only one Higgs scalar doublet the fermion mass matrices remain arbitrary. In order to become more predictive about the origin of masses and mixing, extra symmetries, continuous [1], or discrete [2], may be imposed. On one hand, among the most motivated extensions of the SM are those with several Higgs doublets [3]. On the other hand, the most general scalar potential even for the simplest case of two doublets is hard to be worked out in detail [4,5]. Hence, in order to study multi-Higgs extensions of the SM it is necessary to impose extra symmetries. An interesting possibility is that these extra symmetries are the same that are also useful for explaining the fermion masses and mixing, avoiding that the Yukawa couplings do not span over two or even five decade of magnitude. Moreover, since all the neutral components of doublets getting a non-zero VEV contribute to the W^\pm and Z masses, the sum of the VEVs squares is equal to $\sim (174 \text{ GeV})^2$ and some VEVs may be smaller than 174 GeV. Neutral Higgs scalars getting small VEVs (i.e., smaller than the electroweak scale) are potentially dangerous since they may imply the existence of light charged or neutral (pseudo)scalars in the model. However, the appearance of these sort of VEVs is almost inevitable when more than two doublets are added to the SM. This situation arises in models in which different Higgs sectors give mass to different charged fermion sectors, as in [6,7]. Thus, it is necessary to search for a mechanism that allows heavy enough scalars or their phenomenological effects be appropriately suppressed.

2. Three doublet model with soft A_4 violating terms

Here we will show several vacuum alignments and see in what cases when some, or all, VEVs are small light scalar may be avoided. This is the inverse of the mechanism considered in Ref. [8]. We consider the case when three $SU(2)$ doublets h_i , $i = 1, 2, 3$, are in an A_4 triplet $h \equiv (h_1, h_2, h_3)$. The most general scalar potential invariant under A_4 and the standard model gauge symmetries is denoted by $V(h)$ and we may, or not, add terms which break the A_4 symmetry softly, V_{soft} . The scalar potential is given by

$$V = V(h) + V_{soft}(h_1, h_2, h_3), \quad (1)$$

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where

$$V(h) = \mu^2 [h^\dagger h]_1 + \lambda_1 ([h^\dagger h]_1)^2 + \lambda_2 [h^\dagger h]_{1'} [h^\dagger h]_{1''} + \lambda_3 [h^\dagger h]_A [h^\dagger h]_A + \lambda_3^* [h^\dagger h]_B [h^\dagger h]_B + \lambda_4 [h^\dagger h]_A [h^\dagger h]_B, \quad (2)$$

which can be written explicitly in terms of h_i as [9]

$$V(h) = \mu^2 \sum_i h_i^\dagger h_i + (\lambda_1 + \lambda_2) \sum_i (h_i^\dagger h_i)^2 + (\lambda_1 - \lambda_2) \sum_{i \neq j} (h_i^\dagger h_i)(h_j^\dagger h_j) + \left(\lambda_3 \sum_{i \neq j} (h_i^\dagger h_j)^2 + \text{H.c.} \right) + \lambda_4 \sum_{i \neq j} (h_i^\dagger h_j)(h_j^\dagger h_i), \quad (3)$$

which was considered in [10]. We add also the terms that violate softly the A_4 symmetry:

$$V_{\text{soft}}(h_1, h_2, h_3) = \mu_1^2 h_1^\dagger h_1 + \mu_2^2 h_2^\dagger h_2 + \mu_3^2 h_3^\dagger h_3 + (v_{12}^2 h_1^\dagger h_2 + v_{13}^2 h_1^\dagger h_3 + v_{23}^2 h_2^\dagger h_3 + \text{H.c.}). \quad (4)$$

