Viscoelastic analysis of interaction between freezing wall and outer shaft wall in freeze sinking

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Abstract

In order to reasonably determine the external load of outer freezing shaft wall, it is necessary to conduct interaction study between frozen soil wall and outer shaft wall. First, based on the characteristic of three-stage compressive deformation of foam board, a relational expression of such radial deformations as frozen soil wall, outer shaft wall, and foam board is established; then based on the homogeneous viscoelastic model of frozen soil wall, with the rigidity in different compression stages of foam board being regarded as a constant, an expression of deformation pressure is derived. According to the research conducted in combination with project examples, the foam board remarkably slowed down the increase in deformation pressure; for steady creep frozen soil wall, the final value of deformation pressure should be effectively reduced; for unsteady creep frozen soil wall, the final value of deformation pressure has nothing to do with the effect of foam board, and tends to be equal to the level external load of frozen soil wall. The research shows that as the depth of alluvium rises, the frozen soil wall under high pressure is to gradually present a unsteady creep characteristic and that the value of level external load of outer shaft wall in extra-thick alluvium can be regarded as equal to permanent level ground pressure.

Keywords: freeze sinking; frozen soil wall; outer shaft wall; interaction; viscoelastic model

1. Introduction

The deformation pressure develops from the interaction between frozen soil wall and outer shaft wall. Therefore, the study of this interaction is very important for the design and safety analysis of outer shaft wall to obtain the growth law and calculation means of deformation pressure.

Chinese scholars have carried out researches on the deformation law and effect factor on frozen soil wall with simulation test and numerical analysis, providing basis for the safety analysis of frozen soil wall and freezing pipe\textsuperscript{1-5}. Based on elastoplastic model and on elastic viscoplastic model, Hu X D, Rong C X, and other scholars studied the external load of frozen soil wall under excavation and unloading condition\textsuperscript{6-7}. Based on elastoplastic model, Sun W R, Song L and other scholars studied the design method of frozen soil wall and shaft wall in common loading theory\textsuperscript{8-9}. However, for the external load of outer freezing shaft wall, i.e. deformation pressure, it is very rare to see analytic theory research, but most of researches are mainly about engineering field measurement.
Through a large number of engineering field measurements of deformation pressure in freeze sinking project, in an amount of freezing shaft, the deformation pressure (also named as freezing pressure) is nearly the same as or even greater than permanent level ground pressure\[^{10-12}\], and that a stern challenge has been raised against the common loading theory of frozen soil wall.

Therefore, based on viscoelastic frozen soil wall model, in view of compression characteristic of foam board between frozen soil wall and outer shaft wall, we intend to deduce a computing formula for deformation pressure and perform theoretical research on the growth law and calculation means of deformation pressure so as to further understand the effect of frozen soil wall in freeze sinking project.

2. Interaction process between frozen soil wall and outer shaft wall

2.1. Compressive deformation characteristic of polystyrene foam board

First, the compression curve of polystyrene foam board commonly used in freeze sinking project is obtained through test (Fig.1).

![Compression curve of polystyrene foam board](image)

The compression ratio in Fig.1 is the ratio of compression deformation to initial thickness\[^{13}\]. From the curve, the compression process can be roughly divided into three phases:

Phase 1: Approximately linear compression phase, with a compression ratio of 0-75%.

Phase 2: Nonlinear compression phase, with a compression ratio approximately from 75% to 95%. The rigidity of foam board dramatically increases with the increase of compression ratio.

Phase 3: Compaction phase. After the compression ratio gets over 95%, the foam board is practically compacted and can be approximately deemed as rigidity.

2.2. Relation of radial deformations such as frozen soil wall, outer shaft wall and foam board

Fig.2 is the schematic diagram of interaction between frozen soil wall and outer shaft wall (take a given radian in cross section). In the Figure, \(a\) is the outer radius of outer shaft wall and can be approximately considered equal to the radius of inner face of frozen soil wall; \(P_0\) the level external load of frozen soil wall; and \(P\) the supporting pressure for frozen soil wall from outer shaft wall and is the counterforce of deformation pressure.

