Reconstruction of impact force on curved panel using piezoelectric sensors

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Abstract

This paper is focused on reconstruction of impact force in an unknown location on a general thin-walled curved structure with a hole. Ten irregularly distributed piezoelectric sensors are attached to the panel for the measurement of response signals. A corresponding finite element model is created in MSC.Marc. The methodology for reconstruction is based on the transfer function approach. The time dependence of impact force is reconstructed and the location of impact is identified on a different coarse mesh and it is compared to the solution on the original mesh for all possible impact locations. Moreover, the influence of interpolation of the transfer functions within elements (or sectors) of the coarse mesh on increase of the accuracy of impact location identification is investigated.

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Keywords: finite element method; deconvolution; transfer function; inverse problem; regularization

Nomenclature

\[d \quad \text{distance between real and reconstructed location}\
\[e \quad \text{piezoelectric constant}\
\[f \quad \text{loading signal (input)}\
\[g \quad \text{transfer function}\
\[h \quad \text{response signal (output)}\
\[t \quad \text{time}\
\[E \quad \text{Young's modulus}\
\[F \quad \text{matrix of loading signal}\
\[G \quad \text{matrix of transfer functions}\
\[H \quad \text{matrix of response signals}\
\[K \quad \text{number of impact locations}\
\[L \quad \text{number of sensors}\
\[M \quad \text{number of impacts at one location}\
\[N \quad \text{number of signal samples}\
\[\delta \quad \text{response reconstruction error}\
\[\varepsilon \quad \text{dielectric permittivity constant}\

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1. Introduction

The safety or functionality of every structure can be significantly affected by defects. These defects can be invisible to surface inspection. Therefore, they must be found prior to any catastrophic scenario. Currently, they are detected by non-destructive techniques like ultrasonic, X-ray, coin tapping or other methods [19], which are time and cost consuming and require the construction to be taken out of service. In contrary, the condition of construction can be evaluated during operation from measurements of sensors placed over the structure. This principle is so-called structural health monitoring.

The identification of impact force and impact location is an important task of such systems and the ideal identification method should identify the impact force, or even the combination of impact forces, on complex structures in real time with low dependence of operating noise.

The hidden defects are very common especially in modern composite materials, such as carbon fiber reinforced epoxy laminates, that are widely used thanks to their high strength and stiffness to weight ratios. Not only is the design process of structures which contain parts from composite materials complicated due to effects such as non-linear behavior [3], specific damage behavior [9], and directional dependence of velocity of propagation of stress waves [15], but, furthermore, composites are highly susceptible to transverse loading, which can cause delamination and cracks in matrix and thus significantly reduce the stiffness or strength of the construction.

The impact identification problems have been studied by many researchers in recent years and several methods were proposed. The often used one is the inversion of forward problem, which can be performed in time, frequency or spectral domain. Direct deconvolution is a well-known ill-conditioned problem and its results are strongly influenced by quality of experimental data, appropriateness of the mechanical model and robustness of employed algorithm. Many researches define the problem rather as a minimization of the difference between measured and modeled responses of the impacted structure. Additional terms and constraints are added to minimization to regulate oscillations in results.

Jacquelin et al. [6] analyzed the deconvolution in time domain. The influence of sensors location and different regularization methods were investigated. Similarly, Gunawan et al. [8] used the time domain. The impact force was approximated by cubic spline and the two-step B-spline regularization method was developed. On the other hand Yan and Zhou [13] used Chebyshev polynomials to represent the impact force and the modified genetic algorithm to solve the minimization problem. Park and Chang [7] determined the system experimentally and investigated several types of impacts. Martin and Doyle [1] used the Fast Fourier Transform to switch into frequency domain and solved the deconvolution directly. Furthermore, Doyle [2] employed the wavelet deconvolution and modeling with FEA. Other researches preferred to work in spectral domain. Hu et al. [12] formulated the minimization with regularization parameter and constraint, which was solved by quadratic programming method. Moreover, different types of sensors were compared and Chebyshev polynomials were employed to reduce the number of unknowns. Atobe et al. [14] used the gradient projection method to solve the minimization problem and compared the determination of the system by experiment or by FEA. Finally Sekine and Atobe [16] formulated the minimization where multiple impacts can be identified. Another possibility is to define the minimization in recursive form in time domain and to use filtering method to solve the investigated problem. Seydel and Chang [4, 5] used smoothing-filter method and investigated the influence of sensor locations and boundary conditions. Similarly, Zhang et al. [11] implemented smoothing-filter algorithm with the possibility of real-time computations.

