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Transition to turbulence in a separated boundary layer with spanwise perturbations

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Abstract

The influence of spanwise perturbations on the transition to turbulence of a strong adverse pressure gradient (APG), laminar separated boundary layer is studied by direct numerical simulation using a high resolution numerical scheme. The perturbations are generated by positioning a discrete roughness element with a sinusoidally varying height in the spanwise direction, close to the inflow. Results indicate that the dynamics of the transition to turbulence mechanism are altered significantly due to the accelerated flow in the openings in the roughness element and the subsequent 3D instability.

Keywords: direct numerical simulation, transition to turbulence, separated flow, roughness, instability

1. Introduction

The efficiency of low pressure turbines is influenced by the extent and size of the separation bubble that forms on the low pressure side of the turbine blade. In general the flow on the low pressure side of the turbine blade is laminar at the leading edge, decelerates due to an adverse pressure gradient, which causes the flow to separate and transitions to turbulence close to the trailing edge. An increase in turbine efficiency requires the ability to control the size of this bubble, without unduly moving the laminar-turbulent transition point upstream. Obviously a lot of study has already been undertaken to understand the influence of small perturbations on the separation bubble. Apart from time dependent fluctuations, which will not be discussed further, time independent fluctuations have been studied\textsuperscript{1}. In general these fluctuations are generated by a tripwire, with or without a spanwise (perpendicular to the flow direction) variation but roughness also falls in this category. Although these perturbations have a lower growth rate than the time dependent fluctuations, they are still important because they are easier to implement and/or they occur naturally on used turbine blades.

The work done on this subject can be broadly divided in two branches, namely a branch that concentrates on the engineering aspect of these perturbations and a branch which is more academic. The difference is mostly due to differences in Reynolds number (high in the first branch and relatively low in the second branch), pressure gradient,
and in the manner the perturbations are imposed (real roughness and trip wires in the first branch and certain blowing and suction profiles in the second).

In this article we would like to contribute in closing the gap between the engineering approach by presenting a direct numerical simulation (DNS) of a boundary layer with a turbine like pressure gradient and Reynolds number. The trip wire is implemented using an immersed boundary method, which adds additional features of real flow configuration to the simulation. This article is subdivided in three parts, namely a short description of the numerical method, a section about the results and conclusions.

### Nomenclature

- **$h$**: Height of the roughness element [m]
- **$L_b$**: Length of the separated region [m]
- **$L_r$**: Streamwise extent of the roughness element [m]
- **$L_x, L_y, L_z$**: Domain length in $x, y, z$ [m]
- **$N_x, N_y, N_z$**: Number of grid points in $x, y, z$
- **$Re$**: Reynolds number $U_{ref} \theta/\nu$
- **$U_{ref}$**: Reference velocity at the inflow [$ms^{-1}$]
- **$x_o$**: Location of the roughness element [m]
- **$\delta^*$**: Displacement thickness [m]
- **$\theta_0$**: Momentum thickness at inlet [m]
- **$\theta_s$**: Momentum thickness at separation [m]
- **$\lambda_r$**: Wavelength of the roughness element [m]
- **$\Lambda_s$**: Pressure gradient parameter at separation $\theta_s^2 \frac{dU_e}{dx}$

### 2. Numerical method

The DNS code, discussed in detail in $^2,^3,^4$, uses a relatively classical fractional-step method to solve the incompressible Navier-Stokes equations expressed in primitive variables.

