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# A synthetic approach to the evaluation of the carrying capacity of complex railway nodes



Gabriele Malavasi <sup>a</sup>, Tatiana Molková <sup>b</sup>, Stefano Ricci <sup>a</sup>, Francesco Rotoli <sup>c,\*</sup>

- <sup>a</sup> University of Rome "La Sapienza", Department of Civil, Building and Environmental Engineering, Transport Area, via Eudossiana 18, 00184 Rome, Italy
- b University of Pardubice, Jan Perner Transport Faculty, Department of Transport Technology and Control, Studentská 95, 532 10 Pardubice, Czech Republic
- European Commission, Joint Research Centre (JRC), Institute for Prospective Technological Studies (JRC-IPTS), Edificio Expo, C/Inca Garcilaso 3, 41092 Seville, Spain

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#### ABSTRACT

The evaluation of carrying capacity of complex railway nodes is a typical problem to be faced in metropolitan areas. This paper initially analyzes a few methods (Potthoff methodology, Probabilistic approach and Deutsche Bahn procedure) for the evaluation of carrying capacity of complex railway nodes. The aim of the article is to investigate commonalities and differences among these methods in order to try (even in the continuation of the research) to identify potential margins of improvement or to formulate a new approach to evaluate the use of stations in a synthetic mode, considering the characteristics and the limits of the existing and analyzed models. The results of the theoretical analysis have been validated by means of applications to typical case studies.

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# 1. Introduction

A complex railway node is a part of railway network characterized by a variable configuration due to the presence of elements as turnouts or crossings in a very limited area; sometimes it may coincide with the railway network. The evaluation of carrying capacity of complex railway nodes is a typical problem to be examined in metropolitan areas where the same infrastructures are used for different services (urban, regional, national, passenger, freight, etc...). The frequency of these services is usually fairly high, constant during specific periods of the day (basic interval schedules) and variable according to season and year (demand configuration). In these circumstances, the most common problems to be considered include the identification of the infrastructural critical elements as well as the definition of the most effective actions for the full exploitation of the carrying capacity.

Models for the evaluation of carrying capacity of complex railway nodes can be allocated to two main categories (Crenca et al., 2005):

• synthetic models: they have a general approach and provide overall information for network planning; they are limited to study specific problems (e.g. selection of partial and particular appropriate interventions),

E-mail addresses: gabriele.malavasi@uniroma1.it (G. Malavasi), Tatiana.Molkova @upce.cz (T. Molková), stefano.ricci@uniroma1.it (S. Ricci), francesco.rotoli@ec.europa.eu, francescorotoli@tiscali.it (F. Rotol.

 micro and macro simulation models: they enable the duplication of the system and therefore they are useful both in the planning phase and in the operational phase., but they require detailed definitions of related components.

The paper analyses a few synthetic methods for the evaluation of carrying capacity (Potthoff approach, Probabilistic methodology and Deutsche Bahn procedure) of complex railway nodes. The final goal of the study is to investigate the common points and differences among these methods in order to try to identify their characteristics and potential margins of improvement. The results of the analysis have been validated by means of experimental measures on typical case studies.

This article attempts to represent the beginning of an extensive research, aiming to analyse several and different analytical methods, also in comparison with other methodologies (e.g. simulation models), with the purpose of comparing and possibly integrating them in a unique new approach to evaluate the use of stations in a synthetic mode.

# 2. State of the art

The assessment of railway capacity is a typical issue of highly utilized networks, since an effective analysis of capacity is crucial not only for evaluations of new investments, but also for an efficient management of the existing infrastructure.

The complexity of the "term" capacity and the benefit of creating a transnational method for its evaluation are highlighted in the UIC Code 406.

<sup>\*</sup> Corresponding author. Tel.: +39 3284188725.

In the last years technical literature has largely addressed this topic (Hansen and Pachl, 2008; Abril et al., 2008; Kontaxi and Ricci, 2009; Kontaxi and Ricci, 2011), presenting also various approaches to the problem and different computer-based tools for supporting and improving the assessment and the management of railway capacity.

Basically capacity evaluation models can be allocated in two main categories: analytic or simulation models.

Analytical methods utilize mathematical expressions with a general approach to obtain overall information for network planning; they provide approximate results offering a first indication on railway capacity in a preliminary phase. Analytical methods may be useful in simple situations and they often lead to useful results without the need of expensive simulations (Hansen and Pachl, 2008); these methodologies can also be used for reference or comparison.

Simulation models, instead, allow us to obtain a more accurate representation of reality by means of duplication of the railway system, but they require detailed definitions of related components. Modern market provides several tools to simulate rail traffic with various approaches (Hansen and Pachl, 2008; Abril et al., 2008).

Since the bottlenecks of a double track railway network are mostly located at or around stations (Yuan and Hansen, 2007; Hansen, 2000; Carey and Carville, 2003) numerous models and tools have been developed to address the evaluation of carrying capacity of complex railway nodes.

In this perspective, example of analytical approaches were already presented by Potthoff in 1965 Potthoff (1963–1972) and by Deutsche Bahn in 1979 Deutsche Bundesbahn (1979); both the methodologies considered the station as a bottleneck allowing an average number of movements depending on its topological structure and on the compatible paths matrix. According to these formulations, Florio and Mussone (1998) suggested the analytical splitting up of complex junctions in elementary nodes and the solution of a problem of optimum to maximise, in iterative form, the overall number of trains in the station.

Based on the queuing theory, in 1985 Wakob (Wakob (1985) and De Kort et al. (1999)) suggested an analytical approach for the analysis of railway nodes extending the Schwanhäußer's method (Schwanhäußer, 1974; Schwanhäußer, 1978) to railway stations and adjacent junctions; he applied a queuing model to predict the waiting time incurred by the simultaneous arrival and random processing of two trains at isolated parts of the infrastructure. In 1999 Wendler Wendler (1999) proposed several extensions to these models based on new approximation algorithms. Afterwards Huisman et al. (2002) developed a solvable queuing network model for railway while Vakhtel (2002) presented a new analytical software system (named ANKE, Analytic Network Capacity Determination) following critical considerations on the newest scientific achievements and the current state of computing techniques.

Even Oetting and Nießen in 2003 Oetting and Nießen (2003) presented a R&D project of Deutsche Bahn aimed to develop a tool package for the medium-and long-term infrastructure planning; algorithms and data used in the tool suite were compatible with the already existing Deutsche Bahn railway operational scientific methods for the short and medium term assessment of network elements (lines and stations). Inter alia, Nießen in 2008 Nießen (2008) presented different methods to compute the capacity characteristics for a "route node"; this last one was modelled with a mathematical approach, using a multiresource queue.

Recently, Lindner (2011) and Landex et al. (2008) assess the applicability of the analytical UIC Code 406 compression method for evaluating stations and junctions capacity.