The location of impact within these methods is often estimated from the minimization of the error between measured and modeled responses along the structure. This can be done by direct search methods [12] or by some other optimization techniques [14]. Another possibility is to use the techniques derived from methods used in acoustic emission (AE) [10, 4], where the difference in arrival time of signal is determined and the location of impact is estimated from velocity of waves. Unfortunately, the determination of exact time of arrival in composite material or complex structures is limited because of the dispersion and reflection of waves on boundaries. The alternative is calculation of distribution of energy in defined time step and the determination of its maximum [7].

Totally different approach is the determination of impact force and force location from models based on neural networks [17]. The model is composed of parallel elements connected by defined relations and trained by preliminary tests.
The output of the model is then set by learned behavior. The weakness of such approach is the necessity of learning period and uncertain reaction of model to not learned impacts.

The above cited papers differ in several features like complexity of geometry, determination of the system model or the type and number of sensors. The impact force was investigated on metal beam [1], metal plate [2, 6, 8], composite plate [5, 7, 11, 12, 13, 14, 16]. The model of the system is defined analytically [4, 11, 13] or determined by FEA [8, 12, 14, 16] or by experiment [7, 14]. Signal is mostly obtained from strain gauges [14, 16], accelerometers [1, 2, 12], simple piezoelectric sensors [12] or from sensor network [7, 13, 20].

2. Discrete convolution and inverse problem

The methodology used in this work is based on the transfer function approach. For a linear system, its response \( h \) to an input \( f \) can be expressed by convolution

\[
f \ast g = h
\]

where \( g \) is so-called transfer function and it represents the characteristics of the system. In order to find the location of impact and to reconstruct the time dependence of the impact force, it is necessary to perform to consecutive steps; a) a calibration procedure, i.e., to perform experimental measurements while recording the corresponding input and response, and to calculate the transfer functions for all combinations of impact locations and sensors, and b) a reconstruction procedure, i.e. to reconstruct the force in each possible location for experimentally measured response for unknown impact and to seek the impact location by minimizing the error of response reconstruction. This approach can be carried out either experimentally or numerically. In this work, a virtual experiment is performed using finite element analysis (FEA). Hence, a discrete deconvolution method is used. All pre- and post-processing algorithms are performed using Matlab scripts.

2.1. Transfer function calculation

Let us consider a system (a structure) with \( K \) impact locations and \( L \) sensors. First, we measure the force in location \( i \) and the corresponding response in sensor \( j \). For discrete system the input and response signals, each consisting of \( N \) samples (assuming constant time increment \( \Delta t \)), can be written as

\[
\mathbf{f}_i = [f_i, f_2, \ldots, f_N]^T
\]

and

\[
\mathbf{h}_j = [h_i, h_2, \ldots, h_N]^T,
\]

respectively. The force vector can be rearranged to matrix \([N \times N]\) for each performed impact (or experiment) \( m = 1 \ldots M \) as

\[
\begin{bmatrix}
0 & \cdots & 0 & f_1 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & f_1 & f_2 \\
\cdots & \cdots & \cdots & \cdots \\
0 & \cdots & f_1 & f_2 & \cdots & f_N \\
0 & \cdots & f_1 & f_2 & \cdots & f_N
\end{bmatrix}
\begin{bmatrix}
g_1 \\
g_2 \\
\vdots \\
g_N
\end{bmatrix} =
\begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_N
\end{bmatrix},
\]

and then a global matrix system can be assembled for up to \( M \) subsequent impacts as

\[
\begin{bmatrix}
\mathbf{F}_1 \\
\vdots \\
\mathbf{F}_M
\end{bmatrix}
\begin{bmatrix}
g_1 \\
\vdots \\
\mathbf{g}_M
\end{bmatrix} =
\begin{bmatrix}
\mathbf{H}_1 \\
\vdots \\
\mathbf{H}_M
\end{bmatrix}.
\]