The model that is used in this study is a flat plate with a streamwise pressure distribution similar to those encountered on the suction side of turbine blades. An almost constant suction velocity is imposed at the upper boundary to match a typical turbine adverse-pressure-gradient (APG). No-slip and impermeable boundary conditions are applied at the wall and the spanwise direction is treated as periodic. At the outflow plane a convective boundary condition is used, with minor adjustment to the exit velocity to ensure global mass conservation. The laminar Hiemenz profile is prescribed at the inflow and steady three-dimensional perturbations are also explicitly added at the inflow for transition to occur in the cases where a two-dimensional roughness is imposed, since otherwise spectral codes along the span, like the one used here, would remain strictly two-dimensional. The time step is adjusted to a constant $CFL = 0.6$, to preserve time accuracy. The Reynolds number based on inlet momentum thickness, $\theta_0$ and streamwise velocity, $U_{ref}$ is $Re_{\theta_0} = 110$. The streamwise, wall-normal and spanwise directions and velocity components are $x, y, z$ and $u, v, w$, respectively.

The simulation domain, shown in Fig. 1(a) ($L_x \times L_y \times L_z$) = 1460 $\times$ 463 $\times$ 770 is discretized in $N_x \times N_y \times N_z = 1537 \times 301 \times 768$ collocation points, comprising approximately 350 million cells. Statistical averages are performed over the homogeneous spanwise direction and time, with the total averaging time is 11000$\theta_0/U_{ref}$, which is equivalent to about 7 flow-through times.

The transition to turbulence scenario is altered by positioning a trip wire close to the inflow. The roughness shown schematically in Fig. 1(b), is modelled using the immersed boundary method$^5,^6,^7$. Here, instead of using interpolation to impose the immersed boundary we estimate the immersed force necessary to achieve zero velocity within the roughness and then add this to the momentum equations. A similar approach was also used in$^7,^8$. The spanwise wavelength $\lambda_r$ of the roughness is varied while the streamwise extent and height of the roughness are held constant at
Table 1. Parameters of the simulations and characteristics of the separated region. $\theta_s$ is the momentum thickness at the separation point, $x_s$ and $x_r$ represent the streamwise location of the separation and reattachment, respectively. $L_b$ is the separation bubble length, $L_b = x_r - x_s$ of the rough case, $L_{b0}$ is the smooth reference case.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\lambda_s/\theta_0$</th>
<th>$Re_{\theta_0}$</th>
<th>$\Lambda_s$</th>
<th>$x_s/\theta_0$</th>
<th>$x_r/\theta_0$</th>
<th>$L_b/L_{b0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smooth</td>
<td>-</td>
<td>161</td>
<td>-0.089</td>
<td>215</td>
<td>746</td>
<td>1</td>
</tr>
<tr>
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<td>$\infty$</td>
<td>168</td>
<td>-0.116</td>
<td>286</td>
<td>544</td>
<td>0.48</td>
</tr>
<tr>
<td>Rough2</td>
<td>258</td>
<td>164</td>
<td>-0.111</td>
<td>285</td>
<td>542</td>
<td>0.48</td>
</tr>
<tr>
<td>Rough3</td>
<td>129</td>
<td>164</td>
<td>-0.111</td>
<td>285</td>
<td>549</td>
<td>0.49</td>
</tr>
</tbody>
</table>

Fig. 1. (a) The numerical setup and the instantaneous visualization of the spanwise vorticity in an adverse pressure gradient turbulent boundary layer. The vortices are colored with the distance to the wall; blue (dark) near the wall, and red near the top. (b) Schematic drawings of the isometric perspective (top) and cross-section (bottom) views of the discrete surface roughness. Not to scale.

$L_r = 36\theta_0$ and $h = 0.7\theta_0$ respectively. For all configurations roughness is located slightly downstream of the inflow, $x_0 = 55\theta_0$ where the flow is still laminar and attached.

The parameters of the numerical experiments are summarized in Table 1. For all cases the pressure gradient parameter at separation point lies in the range $-0.171 < \Lambda_s < -0.083$ as suggested for laminar separation. Furthermore, the Reynolds number at the separation point is approximately 160, indicating that the separation occurs in the laminar region for all cases.

