Available online at www.sciencedirect.com
SciVerse ScienceDirect
Procedia Engineering 15 (2011) 4456 – 4460
Procedia Engineering
www.elsevier.com/locate/procedia

Advanced in Control Engineeringand Information Science

Improved delay-dependent stability criteria for discrete-time stochastic neural networks with time-varying delays

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Abstract

This letter, investigates the problem of mean square exponential stability for a class of discrete-time stochastic neural network with time-varying delays. By constructing a appropriate Lyapunov-Krasovskii functional, combining the stochastic stability theory, and the convex theory method, a delay-dependent exponential stability criteria is obtained in term of LMIs. Finally, a numerical example is exploited to show the usefulness of the results derived.

Keywords: Delay-dependent stability; Neural network; Time-varying delay; Lyapunov-Krasovskii; Linear matrix inequalities.

1. Introduction

Recent years have witnessed a growing interest in investigating neural networks, this is mainly to the great potential applications in varies areas such as signal processing; pattern recoganization; static image processing; associative memory and combinatorial optimization [1]. As is known to all, dynamical behaviors of neural networks are key to the applications, and the achieved applications heavily depend on the dynamic behaviors of equilibrium point for neural network, therefore, stability is one of the most important issues related to such behavior.

It is worth pointing out that most neural networks are concerned with continuous-time cases. Since discrete-time neural networks play a more important role than their continuous-time counterparts in

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doi:10.1016/j.proeng.2011.08.837

today's digital life, moreover, in implementing and applications of neural networks, discrete-time neural networks also take a more crucial key than their continuous-time counterparts [2]. Therefore, both analysis and synthesis problem for discrete-time neural networks have been extensively studied and a great number of important results have been reported in the literature [3-4]. But it is our observation that there still exist room for further improvement by constructing rational Lyapunov functionals which motivates the present study..

2. Problem formulation an preliminaries

Consider the following discrete-time stochastic neural networks (DSNNs) with time-varying delays:

$$x(k+1)=Cx(k)+Af(x(k))+Bf(x(k-\tau(k)))+\delta(k,x(k),x(k-\tau(k)))\omega(k) \quad (2.1)$$

where $x(t)=\left[x_1(k),x_2(k),\cdots,x_n(k)\right]^T\in\Re^n$ is the neuron state vector.

$C=diag(c_1,c_2,\cdots,c_n)$ with $|c_i|<1$, described the rate with which the ith neuron will reset its state in isolation when disconnected from networks and external inputs. $\tau(k)$ is time-varying delay and satisfies $0<\tau_1\le\tau(k)\le\tau_2$.

Assumption 1. For any $x,y\in\Re,x\neq y$,

$$l_i^-\le\frac{f_i(x)-f_i(y)}{x-y}\le l_i^+ \quad (2.2)$$

Assumption 2. There exist a constant matrix $G\ge 0$, and is assumed to satisfy

$$\delta^T(k,x(k),x(k-\tau(k)))\delta(k,x(k),x(k-\tau(k)))\le\begin{bmatrix}x(k)\\x(k-\tau(k))\end{bmatrix}^T G\begin{bmatrix}x(k)\\x(k-\tau(k))\end{bmatrix}$$

where $G=\begin{bmatrix}G_1 & G_2\\ * & G_3\end{bmatrix}$..

3. Main result

Theorem 1. Suppose that Assumption (1-2) hold. Then the DSNNs (2.1) is exponential stable in the mean square if there exist positive definite matrices P,Q_1,E_1,E_2,Z, diagonal matrices $D_i>0,i=1,2$, $K>0$, $L>0$, and positive scalars $\varepsilon>0$ such that following LMIs hold:

$$P \le \rho I \qquad Z \le \rho_1 I \qquad Q = \begin{pmatrix} Q_{11} & Q_{12} \\ * & Q_{13} \end{pmatrix} > 0 \tag{3.4}$$

$$\begin{pmatrix} \Xi_{11} & \rho G_2 + (\tau_2-\tau_1)\rho_1 G_2 & 0 & 0 & \Xi_{15} & C^T PB + (\tau_2-\tau_1)(C-I)^T ZB \\ * & \Xi_{22} & 2\mu Z & \mu Z & 0 & \Xi_{26} \\ * & * & -E_1 - 2\mu Z & 0 & 0 & 0 \\ * & * & * & -E_2 - \mu Z & 0 & 0 \\ * & * & * & * & \Xi_{55} & A^T PB + (\tau_2-\tau_1) A^T ZB \\ * & * & * & * & * & \Xi_{66} \end{pmatrix} < 0 \tag{3.5}$$