The system is then represented by all solutions for each combination of \( i \) and \( j \). However, each system of algebraic equations in (5) is overdetermined and ill-posed. If we rewrite (5) concisely as

\[
\mathbf{F}_i \mathbf{g} = \mathbf{H}_j,
\]

the solution can be obtained by various methods, for example by simple pseudoinversion

\[
\mathbf{g} = (\mathbf{F}_i^T \mathbf{F}_i)^{-1} \mathbf{F}_i^T \mathbf{g},
\]

by minimizing the residuum using least squares method

\[
\min \left\{ \| \mathbf{F}_i \mathbf{g} - \mathbf{H}_j \| \right\},
\]

by quadratic programming techniques

\[
\min \left\{ \frac{1}{2} \mathbf{g}^T \mathbf{P} \mathbf{g} + \mathbf{Q}^T \mathbf{g} \right\} \quad \mathbf{P} = \mathbf{F}_i^T \mathbf{F}_i, \quad \mathbf{Q} = -\frac{1}{2} \mathbf{H}_j \mathbf{F}_i
\]

or others. Nonetheless, to avoid unrealistic oscillations of the solution, it is advisable to use adequate regularization technique that imposes additional condition on the solution. In this work, the Tikhonov regularization [18].
\[
\min\left\{ \|Fg - H\|^2 + \lambda^2\|g\|^2 \right\} \tag{10}
\]
is used, where the additional term, compared to least square method, means that the norm of the solution will be minimized too. A proper choice of the parameter \( \lambda \) is needed to balance the ratio between the standard residuum and the oscillations.

### 2.2. Impact force reconstruction

When all transfer functions \( g_{ij} \) are known, we can attempt to reconstruct the unknown input signal from measured responses only. The response again with \( N \) samples as in (3) is obtained in all \( L \) sensors or only in a subset of sensors. Now, the transfer function for each combination of \( i \) and \( j \) must be rearranged to matrix \([N \times N]\) so that

\[
\begin{bmatrix}
0 & \cdots & 0 & g_1 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & g_{N-1} & g_N
\end{bmatrix}
\begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_N
\end{bmatrix}
= \begin{bmatrix}
h_1 \\
h_2 \\
\vdots \\
h_L
\end{bmatrix}
\tag{11}
\]

and then a global system for all selected sensors can be assembled as

\[
Gf = H. \tag{12}
\]
The solution can be performed for each possible (suspected) impact location \( i \). Again, the problem (12), written concisely as

\[
Gf \approx H, \tag{13}
\]
is overdetermined and ill-posed. Moreover, as the impact force is always non-negative (assuming non-sticking impact), additional inequality constraint

\[
f_i \geq 0 \tag{14}
\]
might be advantegous [20]. In this work, the Tikhonov regularization is used again without the inequality constraint. Hence, the solution \( f \) is found from

\[
\min\left\{ \|Gf - H\|^2 + \lambda^2\|f\|^2 \right\}. \tag{15}
\]

### 2.3. Impact location search strategy

To find the real location (or at least a good estimate) of the unknown impact, it is necessary to seek the location \( i \) which produces the smallest error \( \delta_i \) between the measured response \((H_i)\) and the response \((H)\) reconstructed using the corresponding solution of (15) as

\[
Gf_i \approx H_i = Gf. \tag{16}
\]
Therefore, the goal is to solve

\[
\min\{\delta_i\}, \quad \delta_i = \|H_i - H\| \tag{17}
\]
The solution \( f \) which minimizes (17) can be sought by various methods, however, in this work, a brute-force search in all locations was conducted to ensure that the global minimum is found.

### 3. Finite element model of panel with piezoelectric transducers

The accuracy of the presented approach is tested on a virtual experiment in this work. A curved thin-walled steel panel with a hole is equipped with \( L = 10 \) irregularly placed piezoelectric transducers (so-called patches). The panel is loaded by known impact force in the direction of normal to the outer surface (see Fig. 1 and Fig. 2). The force is represented by Gauss function as shown in Fig. 3. Only one impact per location is used \((M = 1)\). The transducers are used to measure the response (Fig. 3) of the panel, whereas the resulting signal is electric potential between its electrodes (voltage).

A simplified FEA model of the panel is created in MSC.Marc using 8-node solid elements (this is necessary for the piezoelectric sensors). Only one layer of elements through the thickness of the panel was used, therefore, the assumed strain option was engaged. The transient analysis used single-step Houbolt time integration scheme with \( N = 500 \) time increments. Each increment was \( \Delta t = 20 \mu s \).