3. Results

In a recent study we systematically investigated the discrete roughness effect on a separated boundary layer development by varying the roughness type, the height, and the location. Our results indicate that the presence of discrete surface roughness increases the turbulent fluctuations in the turbulent boundary layer and it shifts the laminar-turbulent transition to some upstream position, and results in a shorter and lower separation bubble as compared to the uncontrolled flow. In that study we also found that the spanwise varying roughness elements alter the laminar separation and turbulent transition in a different manner than the two-dimensional roughness elements. The reader is referred to that work for details of the basic flow statistics. In this study we would like to investigate in detail the transition mechanism.

This section begins with the effect of roughness in controlling the separation bubble and the mean flow parameters, and followed by a visual assessment of the flow field to identify the different structures that emerge behind the roughness in various regions of the flow. Furthermore, the stability characteristics of the mean flow are evaluated in terms of local temporal linear stability analysis based on the Orr-Sommerfeld equation.

4. Streamwise evolution of the mean flow

The streamwise evolution of the mean flow parameters are presented in Fig. 2 and are compared with the smooth reference case. In these figures, the solid lines correspond to the separated region ($U(x, y = 0, z) < 0$) and vertical shaded area denotes the streamwise extent of the roughness field.
To assess the nature of the flow within the boundary layer, the streamwise variation of the shape factor, $H$, is depicted in Fig. 2(a) in semi-log plot. For cases with roughness, the shape factor, and hence the profile loss, is reduced significantly compared to the smooth case. The vertical displacement of the boundary layer is squeezed, indicating that the boundary layer is accelerated downstream of the roughness. Figure 2(b) shows the evolution of the skin friction coefficient, $C_f$, which gives a quantitative measure of the length of the separated region, which has significantly decreased due to roughness. It is apparent that due to the amplified disturbances upstream and accelerated flow through the openings of the roughness, the separation point moves slightly downstream while transition and reattachment are promoted significantly. The wall-normal extension of the separation bubble decreased considerably by 80% from the smooth to the rough case.

The shape and extent of the separation bubble depend largely on the transition mechanism. The evolution of the maximum turbulent kinetic energy, $T$, (normalized by the local free-stream velocity squared $U_e^2$) shown in figure 3(a) complements these observations. Note that steady three-dimensional perturbations exist at the inflow for smooth and 2D rough cases for transition to occur. Due to the absence of the these perturbations at the inflow in the case of the 3D surface roughness the turbulent intensity is zero at the inlet. Perturbations grow when the flow passes through the roughness elements. However, these perturbations are not unstable in the attached laminar boundary layer and decay quite rapidly until reaching a point slightly upstream of the separation point, when the perturbations start to grow. The fast linear growth of the disturbances is then observed soon after the separation, while the non-linear growth sets in approximately at the maximum bubble height.

Alam and Sandham\textsuperscript{10} observed that the viscous Tollmien-Schlichting (TS) instability of the reversed flow along the wall may dominate the inviscid instability of the separated shear layer when the reversed flow velocity exceeds...
The ratio of the wall-normal location of the maximum turbulent intensity to the inflection point is shown in Fig. 3(b). The profiles have an inflection point imposed by the APG, which is the precursor of the separation and transition. The location of the disturbance growth relative to the inflection point indicates that the disturbances are not amplified through a viscous (TS) instability mode, in which the maximum amplification is located close to the wall but through an inviscid Kelvin-Helmholtz (KH) instability mechanism of the separated shear layer. By using a 3D roughness pattern, a significant attenuation of the KH wave amplitude is observed as shown in Fig. 3(a) which is followed by the sudden growth of the streamwise fluctuations due to inviscid instability in the separated shear layer. It is interesting to note the effect of short wavelength case (Rough2) which slightly delays the amplification of the KH instability as compared to the long wavelength case (Rough3). The results, however, show that the wavelength of the roughness is not important for separation delay.