Regarding simulation tools, as detailed by Yuan (2006), macroscopic models replicate real train operation on the basis of a macroscopic network, e.g. SIMONE (Middelkoop and Bouwman, 2001), while microscopic simulation models, e.g. RailSys (Radtke and Hauptmann, 2004) and OpenTrack (Nash and Huerlimann, 2004), reproduce real train operations according the scheduled arrival and departure times, the dispatching rules, the track configuration, the train dynamic characteristics and the signalling system. Extensive analysis of railway simulation tools are presented by Abril et al. (2008) and Kontaxi and Ricci (2009, 2011).

#### 3. Evaluation of carrying capacity of a complex railway node

This article analyzes three methods for the evaluation of carrying capacity of a complex railway node:

- Potthoff method (Potthoff, 1963-1972) (Potthoff, 1960).
- Probabilistic method, (Corazza and Musso, 1991).
- Deutsche Bahn method (Deutsche Bundesbahn, 1979).

Two of the described approaches (Potthoff and DB) are based on a probabilistic concatenation of train paths, while Corazza and Musso derive capacity parameters from the sets of compatible routes of a station. Although the methods are quite old, the principles and the concepts on which they are based are still widely valuable. The authors have purposely chosen to start the research from basic and simple models for capacity evaluation to try to understand and to consider the foundations and the evolution of the synthetic methods. In particular the Deutsche Bahn procedure and the Potthoff methodology are supposed to observe and analyse the problem from two different prospective: the university point of view (Potthoff) and the industrial practice (DB).

The different results obtained by the application of these methods to two different stations (Frattamaggiore in Italy, and Huersko in the Czech Republic) are analysed; moreover in paragraph 4.4 we have also reported a larger investigation on a real case (Praha Masarikovo).

# 3.1. Potthoff method (Potthoff, 1963-1972)

This method assumes that trains could arrive at any instant of an assigned *T* time period with the same probability; it does not require an assigned timetable because the Potthoff methodology is based on a global quantitative analysis of the traffic in the period *T*. Its great advantage is the simplicity of application.

The first step of this method is the topological analysis of a station layout in order to identify, to analyse and above all to compare all possible routes. The logical structure allowing this comparison is a matrix in which rows and columns correspond to the routes listed in the same order. Each element of the matrix represents the compatibility or incompatibility status of routes (See Fig. 1).

The symbols in the Table 1 are explained below:

- "." compatible routes;
- "a" comparison of a route with itself;
- "x" intersection of routes;
- "z" converging routes;
- "s" diverging routes;

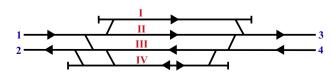


Fig. 1. Frattamaggiore Station.

**Table 1**Compatibility matrix for the Frattamaggiore Station.

	1 – I	1 - II	1 – IV	4 - III	4 - IV	III – 2	IV - 2	I – 3	II - 3	IV - 3
1 – I	a	S	S					d		
1 – II	S	a	S				•		d	•
1 – IV	S	S	a		u	X	u		•	d
4 - III			•	a	S	d	•		•	х
4 - IV			u	S	a		d		•	u
III – 2			X	d		a	Z		•	•
IV – 2			u		d	Z	a			
I – 3	d		•				•	a	Z	Z
II – 3		d	•				•	Z	a	Z
IV – 3			d	X	u			Z	Z	a

- "d" consecutive routes;
- "u" frontal collision routes:

The following step is to introduce the number of movements concerning each route (See Table 2).

The average number of compatible routes is determined by an empirical equation as suggested by Potthoff:

$$n_{med} = \frac{N^2}{\sum (n_i * n_j)} \tag{1}$$

where

*N*: total number of movements ( $N = \sum n_i = \sum n_i$ ).

 $n_i$ : number of movements concerning the route i.

 $n_i$ : number of movements concerning the route j.

 $\Sigma$ : it is extended to all couples of incompatibles routes.

Therefore, to determine  $n_{med}$ , the respective weight  $\Sigma(n_i*n_j)$  is assigned to each element of the matrix containing an incompatibility symbol (a value equal to zero is assumed for the cells representing compatibility of routes,); moreover all the elements of the matrix are added (see Table 3). The complex node could notionally be represented by an intersection of two lines;  $n_{med}$  trains simultaneously circulate in the node and each simultaneous movement occupy the station for a  $t_{med}$  time.

The total time B of occupation and the coefficient U of utilization of the station are determined by the following equations:

$$B = \frac{N}{n_{med}} * t_{med} \tag{2}$$

$$U = \frac{B}{T} = \frac{N}{n_{med}} * \frac{t_{med}}{T}$$
 (3)

For the calculation of the occupation times of the routes (according to the station and rolling stocks characteristics), it is possible to arrange the data within another matrix (see Table 4). In particular, when i=j, the element  $t_{ij}$  of this matrix represents the occupation time of route i, while if  $i\neq j$ ,  $t_{ij}$  represents the period during which the route j may not be run because a train is moving on the incompatible route i (interdiction time); usually  $t_{ij}\neq t_{ji}$ . The average occupation time ( $t_{med}$ ) is obtained by a weighted average of all these times (4).

Each element  $t_{ij}$  of the matrix is referred to a hypothetical number of possible events  $n_i * n_j$  (which represents the weight of the considered element):

$$t_{med} = \frac{\sum (n_i * n_j * t_{ij})}{\sum (n_i * n_j)}$$
 (4)

**Table 2** Number of movements for each route.

Route	1 – I	1 - II	1 - IV	4 - III	4 - IV	III – 2	IV - 2	I – 3	II – 3	IV - 3
Number of movements	56	55	0	112	8	112	8	56	55	0

**Table 3**  $\Sigma(n_i * n_i)$  calculation.

_(	,,										
31	36 3	080	0	0	0	0	0	3136	0	0	
30	80 3	025	0	0	0	0	0	0	3025	0	
0	0		0	0	0	0	0	0	0	0	
0	0		0	12544	896	12544	0	0	0	0	
0	0		0	896	64	0	64	0	0	0	
0	0		0	12544	0	12544	896	0	0	0	
0	0		0	0	64	896	64	0	0	0	
31	36 0		0	0	0	0	0	3136	3080	0	
0	3	025	0	0	0	0	0	3080	3025	0	$\Sigma(n_i * n_j)$
0	0		0	0	0	0	0	0	0	0	90980

The method allows also to calculate the total delay  $(\Sigma R_{ij})$  generated in the node as the sum of the delays related to each incompatibility between two routes i and j.

The ratio between  $\Sigma R_{ij}$  and  $n_{med}$  represents the total delay obtained considering movements of  $n_{med}$  trains simultaneously. Thus, it has to be verified that:

$$T \geqslant \frac{N}{n_{med}} * t_{med} + \frac{\sum R_{ij}}{n_{med}}$$
 (5)

In practice, to calculate the delays related to each incompatibility and their sum  $(\Sigma R_{ij})$ , Potthoff analyzes the scheme of a simple node.

When a train crosses the node, it forbids movements on the other line from the time in which the signal is placed to the free aspect until the instant in which the end of the train passes a given point (A or B in Fig. 2), disengaging completely the simple node.