We will consider, for the sake of simplicity, that all VEVs and dimensionless parameters are real and use the A_4 identity $1 + \omega + \omega^2 = 0$. Defining $\partial V / \partial v_i = t_i$, we have

$$\begin{aligned} t_1 &= v_1 \left[\mu_1^2 + \mu^2 + \lambda v_1^2 + \frac{1}{2} \lambda' (v_2^2 + v_3^2) \right] + v_{12}^2 v_2 + v_{13}^2 v_3, \\ t_2 &= v_2 \left[\mu_2^2 + \mu^2 + \lambda v_2^2 + \frac{1}{2} \lambda' (v_1^2 + v_3^2) \right] + v_{12}^2 v_1 + v_{23}^2 v_3, \\ t_3 &= v_3 \left[\mu_3^2 + \mu^2 + \lambda v_3^2 + \frac{1}{2} \lambda' (v_1^2 + v_2^2) \right] + v_{13}^2 v_1 + v_{23}^2 v_2, \end{aligned} \quad (5)$$

where $\lambda = \lambda_1 + \lambda_2$, $\lambda' = 2\lambda_1 - \lambda_2 + \lambda''$, and $\lambda'' = 2\lambda_3 + \lambda_4$. The constraint equations, which minimize the scalar potential, are $t_i = 0$, for fixed i , can be solved in several ways as we will show below. Notice that all v_i may be different from zero but may be arbitrarily small.

2.1. First case: model with some $\mu_i^2 > 0$

An example of this case is when $\mu^2 < 0$ and $\mu_1^2 < 0$ while $\mu_{2,3}^2 > 0$. We can obtain hierarchies among the VEVs. Let us suppose that one of the VEVs (v_1) is larger than the other two (v_2, v_3) and that the products $v_{12}^2 v_2, v_{13}^2 v_3$ are negligible compared to the first term of t_1 , and also that $\mu_2^2 + \mu^2 + \lambda v_2^2 + (\lambda'/2)[v_1^2 + v_3^2] = 0$, $\mu_3^2 + \mu^2 + \lambda v_3^2 + (\lambda'/2)[v_1^2 + v_2^2] = 0$, we have from (5)

$$v_1 \approx \sqrt{-\frac{\mu^2 + \mu_1^2}{\lambda}}, \quad v_2 \approx -\frac{v_{12}^2}{v_{23}^2} v_1, \quad v_3 \approx -\frac{v_{13}^2}{v_{23}^2} v_1, \quad (6)$$

with $v_{12}^2, v_{13}^2 < 0$ and $v_{23}^2 \gg |v_{12}^2|, |v_{13}^2|$ we have consistence with the condition $v_2/v_1, v_3/v_1 \ll 1$.

On the other hand if still $v_1 \gg v_2, v_3$, with $\mu_{2,3}^2$ positive but now arbitrary and $\mu_{2,3}^2 > |\mu^2|, v_1^2 \gg |v_{12}^2|, |v_{13}^2|$, we have that v_1 as in (6) but now

$$v_2 \approx -\frac{2v_{12}^2}{2\mu^2 + \lambda'v_1^2 + 2\mu_2^2} v_1, \quad v_3 \approx -\frac{2v_{13}^2}{2\mu^2 + \lambda'v_1^2 + 2\mu_3^2} v_1. \quad (7)$$

Under these circumstances we have always a hierarchy in the VEVs as required in the model of [6]: $v_1 \gg v_2 \gg v_3$. If the triplet that is being considered contributes to the generation of the u -type quark masses, the mass spectrum of the neutral scalar sector has a standard-model-like scalar which mass is of the order of v_1 , and two heavy scalars which masses are dominated by μ_2 and μ_3 . Again, the solutions (7) are consistent with the assumed condition that $v_{2,3}$ are smaller than v_1 . Hence, assuming the $|v_{12}^2|$ and $|v_{13}^2|$ smaller than $|\mu_2^2|$ and $|\mu_3^2|$, this model is a three doublet generalization of the two doublet model of Ref. [11].

2.2. Second case: model with all $\mu_i^2 < 0$

Now, Eqs. (5) are solved by using the conditions $\mu_{1,2,3}^2 < 0$ and the resulting scalar mass spectra are as follows. After using the conditions $t_i = 0$ in Eqs. (5), we obtain the mass matrices of each charge sector. For the neutral pseudoscalar sector, in the basis (a_1, a_2, a_3) , the mass matrix is

$$M_a^2 = \begin{pmatrix} -\frac{2\lambda_3 v_1 (v_2^2 + v_3^2) + v_2 v_{12}^2 + v_3 v_{13}^2}{v_1} & 2\lambda_3 v_1 v_2 + v_{12}^2 & 2\lambda_3 v_1 v_3 + v_{13}^2 \\ -\frac{2\lambda_3 v_2 (v_1^2 + v_3^2) + v_1 v_{12}^2 + v_3 v_{23}^2}{v_2} & 2\lambda_3 v_2 v_3 + v_{23}^2 & \\ & & -\frac{2\lambda_3 v_3 (v_1^2 + v_2^2) + v_1 v_{13}^2 + v_2 v_{23}^2}{v_3} \end{pmatrix}. \quad (8)$$