5. Flow visualization

The streamwise evolution of the instantaneous streamwise, wall-normal and spanwise velocities at a plane close to the wall is presented in Fig. 4. The roughness element located upstream of the bubble does not cause the flow to transition, but it is responsible for generating perturbations that hasten the reattachment, $x_r$ of the separation bubble as presented in table 1 for all cases. The most striking feature is the very ordered structure of the flow during the transition phase in the cases that a 3D roughness element is used. This is particularly the case when $\lambda_r = 129\theta_0$, and especially considering the $v$ and $w$ velocity components. The flow with a spanwise uniform roughness element does not show any order and there is even no sign of spanwise uniform shedding vortices. Contrary to that it can be deduced from the $u'$ figure in Fig. 4(b) that there is some coherent spanwise structure before the flow becomes fully turbulent (around $x/\theta_0 = 800$), which is the clearest example, among the three, of a pure KH instability. The streamwise velocity in the case of $\lambda_r = 129\theta_0$ shows large streaky structures, extending over the whole streamwise length, whose origin can be traced back to the roughness element. The same is less clear for the other two cases.
A representation of vortical structures that develop in rough cases are visualized in Fig. 5. The isosurfaces of the second-invariant of the velocity gradient tensor coloured with the distance from the wall for the 2D and 3D cases are shown as viewed from the top and isometric perspective. Results indicate that the three-dimensional roughness located upstream of the bubble does not cause the flow to transition immediately, but is responsible for generating perturbations that hasten the reattachment of the separation bubble. The flow after the trip element is still laminar, and perturbations due to the trip element hardly grow until the bubble starts to form. It is seen that transition to turbulence starts approximately at the same streamwise location in all cases. However, the transition scenario is distinctly different.

The flow oscillates in spanwise direction like a waveform for 3D roughness cases. Since the roughness is imposed on the problem as a sinusoidal function in the spanwise direction, the effect comes from this sinusoidal roughness element continues to influence to the flow structures up to the fully turbulent region. After that region, the flow loses its history. A three-dimensional disturbance with a fixed spanwise wavelength is induced due to the roughness. In fact, it is shown that such a disturbance generates very small vortices. This vortices interact nonlinearly with the large amplitude KH vortices of the separated shear layer, producing oblique 3D vortices with the same spanwise wavelength as the one of the roughness elements. The vortices exhibit peaks and valleys, i.e., regions of enhanced and reduced wave amplitude in spanwise direction, half a spanwise wavelength apart, as shown in Fig. 5(c). At the peak location, the breakdown of the instantaneous high-shear layer vortices into smaller vortices is observed. Watmuff has observed hairpin-like structures in the separated shear layer, which provide a mechanism for the wall-normal exchange of momentum. These hairpin-like structures are also apparent in the transition and reattachment region.
of the separated boundary layer as shown in the figure. A qualitatively similar formation is observed for the long wavelength case.

The underlying transitional mechanisms due to 3D and 2D roughness are distinctly different. Due to more intense and larger amplitudes of the disturbances in 2D case, as shown in Fig. 5(a), the vortices undergo a more rapid breakdown. This finding suggests that 3D roughness is probably involved in the attenuation of the instability.

In order to identify the effect of 3D roughness on the dynamics of the transition, the contour of the streamwise velocity fluctuation, \( u' \) which identifies the high wave-number disturbances upstream of the separation, is visualized in a closed-up view in Fig. 6. The flow in between the roughness is restricted by the small interstitial gaps. The spanwise gradient of the streamwise velocity \( dU/dz \) is negative in the left side of the opening and positive in the right, it is easy to expect that the streamwise vorticity though very small, is generated in the openings positive on one side and negative on the other. The spanwise vortex as in the form of 2D KH vortex in a separated shear layer is tilted downstream with the generation of these positive streamwise vorticity in the region where \( dU/dz < 0 \) and tilted upstream in the region where \( dU/dz > 0 \). This implies that the spanwise vortex evolves into a \( \Lambda \)-shaped vortex as shown in Figs. 5(b) and (c).