If, we assume the following hypothesis:

- the occupation time t<sub>1</sub> and t<sub>2</sub> (respectively for trains running on line 1 and 2) are constant during the period T (homogeneity of services on each line).
- a FIFO service discipline (first in, first out) for the node,
- a constant probability density function for the arrival of trains during the period *T*:

$$f(t) = \frac{1}{T} \tag{6}$$

an arriving train X which finds the node occupied, should wait a time variable from 0 to the occupation time related to the train Y on the other line; therefore, neglecting the effects of the variable acceleration in the departure and braking phases, X would accumulate

Table 4 Matrix of occupation/interdiction times.

$t_{ij}$ (min)	1 - I	1 – II	1 – IV	4 - III	4 – IV	III – 2	IV - 2	I – 3	II – 3	IV - 3
1 – I	3.70	1.95	1.37	0	0	0	0	0	0	0
1 – II	0.93	1.73	0.70	0	0	0	0	0	0	0
1 – IV	1.85	1.85	3.93	0	4.11	1.47	0	0	0	0
4 – III	0	0	0	1.45	0.64	0	0	0	0	0.64
4 – IV	0	0	3.60	1.54	3.34	0	0	0	0	0
III – 2	0	0	0.92	0.82	0	1.31	1.31	0	0	0
IV - 2	0	0	2.06	0	1.80	2.83	2.83	0	0	0
I – 3	2.64	0	0	0	0	0	0	3.07	3.07	3.07
II – 3	0	0.96	0	0	0	0	0	1.28	1.28	1.28
IV - 3	0	0	2.46	2.64	2.64	0	0	3.07	3.07	3.07

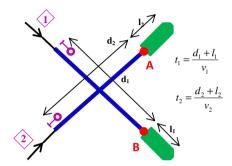


Fig. 2. Scheme of a simple node.

an average delay equal to  $t_1/2$  or  $t_2/2$  depending upon the lines in which the trains X and Y are running.

A train travelling on line 1 could encounter a red aspect signal at the node with probability:

$$p_1 = n_2 * \frac{1}{T} * t_2 \tag{7}$$

where  $t_2/T$  represents the probability to find the node occupied by a train of line 2 and  $n_2$  represents the number of times that this incompatibility occurs during the period T.

So, each train on line 1 suffers a delay equal to:

$$R_1' = p_1 * \frac{t_2}{2} = \frac{n_2 * t_2^2}{2 * T} \tag{8}$$

while the total delay of all the trains  $(n_i)$  running on line 1 is:

$$R_1 = \frac{n_1 * n_2 * t_2^2}{2 * T} \tag{9}$$

In the same way, it is possible to calculate the total delay for the trains on line 2:

$$R_2 = \frac{n_1 * n_2 * t_1^2}{2 * T} \tag{10}$$

Considering now a complex node, each incompatibility between two routes i and j can be represented by a simple node; consequently, the associated delays will be:

$$R_{ij} = \frac{n_i * n_j * t_{ij}^2}{2 * T} \quad \text{and} \quad R_{ji} = \frac{n_i * n_j * t_{ji}^2}{2 * T}$$
 (11)

# 3.2. Probabilistic method Corazza and Musso, 1991

This method assumes that trains could arrive at any instant of an assigned time period with the same probability; however, it considers movements as stochastic events. In the assigned time period, T, the probability that a train uses a route i is assumed equal to:

$$p_{i} = \frac{n_{i} * t_{i}}{T}$$

$$p_{i} = \sum p_{i\nu}$$

$$(12)$$

$$p_{i} = \sum p_{iv} \tag{13}$$

where:

 $t_i$ : occupation time of the route i.

 $n_i$ : number of movements concerning the route i.

v: number of trains which can simultaneously circulate with i( $v = 0,1,2,...,v_{\text{max}}$ ); it is defined by the v-tuples of routes compatible with i.

In practice, the probability to have a movement on route i is given by the sum of several addends:

- the probability that only 1 train uses the route i,
- the probability that 1 train moves along the route i and another train simultaneously utilizes a compatible route (couples of compatible routes).
- the probability that a movement on route i occurs simultaneously with other two circulations on two routes compatible between them and with i (triplets of compatible routes).
- ullet the probability of simultaneous circulation on the  $v_{
  m max}$ -tuple of compatible routes.

The compatibility matrix allows the identification of all the couples of compatible routes. It is also possible to create another matrix in which these couples are reported in columns and single routes are represented in rows; each element of this matrix denotes the compatibility or incompatibility status of the triplet of routes (Fig. 3). With the same process, it is possible to constitute the matrixes for all the possible *n*-tuples (quadruplets, quintuplets and so on, up to the  $v_{\rm max}$ -tuples) of compatible routes.

The results of the above described procedure can be presented graphically in a tree (Fig. 4). Starting from a vertex with index zero, we represent a node for every route and we link these points with zero. Subsequently, from each vertex we draw arcs and nodes for all the couples of routes compatible with it and so forth (for the terns, quadruplets, etc...). Obviously, if node 1 is linked with the vertex 1-4, then the vertex 4-1 is not represented to avoid unnecessary repetitions; the tree is asymmetric.

Regarding the formula (13), it is possible to calculate the  $p_{iv}$  values through the Calculation of Probability. For example, the probability to have a train using route i during the period T, is:

$$p_i^* = \frac{n_i * t_i}{T} \tag{14}$$

The superscript \* indicates that the value of probability is not only referred to single movements of trains on the route i, but also to all the contemporary movements on other compatible routes; it represents the sum of all the components  $p_{iv}$ . In order to evaluate these elements, it is necessary to start from the  $v_{\rm max}$ -tuple.

#### **FRATTAMAGGIORE** Villa Literno 10 1 - II 1 - IV 4 - III 4 - IV III - 2 I - 3 II - 3 IV - 3 1 - I s 2 1 - II d a S 1 - IV 3 d a u u 4 - III а d s x 4 - IV 5 u a d u III - 2 d d u a a 7. d 7. a 7 d 11 a 3-4 6-8 6-9 6-10 7-8 1-5 1-6 1-10 2-4 2-5 2-6 2-7 2-8 3-8 3-9 4-7 4-8 4-9 5-6 2-10 1-5-9 1-6-9 1-6-10 1-7-9 1-7-10 2-4-7 2-4-8 2-5-6 2-5-8 2-6-8 2-6-10 2-7-8 2-7-10 3-4-8 3-4-9 4-7-8 4-7-9 5-6-8 1\_4\_7

Fig. 3. Couples, terns and quadruplets of compatible routes for the Frattamaggiore Station.

