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Improved method of node importance evaluation based on node contraction in complex networks

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Abstract

To solve the problem of node importance evaluation in complex networks, an improved method of node contraction based on agglomeration is presented. To start with, methods to evaluate the node importance in complex networks are analyzed. And secondly, evaluation of edge importance is introduced on the basis of method of node contraction. Node importance is defined as weighted sum of importance of node itself and edges that connected with node. Improved method to evaluate node importance considers the connection character of the node, and moreover, by regulating the coefficient, we can adjust the influence of edge on node importance. Finally, an illustrative example verifies the feasibility and validity of the improved method.

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Keywords: complex networks; node importance; characteristics of node connection; node contraction

1. Introduction

As an important research area of complexity science, complex networks have been attached much importance to from a wide range of disciplines in recent years. Along with the rise of complex networks research, the study of invulnerability of complex networks, which is one of the most important directions of complex networks, is of great theoretical significance and application value and has been a challenging frontier topic[1-2]. The non-homogeneous nature of topology structure determines the difference of node importance in complex networks[3]. Research has shown that, networks of different topology structures have different invulnerability under different kinds of attacks. Scale-free network is stronger in fault tolerance compared with random network under random failures, but under intentional attacks, scale-free network appears to be more frangible. Therefore, mining the "critical nodes" by node importance evaluation will be a crucial measure to improve the invulnerability of complex networks.

At present, methods of node importance evaluation in complex networks cover analysis methods of social network, systems science and information search area. Among them, methods of social network

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analysis are on the basis of the idea that "importance is equivalent to significance," and typical evaluation methods include node degree, betweenness, closeness and eigenvector[4-6] etc. Methods of systems science are on the basis of the thinking that "destruction is equivalent to importance", and typical evaluation methods include method based on the shortest path between source point and collecting point, and method of node deletion based on number of spanning trees[7-8] etc. Methods of information search area take into account more factors, and node importance lies not only on the degree, but also on importance of adjacent nodes. Typical methods include PageRank algorithm, HITS algorithm [9-10] etc.

The method of node importance evaluation based on node contraction in complex networks is proposed in [11], which assumes that the nodes are on a normal condition, by contracting the edges connected with the node, the most important node is the one whose contraction results leads to the largest increase of the networks agglomeration. The method takes into account node degree and the number of the shortest paths passing through the node, and avoids disadvantage of methods based on node degree and node deletion. Moreover, the method is more intuitionistic and effective compared with method based on betweenness, and it is the same with the large scale network. However, the definition of node importance of the method does not consider the importance of edges connected with the node, but the fact is that node importance lies not only on its own importance, but also on the characteristics of node connection[12].

To solve this problem, evaluation of edges importance connected with the node is introduced on the basis of the method based on node contraction. Node importance is expressed as the weighted sum of the importance of the node itself and the edges. Improved evaluation algorithm takes into account the connection character of the node, and we can adjust the influence of edge on node importance by regulating the coefficient.

2. The method of node contraction based on agglomeration

Supposing that v_i is a node in the network $G=(V,E)$, the so-called node contraction syncretizes node v_i and other k_i nodes that connected with v_i into a new node v_i', which takes place of the primary k_i+1 nodes, and edges connected with k_i+1 nodes originally turn to the new node v_i' now.

The network after the operation of node contraction is expressed as G'. Node contraction can be understood intuitively as that node v_i and adjacent k_i nodes are agglomerated as one node. If the node v_i is a core node, after node contraction, the network will contract together much better. A special case is the star network: after the contraction of the centre node, only one node remains. And if the node v_i is not a core node, the agglomeration of the network will change slightly, such as end nodes of the chain network.

To describe the agglomeration of the network more objectively after node contraction, the definition of agglomeration is depicted in [11]. Supposing that number of nodes in the network is N, the distance between node v_i and node v_j is d_{ij}, and the average path length of the network is noted as $L(G)$, then the agglomeration of the network $\partial(G)$ is defined as follow:

$$\partial(G)=\frac{1}{N\cdot L(G)}=\frac{N-1}{2\sum_{i>j}d_{ij}} \tag{1}$$

When $N=1$, the agglomeration of the network reaches the maximum value 1, and $0<\partial(G)\le 1$.

According to the formula (1), the importance of the node v_i, $IMC(v_i)$ is defined as follows:

$$IMC(v_i)=1-\frac{\partial(G)}{\partial(G'(v_i))} \tag{2}$$

Where, $G'(v_i)$ denotes the new network after contracting the node v_i.

From (1) and (2), we can get that:

$$IMC(v_i) = 1 - \frac{(N - k_i) \cdot L(G'(v_i))}{N \cdot L(G)} \quad (3)$$

From (3) we can see that the importance of the node v_i in the network lies on two factors: one is its degree k_i, the other is its position. The higher the degree is and the more critical the position of the node is, the greater the agglomeration is, and the more important the node is accordingly.

3. Improved method of node importance evaluation

On the basis of node contraction based on agglomeration, evaluation of edges importance is introduced. Firstly, take the edges in the network G as new nodes, and the links among the edges as edges, then the network G is converted to network G^*. Secondly, calculating the importance of each node in the network G^* applying the method of node contraction. Finally, calculating the weighted sum of the node importance in the network G and importance of the corresponding nodes in the network G^*, namely:

$$IMC(v_i) = \alpha \cdot IMC_n(v_i) + \beta \cdot \sum_{j \in S} IMC_e(v_j) \quad (4)$$

Where, $IMC_n(v_i)$ denotes the importance of node v_i in the network G, $IMC_e(v_j)$ denotes the importance of corresponding nodes that are connected with v_i in the network G^*, and S denotes the corresponding nodes set in the network G^*.