We will denote the eigenvectors of the matrix in Eq. (8) as G^0 for the would be Goldstone boson, and $A_{1,2}^0$ for the two physical pseudoscalars. In fact, the mass matrix in Eq. (8) has $\text{Det } M_a^2 = 0$. Here and below we will not show the eigenvectors explicitly.

In the neutral scalar sector, in the basis (h_1^0, h_2^0, h_3^0) , we have the mass matrix

$$M_s^2 = \begin{pmatrix} \frac{2\lambda v_1^3 - v_2 v_{12}^2 - v_3 v_{13}^2}{v_1} & \lambda' v_1 v_2 + v_{12}^2 & \lambda' v_1 v_3 + v_{13}^2 \\ \frac{2\lambda v_2^3 - v_1 v_{12}^2 - v_3 v_{23}^2}{v_2} & \lambda' v_2 v_3 + v_{23}^2 & \\ \frac{2\lambda v_3^3 - v_1 v_{13}^2 - v_2 v_{23}^2}{v_3} & & \end{pmatrix}. \quad (9)$$

The respective eigenvectors of M_s^2 are denoted by $S_{1,2,3}^0$, and one of them may or may not correspond to the SM Higgs field. From (9) we obtain $\text{Det} M_s^2 \neq 0$ hence this sector has no Goldstone bosons as it must be. However, it is not obvious if there exist or not a light scalar when all VEVs, but not the v s, are small.

For the charged scalar sector we have, in the basis (h_1^+, h_2^+, h_3^+) ,

$$M_c^2 = \begin{pmatrix} -\frac{\lambda'' v_1 (v_2^2 + v_3^2) + 2(v_2 v_{12}^2 + v_3 v_{13}^2)}{2v_1} & \frac{1}{2} \lambda'' v_1 v_2 + v_{12}^2 & \frac{1}{2} \lambda'' v_1 v_3 + v_{13}^2 \\ -\frac{\lambda'' v_2 (v_1^2 + v_3^2) + 2(v_1 v_{12}^2 + v_3 v_{23}^2)}{2v_2} & \frac{1}{2} \lambda'' v_2 v_3 + v_{23}^2 & \\ -\frac{\lambda'' v_3 (v_1^2 + v_2^2) + 2(v_1 v_{13}^2 + v_2 v_{23}^2)}{2v_3} & & \end{pmatrix}. \quad (10)$$

With the respective mass eigenvectors being denoted by $(H_{1,2,3}^+)$. The mass matrix in Eq. (10) has $\text{Det} M_c^2 = 0$ and hence this sector has a charged Goldstone boson.

Below we will find realistic mass spectra by diagonalizing the mass matrices given in (8), (9) and (10). We assume also that all VEVs are positive.

Consider the case when $\mu_i^2 \neq v_{ij}^2 \neq 0$, $i, j = 1, 2, 3$; $i \neq j$ and $\mu^2 \gg v_{ij}^2, \forall i, j$. For the sake of simplicity we assume also that $v_1 \ll v_2 = v_3 = v$ and dubbed $r = v_1/v \ll 1$ and $v_{12}^2 = v_{13}^2$. Under these conditions the pseudoscalar masses from Eq. (8), are:

$$\begin{aligned} m_{a1}^2 &= 0, \\ m_{a2}^2 &= -2\lambda_3(2+r^2)v^2 - \left(\frac{2}{r} + r\right)v_{13}^2, \\ m_{a3}^2 &= -2\lambda_3(2+r^2)v^2 - rv_{13}^2 - 2v_{23}^2, \end{aligned} \quad (11)$$