Considering the Frattamaggiore Station (see Figs. 3 and 4), the probability of contemporary movements on the quadruplet of compatible routes 1–4–7–9 is obtained by multiplying the probabilities of movements on each route:

$$p_{(1,4,7,9)} = p_1^* * p_4^* * p_7^* * p_9^*$$
 (15)

Furthermore, based on the (13), it is possible to calculate the probability of contemporary movements on the terns and then on the couples of compatible routes. For instance, we can observe for the tern 1–4–7 and the couple 1–4:

$$p_i^* = \sum p_{i\nu} \Rightarrow (p_1^* * p_4^* * p_7^*) = p_{(1,4,7,9)} + p_{(1,4,7)} \Rightarrow p_{(1,4,7)} = (p_1^* * p_4^* * p_7^*) - p_{(1,4,7,9)}$$
(16)

$$p_{(1,4)} = (p_1^* * p_4^*) - p_{(1,4,7,9)} - p_{(1,4,7)} - p_{(1,4,9)}$$

$$\tag{17}$$

Finally, the probability to have a movement on the route 1 without any circulations on other compatible routes will be:

$$p_{(1)} = p_1^* - p_{(1,4,7,9)} - p_{(1,4,7)} - p_{(1,4,9)} - p_{(1,7,9)} - p_{(1,4)} - p_{(1,7)} - p_{(1,9)}$$

$$(18)$$

The sum of all the elements  $p_v$  provides the probability to have at least 1 train in the system during T while the time of use of the station is represented by:

$$t = T * \sum p_{\nu} \tag{19}$$

Moreover, the average number of compatible routes is determined considering the probability to have 1,2,3...,  $v_{\rm max}$  trains in the system:

$$n_{med} = \frac{\sum v * p_v}{\sum p_v} \tag{20}$$

Thus, the traffic in the station can be represented by a sequence of  $N/n_{med}$  events; each event is characterized by  $n_{med}$  trains (simultaneously circulating in the complex node) and it will last a time  $t_{med}$ , equal to:

$$t = \frac{N}{n_{med}} * t_{med} \Rightarrow t_{med} = \frac{T * \sum v * p_v}{N}$$
 (21)

while the average time between two events will be:

$$\Delta t_1 = \frac{T - t_1}{N} n_{med} = T * \left(1 - \sum p_{\nu}\right) * \frac{n_{med}}{N}$$
 (22)

Appendix A2 of Corazza and Musso (1991) reports a quite simple and useful matrix procedure (see also Table 12) to calculate  $p_v$ ,  $\Sigma p_v$  and then all the other descripted parameters ( $n_{med}$ ,  $t_{med}$ , etc.).

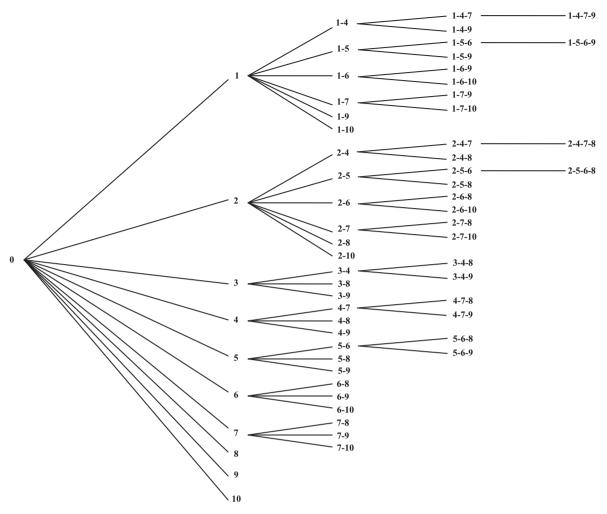


Fig. 4. Tree of the sets of compatible routes for the Frattamaggiore Station.

# 3.3. Deutsche Bahn method (Deutsche Bundesbahn, 1979)

Here we refer to the analytical process described in the guidelines for the calculation of the capacity of nodes proposed by the German Railways (April 1, 1979). The method leads to the same results obtained with the Potthoff methodology although different factors are considered. In fact, it calculates the total time (B) and the coefficient of use of station (h) as parameters to evaluate the carrying capacity of the system.

Firstly, the procedure determines the index k expressing the probability with which the movements relating to the complex node are mutually exclusive:

$$k = \sum \frac{n_i * n_j}{N^2} \tag{23}$$

In (23), the summation is referred only to the couples of incompatible routes i and j. The total time B of occupation represents the period during which the node is occupied by movements reciprocally exclusive; it is calculated as follows:

$$B = \frac{\sum (n_i * n_j * t_{ij})}{N} \tag{24}$$

In the previous formula,  $t_{ij}$  is the blocking time in which a movement on route i forbids movements on j. However, the time B can also be calculated in function of the average blocking time E(t):

$$E(t) = \frac{\sum t_{i_j} * n_i * n_j}{\sum n_i * n_i}$$
 (25)

$$B = k * N * E(t) \tag{26}$$

Moreover, the procedure calculates the coefficient h, representing the utilization of the node:

$$h = \frac{B}{T} \tag{27}$$

The guidelines suggest for the index h, a maximum allowable value of 0.7 in the most favourable situations (minimal delays on the arrival with no priority between routes) and a lower value of 0.45 in the most unfavourable scenarios.

Another useful parameter described in this methodology is the average tolerance time, E(r):

$$E(r) = \frac{(T-B)}{(k*N)} \tag{28}$$

It represents the maximum value of the blocking time  $t_{ij}$  to avoid that a movement on i delays the successive movement on j.

As it is possible to observe, Deutsche Bahn procedure and Potthoff method propose the same expressions for the indexes B and U–h; however, the DB technique introduces (in the calculation of interdiction times), the priority relationships between couples of movements on compatible routes (G: same priority; V: higher priority; N: lower priority). The interdiction time  $p_{ij}$  that a train on route i imposes to trains on j will be:

$$p_{ij} = \frac{n_i * n_j * (t_{i_j} + d_{ij})^2}{2T}$$
 (29)

$$P_b = \sum p_{ij} \tag{30}$$

where:

 $d_{ii}$  =  $t_{ii}$  in case of higher priority V.

 $d_{ii} = -t_{ii}$  in case of lower priority N.

 $d_{ii}$  = 0 in case of same priority G.

The carrying capacity of a station is expressed by the maximum number of daily movements in the node with determinate operation standards.

The average number of trains in the waiting queue  $L_z$  is the parameter used to evaluate the operation quality; the maximum carrying capacity is reached when an average of 60% of trains waits in the queue before entering the node ( $L_z = 0.6$ ) and its value is extrapolated from the number N of planned or observed movements during the period T, using a factor x:

$$N_z = \frac{N * x * 1440}{T} \tag{31}$$

The extrapolating factor x is obtained by the following condition:

$$L_Z = \frac{k * P_b * x^2}{T - x * B} \tag{32}$$

and so, x will be equal to:

$$x = -\frac{B*L_z}{2*k*P_b} + \sqrt{\frac{T*L_z}{k*P_b} + \left(\frac{B*L_z}{2*k*P_b}\right)^2}$$
 (33)

Obviously if x < 1 it could be opportune to reduce the number of movements.