Let $\rho = \alpha / \beta$, it denotes the proportional coefficient of node importance to edges importance. When $\beta = 0$, that is the method of node contraction only considering the node importance. And when $\alpha = 0$, that is the method of node contraction only considering the edges importance. By regulating the coefficient ρ, we can adjust the influence of edges on node importance.

4. Steps of the algorithm and complexity analysis

Here are steps of improved method of node contraction based on agglomeration:

Step1 Calculating the node importance $IMC_n(v_i)$ in the network G.

Step2 Calculating the node importance $IMC_e(v_j)$ in the network G^*.

Step3 According to (4), calculating the node importance $IMC(v_i)$ in the network G.

As can be seen above, the core of the improved method is node contraction based on agglomeration, the time complexity of which is $o(N^3)$ according to [11]. Since it is necessary to consider the importance of edges that connected with the node, that is to say we have to calculate the importance the importance of each node in G^*. Both of them have the same time complexity, consequently, the time complexity of the improved method is $o(N^3)$.

5. An illustrative example

As shown in figure 1(a), the network G contains 10 nodes and 10 edges. Take the edges in the network G as new nodes, and the links among the edges as edges, then the network G is converted to network G^*, as shown in figure 1(b).

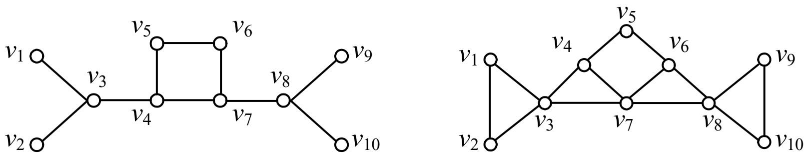

Fig. 1. (a) Topology of network *G*; (b) Topology of converted network *G**

The importance of each node can be obtained by applying the method of node contraction and the improved method respectively. Supposing the weighted coefficient $\rho = 5$, evaluation results are shown in table 1.

Table 1. Results of node importance evaluation

Node	Method of node Contraction	Improved method	Node	Method of node Contraction	Improved method
1	0.1492	0.1771	6	0.2005	0.2768
2	0.1492	0.1771	7	0.4706	0.6489
3	0.4454	0.5685	8	0.4454	0.5685
4	0.4706	0.6489	9	0.1492	0.1771
5	0.2005	0.2768	10	0.1492	0.1771

As is shown in table 1, the evaluation results gained from the method of node contraction and the improved method accord well, and it indicates that ranking result of node importance accords with that of edges importance. By regulating the weighted coefficient, we can analyze the influence of edge properties on node importance, as shown in figure 2.

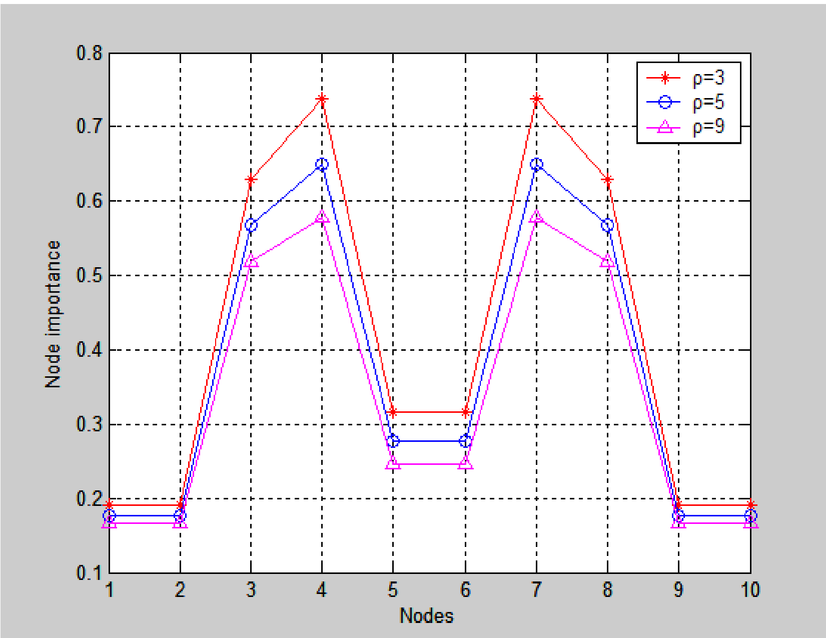

Fig. 2. Compassion of node importance with different coefficients

As shown in figure 2, the influence of different ρ on the node importance is reflected in the variety of relative importance of the node. Figure 2 shows that, with the increasing of the weighted coefficient ρ, the relative importance of the node decreases gradually. For instance, the relative importance of the nodes v_4 and v_7 to the nodes v_3 and v_8 decreases with the increasing of ρ.

6. Conclusion

Node importance evaluation in complex networks is an important part of invulnerability research. In this paper, we analyze the methods of node importance evaluation firstly, then, the method of node contraction based on agglomeration is improved. Node importance evaluation considers not only the importance of node itself, but also the characteristics of node connection. Moreover, the weighted coefficient is introduced. By regulating the coefficient, we can adjust the influence of edge on node importance. Finally, an illustrative example verifies the feasibility and validity of the improved method.

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