![Schematic diagram of interaction between frozen soil wall and outer shaft wall](image)
Due to existence of foam board, the radial displacement of inner face of frozen soil wall should be calculated according to the following formula:

\[ u = \Delta \delta + u_{sw} \]  

(1)

where \( u \), \( u_{sw} \) and \( \Delta \delta \) are radial displacement of inner face of frozen soil wall, radial displacement of outside face of outer shaft wall, and compression deformation of foam board, respectively. In phase 1, the deformation pressure is usually less than 0.5MPa and it is approximately considered that \( u_{sw} = 0 \), thus \( u = \Delta \delta \); in phase 3, the foam board is practically compacted, \( d(\Delta \delta)/dt = 0 \), thus \( d(u)/dt = d(u_{sw})/dt \).

3. Viscoelastic rheological analysis of deformation pressure

3.1. Viscoelastic rheological model of frozen soil wall

The interaction between frozen soil wall and outer shaft wall can be simplified to be axisymmetric problem. Hypothesis:

1) According to the average temperature, the frozen soil wall is regarded as homogeneous and viscoelastic thick cylinder, and the dilatancy is neglected.

2) The ratio of outside diameter to inside diameter of frozen soil wall is no less than 3, and the effect on the external load of frozen soil wall caused by shaft excavation is neglected.

3) The shaft wall is considered to be linear elastic body, and the rheology is out of consideration.

Generalized Kelvin model (Fig.3) is adopted to describe the rheological characteristic of frozen soil. This model can simulate instantaneous elastic strain and steady creep, and its rheological constitutive equation under three dimensional stress patterns are as follow:

\[
\frac{H}{G_1 + G_2} \hat{s}_{ij} + s_{ij} = \frac{2G_1H}{G_1 + G_2} \hat{e}_{ij} + \frac{2G_1G_2}{G_1 + G_2} e_{ij} \quad (i, j = r, \theta, z)
\]

(2)

where, \( s_{ij} \) and \( e_{ij} \) are the deviatoric tensor of stress and deviatoric tensor of strain, respectively; \( G_1 = \frac{E_1}{2(1 + \nu_1)} \), \( G_2 = \frac{E_2}{2(1 + \nu_2)} \), and \( H = \eta \); \( E_1, E_2, \eta \) the elastic module of Hooke body, the elastic module of Kelvin body, and the viscosity coefficient of Kelvin body, respectively; \( \nu_1 \) and \( \nu_2 \) are Poisson’s ratio of Hooke body and Kelvin body, respectively.

3.2. Viscoelastic rheological equation of frozen soil wall
According to document [14], under the ground pressure $P_0$ evenly distributed in distant field and supporting pressure $P$, based on Formula (2), the rheological equation of inner face of viscoelastic surrounding rock mass in circular cross-section tunnel can be derived as following:

$$-T_{rel} \ddot{P} + P_0 - P = 2G_0 T_{rel} \frac{\dot{u}}{a} + 2G_\infty \frac{u}{a} \tag{3}$$

where, $u$ is the radial displacement of inner face of surrounding rock mass; $a$ the radius of the excavated section of passageway; $G_0$ and $G_\infty$ the instantaneous shear modulus and long-term shear modulus of surrounding rock mass, respectively; $G_0 = G_1$, and $G_\infty = \frac{G_1 G_2}{G_1 + G_2}$; $T_{rel}$ the slack time, $T_{rel} = \frac{H}{G_1 + G_2}$.

Based on foresaid hypothesis, on the condition that the external load of frozen soil wall is equivalent to the ground pressure evenly distributed in distant field, it is obvious that the relation between the displacement and supporting pressure of inner face of frozen soil wall can be expressed in Formula (3). Where, both $P$ and $u$ are unknown quantities relating to time, complementary equations are needed to solve.