#### 4. Cases of study

In the following, two simple case studies are presented. The authors have intentionally searched simplicity in the applications at this stage, to guarantee an easier and better understanding of the procedures (rapid to be reproduced by the reader), and also because complex and more articulated stations could obscure some details and information, rather clear in easy examples; however to better display the concrete applicability of the methods a larger investigation on a real case ('Praha Masarykovo') has been reported in paragraph 4.4.

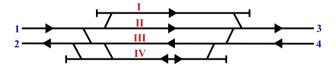


Fig. 5. Frattamaggiore Station.

## 4.1. Application to Frattamaggiore Station

The three methods have been applied to a small Italian railway station (Frattamaggiore Station, Fig. 5) in order to investigate common points and differences among them. Many applications have been carried out to identify the differences in the results, changing the number of movements on holding tracks and on running tracks as showed in Table 5 (the actual situation according to the available data is represented by the hypothesis 1). The values  $t_{ii}$  have been evaluated considering that a train, in clearing each section of the route, releases the locking affecting that section (sectional route locking); the occupation/interdiction times (see Table 6) have been calculated on the basis of the length of the routes/sections and of the allowed speeds (obtained from the schematic plan of the station). It has been assumed a total operational time of 1200 min (20 h per day) while a dwelling time of 60 s has been considered, where applicable: in particular ES and IC trains (about 33% of total trains) do not stop in this station, so the occupation times have been calculated as weighted means of occupation times with and without stop ("EC/IC" occupation time \* 1/3 + "Reg" occupation time \*2/3).

The results of these applications are presented in the following tables and graphics: (See Figs. 6–8) and (See Tables 7–10).

Further comments:

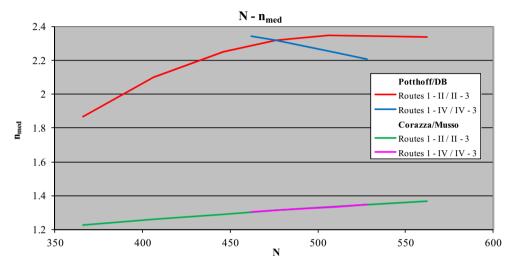
- Changing the total number N of movements, the parameters change in different ways in the various methods; in particular, the Probabilistic method produces lower  $n_{med}$  values and higher  $t_{med}$  and utilization (U) values for the same initial data. This could in part be explained noting that in the calculation of probability, the method considers only the occupation times (not all the interdiction times per each couple of incompatible routes), not allowing to perfectly represent the sectional route locking configuration (for a better understanding, see also Table 12 in the next paragraph and the matrix procedure in appendix A2 of Corazza and Musso, 1991).
- Utilizing the Corazza/Musso approach, any increase in the number of movements (either on the main or lateral tracks) leads to an increase in the value of  $n_{med}$ ; applying the Potthoff method or the Deutsche Bahn procedures, intensifications of  $n_i$  on lateral routes imply reductions in the values of  $n_{med}$ , while a growth of the number of trains on the running tracks entails, at first, higher values of the average number of compatible routes and lastly, a decrease of this parameter.
- Rising the number of movements on lateral tracks (subjected to more incompatibility situations/points) all the methods show an increase in the value of  $t_{med}$ ; On the contrary, intensifications of  $n_i$  on running tracks produce reductions of the average

**Table 5** Traffic hypothesis (Hp. 1–10).

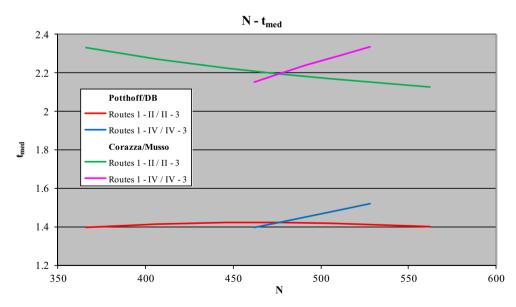
Routes	Traffic hyp	oothesis								
	Hp. 1	Hp. 2	Hp. 3	Hp. 4	Hp. 5	Hp. 6	Hp. 7	Hp. 8	Hp. 9	Hp. 10
1 – I	56	56	56	56	56	56	56	56	56	56
1 – II	55	98	70	40	20	0	55	55	55	55
1 - IV	7	7	7	7	7	7	33	24	15	0
4 – III	112	112	112	112	112	112	112	112	112	112
4 - IV	8	8	8	8	8	8	8	8	8	8
III – 2	112	112	112	112	112	112	112	112	112	112
IV - 2	8	8	8	8	8	8	8	8	8	8
I – 3	56	56	56	56	56	56	56	56	56	56
II – 3	55	98	70	40	20	0	55	55	55	55
IV - 3	7	7	7	7	7	7	33	24	15	0
Total	476	562	506	446	406	366	528	510	492	462

**Table 6**Matrix of occupation/interdiction times.

$t_{ij}$ (min)	1 – I	1 – II	1 – IV	4 – III	4 – IV	III – 2	IV - 2	I – 3	II – 3	IV - 3
1 – I	3.80	1.55	0.97					0.00		
1 – II	0.90	1.95	0.61						0.00	
1 – IV	1.45	1.45	4.03		4.21	1.47	0.00			0.00
4 – III				1.67	0.61	0.00				0.61
4 - IV			3.70	1.54	3.44		0.00			0.00
III – 2			1.22	1.06		1.56	1.56			
IV - 2			2.16		1.90	2.93	2.93			
I – 3	2.74							3.17	3.17	3.17
II – 3		1.20						1.54	1.54	1.54
IV – 3			2.56	2.74	2.74			3.17	3.17	3.17



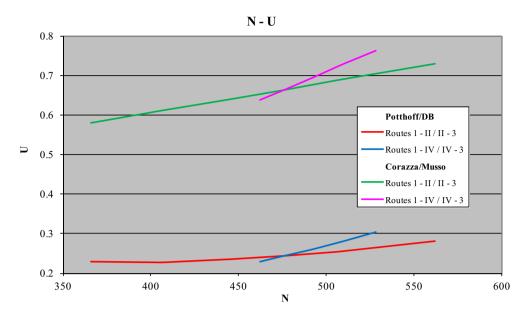
**Fig. 6.** Potthoff/DB and Corazza/Musso:  $N - n_{med}$ 



**Fig. 7.** Potthoff/DB and Corazza/Musso:  $N-t_{med}$ .

occupation time utilizing the Corazza/Musso approach, while applying the other two methodologies we can observe at first higher values of this index and lastly a decrease of it.

- The probabilistic method derives capacity parameters from the sets of compatible routes of a station; it identifies and analyzes the different n-tuples of compatible routes saturating the node but on the other side, it considers only the occupation times of each route (see above). Instead, the topologic analysis of the station in the other two procedures is limited to the examination
- and individuation only of the couples of compatible or incompatible routes; it is possible to vary the number of movements on every route but regardless of the *n*-tuples of permitted simultaneous movements.
- The Deutsche Bahn method produces the same B and U values like Potthoff but unlike Potthoff, it introduces in the calculation of the interdiction time, the priority relationships between couples of movements on compatible routes. A further and deeper analytical comparison between these two methods is reported in the Annex.