Nodal links were used on nodes on the outer electrodes of sensors to induce constant electric potential across the electrodes, while the latter nodes were grounded. The mesh consisted of 800 nodes (380 defining the outer surface nodes – impact locations), 342 elements (332 for steel and 10 for patches). The material characteristics are shown in Tab. 1.
Table 1. Material properties used in finite element analysis.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\rho$ [kg/m$^3$]</th>
<th>$E$ [GPa]</th>
<th>$\nu$ [-]</th>
<th>$\varepsilon_{31}$ [C/m$^2$]</th>
<th>$\varepsilon_{32}$ [C/m$^2$]</th>
<th>$\varepsilon_{33}$ [C/m$^2$]</th>
<th>$\varepsilon$ [F/m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7850</td>
<td>210</td>
<td>0.3</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Piezoelectric</td>
<td>7800</td>
<td>62</td>
<td>0.3</td>
<td>5</td>
<td>5</td>
<td>-12</td>
<td>$1 \times 10^{-10}$</td>
</tr>
</tbody>
</table>

Fig. 1 FEA model of panel with sensors (left) and detail of one sensor with nodal links (right).

Fig. 2 Surface of the panel with normals in all impact locations (left) and surface with reduced number of elements (right).

Fig. 3 Example of input (left) and corresponding output (right).
4. Identification of impact location and reconstruction of force on coarse mesh

The corresponding input and output signals were calculated on the original mesh and these were considered as the exact solution within the virtual experiment. Then, a simplified coarse mesh was created (Fig. 2) and the reconstruction was performed using only the responses and transfer functions corresponding to the simplified mesh. This mesh consisted of 55 nodes (impact locations) and 39 elements (or sectors to avoid confusion with finite elements).

The distance \( d \) between the exact impact location and the location found using (17) on the coarse mesh data is plotted in Fig. 4. The maximum distance difference is \( d_{\text{max}} = 1.353 \text{ m} \) (at node \([-638, 98, 777] \text{ mm}\)) and the mean difference is \( d_{\text{mean}} = 0.153 \text{ m} \), while the mean diameter (size) of the sectors is 0.390 m. For impacts at nodes that belong to the coarse mesh, the error was negligible.

Fig. 4 Values of distance between real impact location and that calculated using transfer functions of coarse mesh only.

5. Interpolation of transfer functions

An attempt was made to increase the accuracy of the reconstruction on the coarse mesh by interpolating the neighboring transfer functions. Each sector of the coarse mesh was subdivided into \( 5 \times 5 = 25 \) subsectors, thus creating new internal nodes. The transfer functions at internal nodes \( q \) are calculated using standard approximation functions for isoparametric quadrilaterals as

\[
g_q = \sum_j (N_j g_j), \quad j = a, b, c, d
\]

where the numbers \( a, b, c, \) and \( d \) correspond to nodes defining the given sector and

\[
N_a = \frac{1}{4}(1-\xi)(1-\eta), \quad N_b = \frac{1}{4}(1+\xi)(1-\eta), \quad N_c = \frac{1}{4}(1+\xi)(1+\eta), \quad N_d = \frac{1}{4}(1-\xi)(1+\eta).
\]

Using this technique, the location error for the worst case at node \([-638, 98, 777] \text{ mm}\) decreased to \( d = 0.667 \). Two other examples are presented herein. If the impact location \((-196, 687, 986) \text{ mm}\) was incorrectly found in a neighboring sector \( (d = 0.328) \), the location error decreased to \( d = 0.036 \text{ m} \) using interpolation. Last, if the impact location \((-87, 125, 1001) \text{ mm}\) was incorrectly found within the same sector \( (d = 0.115) \), the error decreased to \( d = 0.076 \text{ m} \).

The variation of distribution of the reconstruction error (17) for real impact location at node \([-196, 687, 986] \text{ mm}\) is illustrated in Fig. 5 for the three mesh configurations. The cross symbol represents the exact impact location and the circle symbol denotes the identified impact location (minimum response error).
Fig. 5 Examples of error surfaces calculated with transfer functions of original mesh (a), coarse mesh (b), and extrapolated (c) data.
6. Conclusions

The methodology for identification of impact location and reconstruction of impact force time dependence was successfully demonstrated on virtual experiment using finite element model. The reliability of the finite element model is to be validated by comparison with real experiment. Of course, secondary difficulties are likely to emerge during the real experiment. For example, it will be difficult to perform impacts with direction normal to surface and to impact the same location if repeated experiments are used (e.g. to measure impacts with different amplitudes, velocities). Also, the real system needs not to be perfectly linear. The non-linearity can be caused by various effects, such as non-negligible damping, plasticity, and structural or sensor damage. Other problems may occur when selecting proper sampling frequency, signal triggering algorithms, number of samples, and also the numerical parameters, such as number of iterations, tolerance values, or the regularization parameter. These effects could be partially suppressed by optimized placement and numbers of both the sensors and training impact locations.

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