3.3. Complementary equation

The existing foam board is to affect rigidity of supporting system, thus affecting deformation of frozen soil wall and the increase in deformation pressure. Based on Formula (1), the following part discusses the relation in different phases between displacement of inner face of frozen soil wall and deformation pressure, and establishes a complementary equation.

**Phase 1 of foam board compression:**

If the compressive rigidity of foam board is $K_1$, then the relation between $P$ and $u$ should be as following:

$$u = \frac{P}{K_1} + u_0 \tag{4}$$

where, $P$ is the deformation pressure (i.e. supporting pressure); $u_0$ the instantaneous displacement of inner face of frozen soil wall, $u_0 = aP_0/(2G_0)$.

**Phase 2 of foam board compression:**

If the equivalent rigidity of outer shaft wall and foam board is $K_2$, the complementary equation is:

$$u = \frac{P - P_1}{K_2} + \frac{P_1}{K_1} + u_0 = \frac{P}{K_2} + \left( \frac{1}{K_1} - \frac{1}{K_2} \right)P_1 + u_0 \tag{5}$$

where $K_2 = \frac{K_s K_c}{K_s + K_c}$, $K_s$ is the average compressive rigidity of foam board in this phase; $K_c$ the radial compressive rigidity of outer shaft wall (pressure necessary to produce unit displacement of outside face of outer shaft wall); and $P_1$ the deformation pressure at the end of phase 1 of foam board compression.

**Phase 3 of foam board compression:**

In this phase, considering that the foam board is practically compacted, therefore, supporting rigidity can be considered to be equal to the radial compressive rigidity of shaft wall, namely $K_3 = K_c$, then the complementary equation is:
\[ u_a = \frac{P - P_1}{K_3} + \frac{P_2 - P}{K_2} + \frac{P}{K_1} + u_0 = \frac{P}{K_3} + \left( \frac{1}{K_2} - \frac{1}{K_3} \right) P_2 + \left( \frac{1}{K_1} - \frac{1}{K_2} \right) P_1 + u_0 \]  \hspace{1cm} (6)

Where, \( P_2 \) is the deformation pressure at the end of phase 2.

From formula (4) to (6), it can be seen that the total radial displacement of frozen soil wall can be expressed uniformly in following Formula:

\[ u = \frac{P}{\lambda} + \theta \]  \hspace{1cm} (7)

where, \( \lambda \) is the compressive rigidity of supporting system in different phases; \( \theta \) the constant related to supporting rigidity, supporting pressure and the instantaneous displacement of frozen soil wall in the previous phase. Formula (7) is the complementary formula with united form.

### 3.4 Formula for viscoelastic deformation pressure

Substitute complementary Formula (7) to Formula (3), solve the compression deformation of foam board in different phases, we have

\[-T_{rel} \dot{P} + P_0 - P = \frac{2G_0 T_{rel}}{a} \left( \frac{\dot{P}}{\lambda} \right) + \frac{2G_\infty}{a} \left( \frac{P}{\lambda} + \theta \right) \]  \hspace{1cm} (8)

After processing, then

\[ \frac{(2G_0 + a\lambda) dP}{(2G_\infty + a\lambda)P - K_1 (aP_0 - 2G_\infty \theta)} = - \frac{dT}{T_{rel}} \]  \hspace{1cm} (9)

Multiplying \( \frac{2G_\infty + a\lambda}{2G_0 + a\lambda} \) on either side of above equation, then

\[ \frac{(2G_\infty + a\lambda) dP}{(2G_\infty + a\lambda)P - K_1 (aP_0 - 2G_\infty \theta)} = - \frac{2G_\infty + a\lambda}{2G_0 + a\lambda} \frac{dt}{T_{rel}} \]  \hspace{1cm} (10)

Introduce parameter \( \beta = \frac{2G_\infty + a\lambda}{2G_0 + a\lambda} \), the equation is solved as below

\[ (2G_\infty + a\lambda)P - (aP_0 - 2G_\infty \theta)\lambda = Ae^{\frac{\beta}{T_{rel}}} \]  \hspace{1cm} (11)

Where, \( A \) is an undetermined integral constant. According to initial condition that \( P = 0 \) when \( t = 0 \), we have \( A = -(aP_0 - 2G_\infty \theta)\lambda \).