**Fig. 8.** Potthoff/DB and Corazza/Musso: N - U.

**Table 7** Potthoff application results.

Potthoff	$\Sigma(n_i*n_j)$	N	$n_m$	$t_m$	$\Sigma(n_i*n_j)/N$	$U_{20h}$	$\Sigma R_{ij}$	$U_{\rm T20h}$
Frattamag	ggiore Static	n						
Hp 1	97756	476	2.32	1.42	205.37	0.24	122.15	0.29
Hp 2	134908	562	2.34	1.40	240.05	0.28	160.60	0.34
Нр 3	109036	506	2.35	1.42	215.49	0.25	134.23	0.30
Hp 4	88276	446	2.25	1.42	197.93	0.23	111.50	0.28
Hp 5	78436	406	2.10	1.42	193.19	0.23	99.52	0.27
Hp 6	71796	366	1.87	1.40	196.16	0.23	90.09	0.27
Hp 7	126356	528	2.21	1.52	239.31	0.30	177.10	0.37
Hp 8	115844	510	2.25	1.49	227.15	0.28	155.99	0.34
Hp 9	105980	492	2.28	1.45	215.41	0.26	137.09	0.31
Hp 10	90980	462	2.35	1.40	196.93	0.23	110.51	0.27

**Table 8**Probabilistic application results.

Corazza-Musso	T	$\Sigma p_{\nu} = U_{20\mathrm{h}}$	$t_1$	N	$n_m$	$t_m$	$\Delta t_1$
Frattamaggiore St	ation						
Hp 1	1200	0.66	796.68	476.00	1.31	2.20	1.11
Hp 2	1200	0.73	875.96	562.00	1.36	2.13	0.79
Нр 3	1200	0.69	824.33	506.00	1.33	2.17	0.99
Hp 4	1200	0.64	769.02	446.00	1.29	2.23	1.25
Hp 5	1200	0.61	732.14	406.00	1.26	2.27	1.45
Нр 6	1200	0.58	695.27	366.00	1.23	2.33	1.69
Hp 7	1200	0.76	916.24	528.00	1.35	2.33	0.72
Hp 8	1200	0.73	874.85	510.00	1.33	2.29	0.85
Hp 9	1200	0.69	833.47	492.00	1.32	2.24	0.99
Hp 10	1200	0.64	764.48	462.00	1.30	2.15	1.23

# 4.2. Applications to Uhersko Station

To further analyse and validate the results obtained by the aforementioned analysis, another interesting application of these three methodologies was carried out to the Czech Uhersko Station (Fig. 9).

Also, in this case for the evaluation of  $t_{ij}$ , we considered that a train, in clearing each section of the route, releases the locking affecting that section (sectional route locking). As above, the occupation/interdiction times (see Table 11) have been calculated on the basis of the length of the routes/sections and of the allowed speeds; again, it has been assumed a total operational time of 1200 min (20 h per day) while an average dwelling time of 60 s

has been considered, where applicable; in particular stopping trains use track III and VI (close to the platform 1 and 2) while running trains utilize track I, II and IV.

To better investigate differences among the Deutsche Bahn procedure and the other two approaches we have considered several scenarios for the DB method. The first scenario is without any priority assignment; in the others we have considered a higher priority for the running tracks (Hp. 2 and Hp. 3), for the routes with stop at the platforms (Hp.4 and Hp.5), or for the other lateral routes (Hp. 6 and 7). The ranking of priority in the various hypothesis and the results of these applications are represented in the following tables and graphics: (See Tables 12–17).

Result comments:

- This application confirms the results of the previous case study; also in this case it is possible to observe that the Probabilistic method produces with the same initial data lower  $n_{med}$  values and higher  $t_{med}$  and utilization (U) values than the Deutsche Bahn or Potthoff procedures.
- In the various priority scenarios the Deutsche Bahn method returns always the same values for h, E(t) and  $E_r$  (not depending from  $P_b$ ). However assigning higher priority to the running tracks (representing also the routes with the higher number of movements), we can observe lower values for  $P_b$  and so higher values for x and  $N_z$ ; on the contrary, assigning a higher priority to the lateral tracks, the application shows a decrease of carrying capacity ( $N_z$ ) and an increase in total delay ( $P_b$ ).

# 4.3. Analysis of the results

The analysis and the comparison of the two applications lead to interesting observations. First of all, it is possible to note the differences between these two (Italian and Czech) stations and the corresponding influence on the carrying capacity. Although very similar in dimension and layout, Frattamaggiore presents four tracks all with platforms while Uhersko counts five tracks and only the external ones are served by platforms. The hypothesized traffic for the Italian station is variable between 366 and 562 movements, while the Czech node is characterized by a higher intensity (N = 666). In both the stations the running tracks represent also the routes with the higher number of movements (reducing the number of conflicts). Albeit Frattamaggiore is mainly characterized

**Table 9**DB application results.

DB	k	E(t)	В	h	$E_r$	$L_z$	T	$P_b$	x	$N_z$
Frattamagg	iore Station									
Hp 1	0.43	1.42	292.10	0.24	4.42	0.60	1200	122.15	2.39	1365.29
Hp 2	0.43	1.40	336.63	0.28	3.60	0.60	1200	160.60	2.09	1407.06
Нр 3	0.43	1.42	305.46	0.25	4.15	0.60	1200	134.23	2.29	1391.22
Hp 4	0.44	1.42	281.68	0.23	4.64	0.60	1200	111.50	2.47	1322.81
Hp 5	0.48	1.42	273.48	0.23	4.80	0.60	1200	99.52	2.53	1234.71
Нр 6	0.54	1.40	273.73	0.23	4.72	0.60	1200	90.09	2.52	1106.25
Hp 7	0.45	1.52	363.46	0.30	3.50	0.60	1200	177.10	1.93	1222.98
Hp 8	0.45	1.49	337.48	0.28	3.80	0.60	1200	155.99	2.08	1270.78
Hp 9	0.44	1.45	312.81	0.26	4.12	0.60	1200	137.09	2.24	1320.46
Hp 10	0.43	1.40	275.02	0.23	4.70	0.60	1200	110.51	2.53	1404.00

**Table 10**Movements on the various routes.

Routes	2 – II	2 - IV	2 - VI	3 – III	3 – I	III - 1	I – 1	II - 4	IV - 4	VI - 4
$n_i$	140	12	14	33	134	33	134	140	12	14

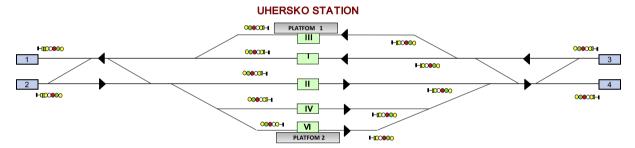


Fig. 9. Uhersko Station.