and $v_{13}^2, v_{23}^2 < 0$ and the sign of λ_3 depend on the magnitude of the soft terms. There is one Goldstone boson as it must be. All the real neutral scalar masses are non-zero:

$$\begin{aligned} m_{s1}^2 &= (2\lambda - \lambda')v^2 - 2v_{23}^2 - rv_{13}^2, \\ m_{s2}^2 &= A + \frac{1}{2r}\sqrt{B}, \quad m_{s3}^2 = A - \frac{1}{2r}\sqrt{B}, \end{aligned} \quad (12)$$

with A and B defined as:

$$\begin{aligned} A &= \left[\lambda(1+r^2) + \frac{\lambda'}{2} \right] v^2 - \left(\frac{1}{r} + \frac{r}{2} \right) v_{13}^2, \\ B &= [(2\lambda r(1+r^2) + \lambda' r)v^2 - (2+r^2)v_{13}^2]^2 + 8r[(\lambda(\lambda' - 2\lambda) + \lambda'^2)r^3 v^4 + (2\lambda + r^4 + \lambda'(1+2r^2))v^2 v_{13}^2]. \end{aligned} \quad (13)$$

Finally, the masses for the charged scalars from Eq. (10), are

$$\begin{aligned} m_{c1}^2 &= 0, \quad m_{c2}^2 = -\frac{1}{2} \left(\frac{2}{r} + r \right) (\lambda'' r v^2 + 2v_{13}^2), \\ m_{c3}^2 &= -\frac{\lambda''}{2} (2+r^2)v^2 - rv_{13}^2 - 2v_{23}^2, \end{aligned} \quad (14)$$

in which we have also only one Goldstone boson, as it must be.

For instance, if the triplet is one of those that contribute to the neutrino masses in [6], i.e., $v \sim 10^{-3}$ GeV and $v_1 \sim 10^{-7}$ GeV, then $r = v_1/v = 10^{-4} \ll 1$ [6]. Notice that, since v 's $\gg v, v_1$ and also r is small, the hierarchy among the scalar masses does not imply a respective hierarchy among the λ s, i.e., they may be of the same order of magnitude. Expanding in r (i.e., $r \ll 1$) we obtain from (11)–(14):

$$\begin{aligned} m_{a2}^2 &\approx -\frac{2}{r}v_{13}^2, \quad m_{a3}^2 \approx -2v_{23}^2 - rv_{13}^2, \\ m_{s1}^2 &\approx -2v_{23}^2 - rv_{13}^2, \quad m_{s2}^2 \approx (2\lambda + \lambda')v^2, \quad m_{s3}^2 \approx -\frac{2}{r}v_{13}^2, \\ m_{c2}^2 &\approx -\frac{2}{r}v_{13}^2, \quad m_{c3}^2 \approx -2v_{23}^2 - rv_{13}^2. \end{aligned} \quad (15)$$

From (15) we see that all pseudoscalar and charged Higgs bosons may be heavy even for small VEVs if the v^2 s are large enough (and negative). Then, when all VEVs are small, i.e., of the order of MeV or less, there is a light real neutral scalar, s_2 .

This is not an artifact of the approximation used in obtaining the square masses in (11), (12) and (14). We have obtained numerically the eigenvalues of the full matrices (8)–(10) i.e., without assuming $v_2 = v_3$ and $v_{12} = v_{13}$, and found that a light real scalar always survives. Just as an example, let us consider one of the triplets $H = (H_1, H_2, H_3)$ related to the neutrino mass generation in the model