Thus, the uniform expression of deformation pressure \( P \) can be gained as

\[ P = \frac{(aP_0 - 2G_\infty \theta)\lambda}{2G_\infty + a\lambda} \left( 1 - e^{\frac{\beta}{T_{rel}}} \right) \]  \hspace{1cm} (12)

The deformation pressure in different compression phases of foam board can be calculated according to Formula (12), but values of \( \lambda \) and \( \theta \) in the equation are different as follows:

**Phase 1:** \( \lambda = K_1 \), \( \theta = u_0 \).
Phase 2: \( \lambda = K_2, \theta = \left( \frac{1}{K_1} - \frac{1}{K_2} \right) P_1 + u_0. \)

Phase 3: \( \lambda = K_3, \theta = \left( \frac{1}{K_2} - \frac{1}{K_3} \right) P_2 + \left( \frac{1}{K_1} - \frac{1}{K_2} \right) P_1 + u_0. \)

Obviously, in condition that permanent supporting rigidity is as \( K_1 = K_2 = K_3 = K, \) Forms of both phase 2 and phase 3 degenerate into the form of phase 1.

3.5. Discussion on deformation pressure

Based on Formula (12), through analysis it can be found that the increase in deformation pressure has following law:

1. Deformation pressure \( P \) changes in exponential order as the time lasts.

2. For steady creep frozen soil wall \( (G_\infty \neq 0) \), when \( t \to \infty \), it is not difficult to prove

\[
P = \frac{aP_0 - 2G_\infty \theta}{2G_\infty + a\lambda} < P_0.
\]

The existing parameter \( \theta \) makes deformation pressure \( P \) depend not only on the mechanical property and external load of frozen soil wall, but also on the history of supporting rigidity and supporting pressure.

3. For unsteady creep frozen soil wall \( (G_\infty = 0) \), \( P \to P_0 \) when \( t \to \infty \), that is, deformation pressure \( P \) will finally tend to be equal to the external load \( P_0 \) of frozen soil wall, and has nothing to do with the history of rigidity change of supporting structure. In other words, the existing foam board only affects the rise rate of the deformation pressure but not its final value.

4. The viscosity coefficient \( H \) of frozen soil only exist in slack time parameter \( T_{rel} \), only affect the rise rate of deformation pressure, and not affect its final value (limit value).

3.6. Calculation approach of deformation pressure

When Formula (12) is used to calculate deformation pressure, the basic approaches are as follows:

1. Determine rheological parameters of frozen soil wall, such as \( G_0, G_\infty, H, \) and \( T_{rel} \);

2. Determine the value of \( \lambda \), that is, the supporting rigidity in different phases: \( K_1, K_2, K_3. \)

3. Determine external load \( P_0 \) of frozen soil wall, which can be calculated according to \( P_0 = (0.012-0.013)D \), expressed in MPa; where, \( D \) is stratum depth, expressed in m.

4. Compute initial displacement \( u_0 \) of inner face of frozen soil wall.

5. Based on the compression ratio cut-off point defined in compression deformation phases of foam board (such as 75% and 95%, see Fig.1), respectively determine the deformation pressure \( P_1 \) and \( P_2 \), and the time \( t_1 \) and \( t_2 \) at the end of phases 1 & 2.