**Table 11** Simplified matrix of occupation/interdiction times.

t <sub>ij</sub> (min)	2 - II	2 – IV	2 – VI	3 - III	3 – I	III – 1	I – 1	II – 4	IV - 4	VI - 4
2 – II	0.91	0.53	0.53	_	_	=	-	_	-	_
2 – IV	1.41	2.92	1.43	_	_	_	_	_	_	_
2 – VI	1.41	1.43	3.94	_	_	_	_	_	_	_
3 – III	_	_	_	3.97	1.33	_	_	_	_	_
3 – I	-	-		0.51	0.88		-	-	-	-
III - 1	_	_	-	1.92	-	2.82	2.82	_	_	-
I – 1	_	_	_	_	0.24	0.60	0.60	_	_	_
II – 4	0.46	_	_	_	_	_	_	0.83	0.83	0.83
IV - 4	_	1.75	_	_	_	_	_	2.78	2.78	2.78
VI – 4	_	_	2.77	_	_	_	_	3.78	3.78	3.78

by a lower average hourly number of movements (variable from 366/20 = 18.3 for hypothesis 6, to 562/20 = 28.1 for hypothesis 2) than Uhersko (666/20 = 33.3) and even though they present comparable values for  $n_{med}$  the Italian node shows mostly a lower utilization coefficient (U or h) than the Czech station applying the Corazza/Musso approach and a higher usage considering the Pothoff or DB procedures.

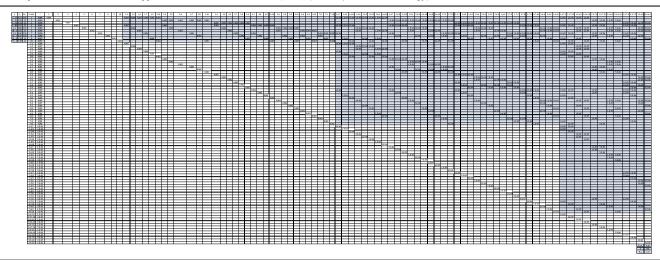
As already highlighted, we can observe that changing the total number N of movements in the station, the results vary in different ways for the three methodologies even if the probabilistic method produces always lower  $n_{med}$  and higher  $t_{med}$  and U values than the other two procedures.

Interesting effects are shown varying the number of movements on different tracks in Frattamaggiore. Utilizing the Corazza/Musso approach, any increase in the number of movements (either on the main or lateral tracks) leads to an increase in the value of  $n_{med}$ ;

applying the Potthoff or the Deutsche Bahn procedures, intensifications of  $n_i$  on lateral routes imply reductions in the values of  $n_{med}$ , while a growth of the number of trains on the running tracks entails, at first, higher values of the average number of compatible routes and lastly, a decrease of this parameter. Moreover, rising the number of movements only on lateral tracks, all the methods show an increase in the value of  $t_{med}$ ; on the other side, intensifications of  $n_i$  on running tracks produce reductions of the average occupation time utilizing the Corazza/Musso approach but applying the other two methodologies we can observe at first higher values of this index and then a decrease of it.

The application to Uhersko, furthermore, indicates that assigning higher priority to the running tracks (representing also the routes with the higher number of movements) the Deutsche Bahn method returns lower values for  $P_b$  and higher values for x and  $N_z$  while considering a higher priority for the lateral tracks, we can

Table 12
Matrix procedure described in the appendix A2 of Corazza and Musso (1991) (Corazza/Musso methodology).



**Table 13** Potthoff application results.

$\Sigma(n_i * n_j)$	N	$n_m$	$t_m$	$\Sigma(n_i*n_j)/N$	T (min)	$U_{20h}$	$\Sigma R_{ij}$	$U_{T20h}$
Potthoff: H	luersko	Station	ı					
188860	666	2.35	0.71	283.57	1200	0.17	82.03	0.20

**Table 14** Probabilistic application results.

T (min)	$\Sigma p_{\nu} = U_{20\mathrm{h}}$	$T_1$	N	$n_m$	$t_m$	$\Delta t_1$
Corazza-N	lusso: Frattamagg	iore Statio	n			
1200	0.74	888	666	1.43	1.91	0.67

observe a decrease in carrying capacity and an increase of total delay.

The above described results show some slight differences among the three methods and suggest some considerations regarding their use in different situations. First of all, it seems clear that the probabilistic method derives capacity parameters from the sets of compatible routes of a station; it analyses the capacity of a station placing more attention on the operation plan or on the group of simultaneous routes able to better saturate the node. In other words, this approach allows us to evaluate, for example, the different results obtained assigning more or less movements to the different *n*-tuples of compatible routes; however since it considers

**Table 16**Ranking of priority in the various scenarios (1: higher priority; 3: lower priority).

Priority levels	1	2	3
Hp 1		No priority	
Hp 2	2-II-4, 3-I-1	2-VI-4, 3-III-1	2-IV-4
Hp 3	2-II-4, 3-I-1	2-IV-4	2-VI-4, 3-III-1
Hp 4	2-VI-4, 3-III-1	2-II-4, 3-I-1	2-IV-4
Hp 5	2-VI-4, 3-III-1	2-IV-4	2-II-4, 3-I-1
Нр 6	2-IV-4	2-II-4, 3-I-1	2-VI-4, 3-III-1
Hp 7	2-IV-4	2-VI-4, 3-III-1	2-II-4, 3-I-1

only the occupation times of each route and not the interdiction times for each couple of incompatible routes, it is not possible to perfectly represent some particular situations as the sectional route locking configuration.

The DB procedure, instead, introduces the priority assignment among itineraries, allowing us to evaluate its effects on the capacity of the station.

Summarizing, it could be possible to utilize a method or the other one on the basis of the aspects of capacity (saturation set of routes, priority assignment, or overall results) that we want to explore or to highlight.

# 4.4. Praha Masarykovo Station

To better analyse and evaluate the relevance and also the concrete applicability of the methods, a larger investigation on a real

**Table 15**Corazza/Musso methodology – usage of time (1200 min per day) per each route.

i	$t_i$ (min)	$n_i$	T (min)	Occupation	time	Interdiction	time	Available time		
				(min)	(%)	(min)	(%)	(min)	(%)	
2 – II	0.91	140	1200	126.9	10.6	353.9	29.5	719.2	59.9	
2 – IV	2.92	12	1200	35.0	2.9	353.9	29.5	811.1	67.6	
2 – VI	3.94	14	1200	55.2	4.6	353.9	29.5	790.9	65.9	
3 – III	3.97	33	1200	130.9	10.9	237.9	19.8	831.3	69.3	
3 – I	0.88	134	1200	118.6	9.9	240.2	20.0	841.2	70.1	
III – 1	2.82	33	1200	93.2	7.8	240.2	20.0	866.6	72.2	
I – 1	0.60	134	1200	80.5	6.7	237.9	19.8	881.6	73.5	
II - 4	0.83	140	1200	115.8	9.7	352.5	29.4	731.7	61.0	
IV - 4	2.78	12	1200	33.3	2.8	341.1	28.4	825.6	68.8	
VI – 4	3.78	14	1200	52.9	4.4	352.5	29.4	794.7	66.2	