The streaks in Fig. 6 represent disturbance paths that are generated as a result of the roughness pattern. This figure clearly shows that the difference between the two wavelengths is due to an increased amplitude of the streak. Due to the acceleration of the flow in the openings of the roughness, the streamwise intensities are amplified. While the initial disturbance amplitudes are different, the long-wavelength induced disturbances are not strong enough to cause a significant difference in the mean flow. So, the mean flow parameters are very similar for the short and long wavelength cases as presented in Figs. 2 and 3.

6. Linear Stability Analysis

To provide support for these computational results, the Orr-Sommerfeld equations that govern the temporal growth of a disturbance are solved following linear stability theory. Using velocity profiles extracted from time- and spanwise-averaged DNS data a linear stability analysis is performed at each streamwise location to determine the amplification rate of locally unstable disturbances. The equation is solved for a given wavenumber \( \alpha \) as an eigenvalue problem in \( \omega \). The solution is an eigenfunction with a corresponding eigenvalue \( \omega = \omega_r + i\omega_i \). The complex phase speed can then be calculated as \( c = \omega/\alpha \). The assumption underlying the Orr-Sommerfeld equation is that the flow is parallel to the
surface, which is not satisfied downstream of separation. Therefore, disturbances with wavelengths much longer than the characteristic length scale of the problem (e.g., the bubble length) are discarded.

By varying the streamwise location where the eigenvalues are calculated, the streamwise distribution of the maximum growth of $c_i/U_{ref}$ is obtained, as shown in Fig. 7(a). In all cases, the flow is stable upstream of the roughness. Slightly upstream of the roughness end, the flow becomes unstable for the rough cases. The maximum growth rate of the unstable modes increases linearly with streamwise direction in the first part of the separated region, and has a maximum downstream of the location of maximum reverse flow, and decreases further downstream. The amplification rates of the most unstable wavelengths are portrayed in Fig. 7(b) for different streamwise locations for the smooth case. There are no unstable modes in the first part of the boundary layer, but they are there as soon as the profiles have inflection points (see Fig. 3(b)). The maximum amplification for the most amplified eigenmode at the end of roughness is found at $\lambda_x/\theta_0 = 42$, which is in the range of wavelengths of the KH vortices obtained from our DNSes and with a convection speed of 0.4$U_{ref}$. This further supports our observations that the vortices generated in the separated shear layer are due to the triggering of the inviscid instability. This mode with the highest amplification is further investigated for three-dimensional waves. Figure 7(c) illustrates the positive contours of the constant growth rate at $x/\theta_0 = 90$, the least stable streamwise location, in the $\lambda_x - \lambda_z$ plane. The spanwise wavelengths of the 3D rough cases are also marked in the same figure as vertical lines. As expected, the maximum growth rate is found for two-dimensional waves while the least unstable spanwise wavelength is around $\lambda_z \approx 110\theta_0$, which is slightly lower than the spanwise wavelengths of our 3D rough studies. So, the spanwise wavelengths considered in this study for the roughness cases are not small enough for transition to take place close to the roughness element, which agrees well with the observations from our DNSes.

7. Conclusions

In this study we have shown that perturbations induced by a roughness element have an important effect on the separation bubble, both the length as the height is reduced and the separation point is moved downstream.

Two different types of roughness elements are used, namely an element with a uniform height in the spanwise direction and one non-uniform element with a sinusoidally varying height. In the latter case two different simulations with two different wavenumbers are done. The wavenumbers chosen do not cause the attached laminar boundary layer to transition. On the contrary, the perturbations induced by these elements actually decay to only start to increase shortly before the flow separates.

In the cases with a non-uniform element a clear correlation exists between the valleys in the roughness element and the instability. The transition process in these cases is characterized by groups of very ordered coherent structures, and although no correlations were obtained it seems that these structures leave a trace until the end of the flow domain. The opposite happens in the case a uniform element is applied, in which case the flow does not show clear coherent structures during the transition process.
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