6. Calculate deformation pressures one by one in three different phases.

4. Case study

4.1. Calculating Parameter

In deep part of auxiliary shaft of Longgu Coal Mine in Juye Coalfield, the frozen soil is at different stresses and temperatures, which respectively present steady and unsteady creep features. Mechanics parameter (see Table 1) should be defined based on testing data of steady creep frozen soil at a depth of 500m.
Table 1. Mechanics parameter of frozen soil

<table>
<thead>
<tr>
<th>$E_1$/MPa</th>
<th>$v_1$</th>
<th>$E_2$/MPa</th>
<th>$v_2$</th>
<th>$\eta$/MPa·h</th>
</tr>
</thead>
<tbody>
<tr>
<td>370</td>
<td>0.28</td>
<td>92.5</td>
<td>0.28</td>
<td>927</td>
</tr>
<tr>
<td>$G_1$/MPa</td>
<td>$G_2$/MPa</td>
<td>$G_0$/MPa</td>
<td>$G_\infty$/MPa</td>
<td>$T_{rel}$/h</td>
</tr>
<tr>
<td>144.53</td>
<td>36.13</td>
<td>144.53</td>
<td>28.91</td>
<td>5.131</td>
</tr>
</tbody>
</table>

Calculated according to quasi-heavy liquid formula $P_0 = 0.013D$, the external load (i.e. permanent level ground pressure) of frozen soil wall at a depth of 500m should be 6.5MPa. In view of the above, the instantaneous displacement of inner face of frozen soil wall can be calculated to be $u_0 = 128.17\,\text{mm}$. Based on design parameters of outer shaft wall (inside radius: 4,600mm, outside radius 5,700mm), in light of compression curve (Fig.1) of foam board, compression ratios 73.7% and 94.7% are respectively taken as cut-off points, supporting rigidities ($K_1$, $K_2$, $K_3$) from Phase 1 to Phase 3 can be calculated accordingly, and $P_1$ and $P_2$ can be defined. Substitute above parameters to Formula (12), then computing formulas in three phases of deformation pressure can be derived, and thus the time cut-off points ($t_1$, $t_2$, see Table 2) divided in compression phase of foam board can be determined.

Substitute foresaid parameters to Formula (12), then computing formulas in three phases of deformation pressure can be derived, and can be used for calculation.

Table 2. Phase dividing and supporting rigidity parameter

<table>
<thead>
<tr>
<th>$K_1$ (MPa·mm$^{-1}$)</th>
<th>$K_2$ (MPa·mm$^{-1}$)</th>
<th>$K_3$ (MPa·mm$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0056</td>
<td>0.1138</td>
<td>1.3539</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$P_1$ (MPa)</th>
<th>$P_2$ (MPa)</th>
<th>$t_1$ (h)</th>
<th>$t_2$ (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.310</td>
<td>2.273</td>
<td>144.53</td>
<td>36.13</td>
</tr>
</tbody>
</table>

4.2. Analysis of deformation pressure of Longgu coal mine auxiliary shaft

As for such different working conditions as with foam board (rigidity experiences three phases of change) and without foam board (rigidity takes radial compressive rigidity of external shaft wall), the limit value of deformation pressure and the time used to reach 95% of this value (see Table 3) are separately calculated under condition of $G_\infty = 28.91\,\text{MPa}$ and 0MPa. Meanwhile, through multiplying formula $H=927\,\text{MPa/h}$ by different factors, gain calculated results of different viscosity coefficients.

When $G_\infty = 28.91\,\text{MPa}$, $G_\infty = 0\,\text{MPa}$, curves of deformation pressure are as shown in Fig.4 and Fig.5, respectively. The abrupt change of deformation pressure in Fig.4 (a) and Fig.5 (a) is induced by abrupt change of support rigidity. If the compression process of foam board is divided into more phases, so as to simulate the change of support rigidity more accurately, the abrupt change of deformation pressure will be lessened and the analysis precision will be improved.