of Ref. [6]. For simplicity we assume that the dimensionless coupling constants are $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 1$. The VEVs are (in GeV) $v_1 = 0.000001$, $v_2 = 0.00002$ and $v_3 = 0.001$, and the soft parameters (in GeV^2) $v_{12}^2 = -100$, $v_{13}^2 = -200$ and $v_{23}^2 = -300$, we obtain the masses for the physical scalars (in GeV): m_{a1} and m_{c1} vanish, they are the Goldstone bosons, $m_{s1} \sim m_{c2} \sim m_{a2} \approx 122.5$, $m_{s2} \approx 2 \times 10^{-3}$, and $m_{s3} \sim m_{c3} \sim m_{a3} \approx 450$. Notice that H_1^0 , with mass m_{s1} may be the neutral scalar which correspond to the SM Higgs scalar, H_{SM} . However, in this case it couples mainly to neutrinos. For the VEVs of the other doublets, denoting the respective masses as (m_{s1}, m_{s2}, m_{s3}) we obtain (in GeV) (65, 84, 466), (5.7, 25, 302), (116, 244, 5895), (98, 137, 230) for the VEVs of the $H''(\Phi'')$, (H') , (Φ') , in the notation of [6], respectively. The triplet with the lightest neutral scalar belong to Φ'' , notwithstanding, their coupling with d -like quarks are suppressed by the factor $1/\Lambda$, see Eq. (4) in Ref. [6].

One of the triplets of doublets has, at the tree level, a light neutral scalar, with mass of the order of the MeV, see $m_{s2} \approx 2$ MeV above. This sort of scalar boson can be discovery by the shift in the energy level in muonic atoms [12]. If the shift caused by scalar exchange is less than the experimental value [13] a scalar mass ≤ 9 MeV is ruled out [14]. This limit however may be evaded in the case of multi-Higgs models and lighter scalar may be allowed because there are mixing among the scalars, see Ref. [15]. It happens also that loop corrections must rise the mass m_{s2} to a value above this limit (see below). We must stress that usually the search for light scalars in nuclear and rare meson decays is based on the assumption that the scalar is the one of the SM [3].

On the other hand, if all v_S vanish and all the VEVs in Eqs. (8)–(10) are of the order of several GeVs, say $\sim 174/\sqrt{3}$ GeV, all Higgs scalars are heavy as well. Notwithstanding, this case is not interesting in the context of the models of [6] and [7] if neutrino masses are included in the latter model.

2.3. Third case: model with no soft A_4 violating terms

The case when $\mu_i^2 = v_{ij}^2 = 0$, $i, j = 1, 2, 3$; $i \neq j$ admits two type of solutions: i) $v_1 = v_2 = v_3$ and ii) $v_1 \neq v_2 \neq v_3$ (and all of them different from zero). In the case i), the relations in (5) are reduced to only one. In this case all mass matrices have all the same form: non-diagonal elements are all equal (denoted by b) and the diagonal elements are also equal to each other (denoted by a). The eigenvalues are $a+2b$ and $a-b$ and $a-b$ (the values of a and b depend on the sector: real, imaginary and charged sector). All real scalars are massive, one on them may, or may not, corresponds to the SM Higgs doublet and the other two are mass degenerated. This case was considered in detail in Ref. [16], see also Ref. [17].

The case ii) is consistent with the simultaneous solution of the constraint equations (Eqs. (5)), $t_i = 0$, only if we have $3\lambda_2 = 2\lambda_3 + \lambda_4$ [i.e., $-\mu^2 = (\lambda_1 + \lambda_2)V^2$ where $V^2 = v_1^2 + v_2^2 + v_3^2$] and $\lambda_1 + \lambda_2 > 0$. The corresponding mass spectrum for the pseudoscalars is:

$$m_{a1}^2 = 0, \quad m_{a2}^2 = m_{a3}^2 = -2\lambda_3 V^2, \quad (16)$$

while in the neutral real scalars we have

$$m_{s1}^2 = m_{s2}^2 = 0, \quad m_{s3}^2 = 2(\lambda_1 + \lambda_2)V^2. \quad (17)$$

In the charged scalars sector we find:

$$m_{c1}^2 = 0, \quad m_{c2}^2 = m_{c3}^2 = -\frac{3}{2}\lambda_2 V^2. \quad (18)$$

From the above results we have the following constraints on the scalar potential parameters: $\lambda_{2,3} < 0$, $\lambda_1 + \lambda_2 > 0$ and $\lambda_2 = (2\lambda_3 + \lambda_4)/3$, which can be satisfied provide

$$\lambda_4 > 0, \quad \lambda_3 < -\frac{\lambda_4}{2}, \quad -\lambda_1 < \lambda_2 < 0. \quad (19)$$