$$\begin{pmatrix} \Xi_{11} & \rho G_2 + (\tau_2-\tau_1)\rho_1 G_2 & 0 & 0 & \Xi_{15} & C^T PB + (\tau_2-\tau_1)(C-I)^T ZB \\ * & \Xi_{22} & \mu Z & 2\mu Z & 0 & \Xi_{26} \\ * & * & -E_1 - \mu Z & 0 & 0 & 0 \\ * & * & * & -E_2 - 2\mu Z & 0 & 0 \\ * & * & * & * & \Xi_{55} & A^T PB + (\tau_2-\tau_1) A^T ZB \\ * & * & * & * & * & \Xi_{66} \end{pmatrix} < 0 \tag{3.6}$$

where

$$\Xi_{11} = C^T PC - P + \rho G_1 + \theta Q_{11} + E_1 + E_2 - 2\theta\Gamma_1 K + 2\theta\Gamma_2 L + (\tau_2-\tau_1)(C-I)^T Z(C-I) + (\tau_2-\tau_1)\rho_1 G_1 - 2\Gamma_1 D_1 \Gamma_2$$

$$\Xi_{15} = C^T PA + \theta Q_{12} + \theta K - \theta L + (\tau_2-\tau_1)(C-I)^T ZA + D_1(\Gamma_1+\Gamma_2)$$

$$\Xi_{22} = \rho G_3 + (\tau_2-\tau_1)\rho_1 G_3 - Q_{11} + 2\Gamma_1 K - 2\Gamma_2 L - 2\Gamma_1 D_2 \Gamma_2 - 3\mu Z$$

$$\Xi_{26} = -Q_{12} - K + L + D_1(\Gamma_1+\Gamma_2) \qquad \Xi_{55} = A^T PA + \theta Q_{13} + (\tau_2-\tau_1) A^T ZA - 2D_1$$

$$\Xi_{66} = B^T PB - Q_{13} + (\tau_2-\tau_1) B^T ZB - 2D_2 \quad \theta = \tau_2 - \tau_1 + 1 \quad \mu = \frac{1}{\tau_2-\tau_1}$$

$$\Pi_1 = [0 \quad I \quad 0 \quad -I \quad 0 \quad 0]^T \qquad \Pi_2 = [0 \quad -I \quad I \quad 0 \quad 0 \quad 0]^T$$

Proof of Theorem 1. Take the following L-K functional candidate as follows

$$v_1(x_k) = x^T(k)Px(k)$$

$$v_2(x_k) = \sum_{i=k-\tau(k)}^{k-1} \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix}^T Q_1 \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix} + \sum_{j=k+1-\tau_2}^{k-\tau_1} \sum_{i=j}^{k-1} \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix} Q_1 \begin{bmatrix} x(i) \\ f(x(i)) \end{bmatrix}^T$$

$$v_4(x_k) = 2\sum_{j=-\tau_2+1}^{-\tau_1+1} \sum_{i=k-1+j}^{k-1} \left\{ \left[f(x(i)) - \Gamma_1 x(i) \right]^T K + \left[\Gamma_2 x(i) - f(x(i)) \right]^T L \right\} x(i)$$

$$v_5(x_k) = \sum_{j=-\tau_2}^{-1-\tau_1} \sum_{i=k+j}^{k-1} \eta^T(i) Z \eta(i) \qquad \eta(i) = x(i+1) - x(i)$$

$$E(\Delta v_1(x_k)) = E\left(x^T(k+1)Px(k+1) - x^T(k)Px(k) \right)$$

$$E(\Delta v_2(x_k)) \le E\Big(x^T(k)\theta Q_{11} x(k) + 2x^T(k)\theta Q_{12} f(x(k)) + f^T(x(k))\theta Q_{13} f(x(k))$$

$$-x^T(k-\tau(k)) - f^T(x(k-\tau(k))) Q_{13} f(x(k-\tau(k))) \Big)$$

$$Q_{11} x(k-\tau(k)) - 2x^T(k-\tau(k)) Q_{12} f(x(k-\tau(k)))$$

$$E(\Delta v_3(x_k)) = E\left(x^T(k)(E_1+E_2)x(k) - x^T(k-\tau_1)E_1 x(k-\tau_1) - x^T(k-\tau_2)E_2 x(k-\tau_2) \right)$$

$$E(\Delta v_4(x_k)) \le E\Big\{ 2\theta\left[f(x(k)) - \Gamma_1 x(k) \right]^T Kx(k) - 2f^T(x(k-\tau(k)))Kx(k-\tau(k))$$

$$+2x^T(k-\tau(k))\Gamma_1 Kx(k-\tau(k)) + 2\theta\left[\Gamma_2 x(k) - f(x(k)) \right]^T Lx(k)$$

$$+2f^T(x(k-\tau(k)))Lx(k-\tau(k)) - 2x^T(k-\tau(k))\Gamma_2 Lx(k-\tau(k)) \Big\}$$