Table 3. Calculated result of deformation pressure

<table>
<thead>
<tr>
<th>Viscosity coefficient</th>
<th>With foam board</th>
<th>Without foam board</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$G_\infty = 28.91,\text{MPa}$</td>
<td>$G_\infty = 0,\text{MPa}$</td>
</tr>
<tr>
<td></td>
<td>$P$ (MPa)</td>
<td>$t$ (h)</td>
</tr>
<tr>
<td>$1H$</td>
<td>4.45</td>
<td>15.8</td>
</tr>
<tr>
<td>$2H$</td>
<td>4.45</td>
<td>31.6</td>
</tr>
<tr>
<td>$5H$</td>
<td>4.45</td>
<td>79.0</td>
</tr>
<tr>
<td>$10H$</td>
<td>4.45</td>
<td>158.0</td>
</tr>
</tbody>
</table>
Based on calculated results in Table 3 and growth curves of deformation pressure in Fig. 4 and Fig. 5, analysis shows that:

1. For $G_c = 28.91 \text{MPa}$, that is, for the steady creep frozen soil wall with or without foam board, the limit values of deformation pressure respectively are 4.45MPa and 5.16MPa. It can clearly be seen that existing foam board not only reduce the increasing speed of supporting pressure, but also reduce the final value of deformation pressure by 13.8%, so that the load bearing condition of outer shaft wall is effectively improved.

2. For $G_c = 0$, that is, for unsteady creep frozen soil wall, no matter whether there is foam board, all limit values of deformation pressure tend to be equal to the external load of frozen soil wall. It can clearly be seen that existing foam board or, in other words, the history change of supporting rigidity can only affect the increasing speed of deformation pressure but not the limit value.

3. Viscosity coefficient only can affect the increasing speed of deformation pressure but not the final value. As the viscosity coefficient grows, the increasing speed of deformation pressure slows down.

The stabilization of frozen soil wall depends on its bearing capacity and external load, and the thickness and average temperature are key factors to affect the bearing capacity of frozen soil wall.

The ground pressure in shallow topsoil (like topsoil within 200m) is relatively small. Usually, higher long-term strength of frozen soil can be obtained only if frozen soil wall has lower average temperature; therefore, frozen soil wall can be in steady creep state, and along with the effect of foam board, the deformation pressure can be effectively reduced.

As thickness of topsoil increases (such as exceeding 400m), the moisture content of deep cohesive soil gradually decreases, making most water in cohesive soil become the hydration water, leading to a very small increase in strength of frozen soil strength with the decrease of temperature. Under high pressure, the frozen soil wall is easy to present more and more obvious unsteady creep characteristic. Under this condition, the function of frozen soil wall is primarily reflected in delaying increment in freezing pressure, saving time for increment in initial bearing capacity of outer shaft wall, the frozen soil wall has no long-term bearing function, and the external load of outer shaft wall will tend to be equal to level external load of frozen soil wall. Therefore, without considering frost heave, the external load of outer freezing shaft wall in extra-thick alluvium can take the external load of frozen soil wall, that is, value of permanent level ground pressure.
5. Conclusion

Based on viscoelastic rheological model, the formula of deformation pressure is derived, and the following conclusions are drawn:

(1) The deformation pressure of steady creep frozen soil wall is finally to be smaller than the level external load of frozen soil wall. The existing foam board can effectively decrease the increasing speed in deformation pressure, decrease the final value, and improve the initial load bearing condition of shaft wall.

(2) The deformation pressure of unsteady creep frozen soil wall finally tends to be equal to the level external load. It can clearly be seen that existing foam board or, in other words, the history change of supporting rigidity can only affect the increasing speed of frozen soil wall deformation pressure but not the final value.

(3) The deep frozen soil wall of freezing shaft in extra-thick topsoil will present more and more obvious unsteady creep characteristic, and the external load of outer shaft wall can take the external load of frozen soil wall, that is, value of permanent level ground pressure.

Acknowledgements

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References