At this level, this situation is unrealistic since it has two massless neutral boson at the tree level, thus at this level it can be considered ruled out unless the coupling of these scalars and light fermions are suppressed. However, we must stress that in the model of [6] it is still possible to avoid massless scalars at the tree level since in the model there are trilinear interactions involving two A_4 triplets and one singlet of $SU(2)$ and A_4 . Denoting the second triplet of doublets by Φ with large VEVs and a scalar singlet ($Y = 0$), ζ , the A_4 symmetry may allow trilinear interactions like $f[h\Phi]_1\zeta$, f is an arbitrary energy scale of the order of the electroweak scale. In this case, the masses related to h are risen up and there is no light real neutral scalars. On the other hand, since there is no symmetry protecting these zero masses fields, small non-zero masses can be generated by radiative corrections. See below.

3. Conclusions

The third case with the solution ii) with just three doublets is not realistic at the tree level. However, as we said before, in the context of models with other doublets and singlets with large VEVs the light scalar becomes heavy by quantum corrections. It is well known that if there exist a symmetry protecting the mass of a particle, say m^2 , the corrections to m^2 are of the order of m^2 . However, if that symmetry does not exist, nothing protects m^2 from become as large as the energy scale where new physic may arise. In particular, scalar masses are difficult to keep smaller than the higher energy scale until which the model is renormalizable [18]. In this vain, we can make some general considerations. 1) For instance if the light scalar say H_l , with mass m_l , couples with a heavy field H_h of mass M_h , and with the coupling constant λ between them, say $\lambda H_l^2 H_h^2$, or if there exist a trilinear interaction involving a singlet ζ , say $f H_h H_l \zeta$, at the 1-loop level we can only say that, assuming that there is no unexpected cancellations the mass of the light scalar gets quantum corrections $m_l < M_h/4\pi$ or $m_l < f/4\pi$ [19]. Model like those in [6,7] have the hierarchy problem as any multi-Higgs extension of the SM. However, once this problem is solve by any mechanism, say extra dimensions, and the heaviest scalar is stable against quantum corrections, this mass becomes a cut off for the lighter scalars, i.e., it is an upper bound for the masses of these scalars. The mixing patterns also do not depend on fine-tuned and *ad-hoc* values of the parameters but are determined, like masses, by the vacuum structure. 2) The stability of

the vacuum sets a lower bound on the scalar mass but it is model dependent. In the context of the SM it is about 7 GeV [3]. This lower limit depend on the quark top mass and, with the known value of this mass, $m_t \sim 172$ GeV, it is not possible to have a light neutral scalar boson anymore. However, in multi-Higgs models with no scalar singlets this lower bound applies to the heaviest Higgs boson mass [20], thus in those models a very light scalar boson may be consistent with the present value of m_t . 3) In realistic models, as that of Ref. [6], beside the mechanism discussed in 1), it is possible that the Higgs scalars having small VEVs interact mainly with neutrinos and besides these interactions may be only through non-renormalizable effective operators that are suppressed by power of the cut off Λ which is of the order of TeV, as is the case of the model of [6]. 4) Last but not least, notice that in the second case the pseudoscalar physical states are heavy enough to not be produced together with its real partner in Z decay thus, there is no contribution to the invisible Z -decay width, even when all the VEVs are small.

In summary, we have shown that under some condition, in a model with three scalar doublets with an A_4 symmetry the scalar mass spectra may have no light scalar even if some of the VEVs are smaller than the electroweak scale. In cases were light scalars are unavoidable at the tree level, these fields may be safe from the phenomenological point of view since they couple mainly to neutrinos and/or they become heaviest due to quantum corrections. The large hierarchy among the fermion masses are now a hierarchy in the VEV structure i.e., of the vacuum whatever dynamics underlies this mechanism. This mechanisms have been implemented using only three $SU(2)$ scalar doublets, but in models involving more doublets, triplets and singlets it will work as well.

A similar model with 3 Higgs doublets and A_4 symmetry has been studied [21]. The difference between our work and theirs are the following: i) we have considered no CP violation, ii) we are in the context of the model of [6] in which all VEVs have to be different from zero and for these reason we have omitted the possibilities in which one or two VEVs are zero.

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