$$E(\Delta v_5(x_k)) = E\left((\tau_2-\tau_1)\eta^T(k)Z\eta(k) - \sum_{i=k-\tau_2}^{k-1-\tau_1} \eta^T(i)Z\eta(i) \right)$$

$$-\sum_{i=k-\tau_2}^{k-1-\tau_1} \eta^T(i)Z\eta(i) = -\sum_{i=k-\tau_2}^{k-1-\tau(k)} \eta^T(i)Z\eta(i) - \sum_{i=k-\tau(k)}^{k-1-\tau_1} \eta^T(i)Z\eta(i)$$

$$\le \xi^T(k)\Pi_1 I_1(\tau(k))\Pi_1^T \xi(k) + \xi^T(k)\Pi_2 I_2(\tau(k))\Pi_2^T \xi(k)$$

$$\xi^T(k) = \left[x^T(k), x^T(k-\tau(k)), x^T(k-\tau_1), x^T(k-\tau_2), f^T(x(k)), f^T(x(k-\tau(k))) \right]^T$$

$$\Pi_1(\tau(k)) = \left[-\frac{1}{\tau_2-\tau_1} - \frac{1}{(\tau_2-\tau_1)^2}(\tau(k)-\tau_1) \right] Z$$

$$\Pi_2\left(\tau(k)\right)=\left[-\frac{1}{\tau_2-\tau_1}-\frac{1}{\left(\tau_2-\tau_1\right)^2}\left(\tau_2-\tau(k)\right)\right]Z$$

Now combining above discussion, we have a upper bound as

$$E\left(\Delta v_k\right)\le E\left(\xi^T(k)\left(\Xi+\Pi_1 I_1\left(\tau(k)\right)\Pi_1^T+\Pi_2 I_2\left(\tau(k)\right)\Pi_2^T\right)\xi(k)\right)$$

Then if we want to have $\Xi+\Pi_1 I_1\left(\tau(k)\right)\Pi_1^T+\Pi_2 I_2\left(\tau(k)\right)\Pi_2^T<0$ for $\tau_1\le\tau(k)\le\tau_2$, which are equivalent to handle following two LMIs by the convex combination theory:

$$\Xi+\Pi_1 I_1\left(\tau_1\right)\Pi_1^T+\Pi_2 I_2\left(\tau_1\right)\Pi_2^T<0 \text{ and } \Xi+\Pi_1 I_1\left(\tau_2\right)\Pi_1^T+\Pi_2 I_2\left(\tau_2\right)\Pi_2^T<0$$

that are equivalent to (3.5) and (3.6) hold. Therefore, if the LMIs (3.4-3.6) hold, we utilize the similar method proposed in the [3], we can know the system (2.1) is mean square exponential stability.

4. Example

Consider the discrete-time stochastic neural network (2.1) with:

$$C=\begin{bmatrix}0.8 & 0\\ 0 & 0.9\end{bmatrix}\quad A=\begin{bmatrix}0.001 & 0\\ 0 & 0.005\end{bmatrix}\quad B=\begin{bmatrix}-0.1 & 0.01\\ -0.2 & -0.1\end{bmatrix}$$

The activation function satisfy Assumption 1 with $\Gamma_1=diag\begin{pmatrix}0 & 0\end{pmatrix}$, $\Gamma_2=diag\begin{pmatrix}0.5 & 0.5\end{pmatrix}$. By the Matlab LMI Control Toolbox, we find a solution to the LMIs (3.4-3.6)

$$P=\begin{pmatrix}22.0242 & -1.4285\\ -1.4285 & 4.1354\end{pmatrix}\quad D_1=\begin{pmatrix}1.2536 & 0\\ 0 & 0.4064\end{pmatrix}\quad D_2=\begin{pmatrix}3.0549 & 0\\ 0 & 1.1534\end{pmatrix}$$

$$K=\begin{pmatrix}0.0094 & 0\\ 0 & 0.0003\end{pmatrix}\quad Z=\begin{pmatrix}2.4709 & -0.2029\\ -0.2029 & 0.8985\end{pmatrix}$$

5. References

[1] A. Cichoki, R. Unbehauen, Neural Networks for Optimization and Signal Processing, Wiley, Chichester, 1993

[2] A. Stuart, A. Humphries, Dynamical Systems and Numerical Analysis, Cambridge University, 1998.

[3] M. Luo, S. Zhong, R. Wang, W. Kang, Robust stability analysis of discrete-tme stochastic neural networks with time-varying delays, Appl. Math. Comput 2009; 209:.305-313.

[4] Y. Tang, J. Fang, M. Xia, D. Yu, Delay-distribution-dependent stability of stochastic discrete-time neural networks with randomly mixed time-varying delays, Neurocomputing 2009; 72:.3830-3838.