**Table 17**DB application results.

k	T( )								
r.	E(t)	В	h	$E_r$	$L_z$	$T_{20h}$	$P_b$	x	$N_z$
giore Station									
0.43	0.71	200	0.17	3.53	0.60	1200	82.03	3.13	2505
0.43	0.71	200	0.17	3.53	0.60	1200	75.93	3.21	2568
0.43	0.71	200	0.17	3.53	0.60	1200	74.10	3.24	2588
0.43	0.71	200	0.17	3.53	0.60	1200	193.06	2.32	1852
0.43	0.71	200	0.17	3.53	0.60	1200	217.50	2.21	1769
0.43	0.71	200	0.17	3.53	0.60	1200	98.54	2.95	2358
0.43	0.71	200	0.17	3.53	0.60	1200	215.67	2.22	1775
	0.43 0.43 0.43 0.43 0.43 0.43	0.43 0.71 0.43 0.71 0.43 0.71 0.43 0.71 0.43 0.71 0.43 0.71	0.43     0.71     200       0.43     0.71     200       0.43     0.71     200       0.43     0.71     200       0.43     0.71     200       0.43     0.71     200       0.43     0.71     200	0.43     0.71     200     0.17       0.43     0.71     200     0.17       0.43     0.71     200     0.17       0.43     0.71     200     0.17       0.43     0.71     200     0.17       0.43     0.71     200     0.17       0.43     0.71     200     0.17	0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53           0.43         0.71         200         0.17         3.53	0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60           0.43         0.71         200         0.17         3.53         0.60	0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200           0.43         0.71         200         0.17         3.53         0.60         1200	0.43     0.71     200     0.17     3.53     0.60     1200     82.03       0.43     0.71     200     0.17     3.53     0.60     1200     75.93       0.43     0.71     200     0.17     3.53     0.60     1200     74.10       0.43     0.71     200     0.17     3.53     0.60     1200     193.06       0.43     0.71     200     0.17     3.53     0.60     1200     217.50       0.43     0.71     200     0.17     3.53     0.60     1200     98.54	0.43         0.71         200         0.17         3.53         0.60         1200         82.03         3.13           0.43         0.71         200         0.17         3.53         0.60         1200         75.93         3.21           0.43         0.71         200         0.17         3.53         0.60         1200         74.10         3.24           0.43         0.71         200         0.17         3.53         0.60         1200         193.06         2.32           0.43         0.71         200         0.17         3.53         0.60         1200         217.50         2.21           0.43         0.71         200         0.17         3.53         0.60         1200         98.54         2.95

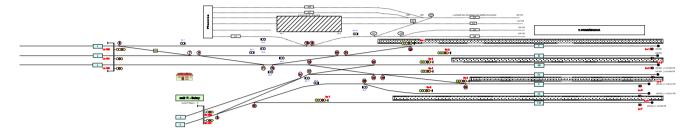


Fig. 10. Praha Masarykovo.

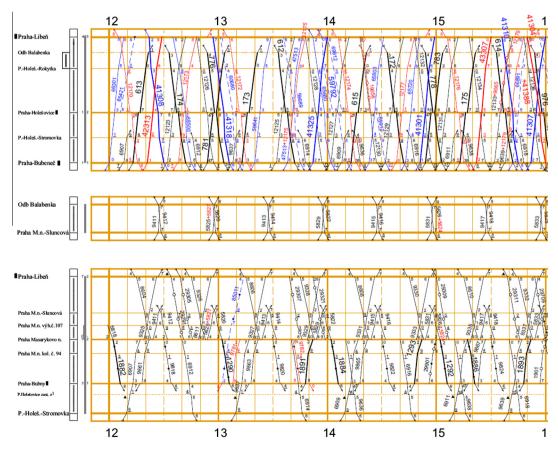
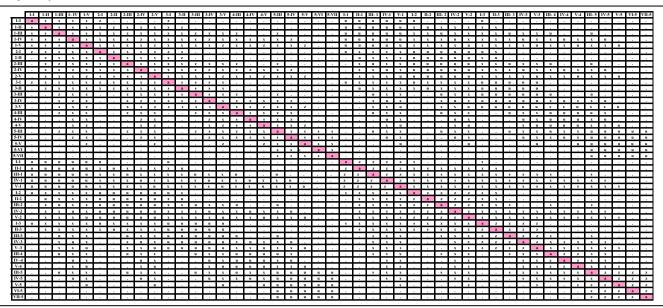


Fig. 11. Extract of the scheduled timetable.

case ('Praha Masarykovo') has been carried out. Among other this application points out the complexity of the three described methods analyzing large stations (e.g. with more tracks). While the Potthoff and DB approaches can be considered linear and the main difference between diverse-sized studies is represented by dif-

ferent datasets and matrixes to be analysed and handled, the probabilistic method could appear a little bit more intricate. However, after identifying the *n*-tuples of compatible routes, applying the matrix procedure for the calculation of the probabilities of movements described in the appendix A2 of Corazza and Musso

**Table 18**Compatibility matrix.



(1991), even the application of this method becomes easier and quite manageable.

The Czech station 'Praha Masarykovo' is represented in the next figure. Even in these cases for the evaluation of  $t_{ij}$ , we have considered that a train, in clearing each section of the route, releases the locking affecting that section (sectional route locking).

In the calculation of the lost times in the start-up and braking phases we have assumed uniformly accelerated motion for the trains (constant acceleration or deceleration) (see Figs. 10).

The number of movements for each route has been evaluated on the basis of the scheduled timetable and operations plan (data 2003) (see Fig. 11, Tables 18–22).

Two different scenarios have been considered: the actual situation (Hp.1, data 2003) and a second scenario (Hp.2, saturation of the node) obtained multiplying the numbers of movements for route  $(n_i)$  by the same constant  $\alpha$  saturating the capacity of the station (so that T = B + R) (5).

The results of the application are presented in the following tables:

#### Result comments:

- The application confirms the results of the previous case studies: the probabilistic method produces with the same initial data lower  $n_{med}$  values and higher  $t_{med}$  and utilization (U) values than the Deutsche Bahn or Potthoff procedures; in particular the results of the Corazza/Musso approach indicate that, assuming an operative period of 20 h (T = 1200 min), the node in the second scenario is slightly oversaturated being the time of use of the station equal to 1235 min.
- This case study tries to point out the complexity of the three described methods analysing large stations; while the Potthoff and DB approaches can be considered linear and the main difference between diverse-sized studies is represented by different datasets and matrixes to be analysed and handled, the probabilistic method could appear a little bit more intricate. However, after identifying the *n*-tuples of compatible routes, applying the matrix procedure for the calculation of the proba-

**Table 19**Simplified matrix of occupation/interdiction times.

t <sub>ij</sub> (min)	3 – I	3 – II	3 – III	3 – IV	5 – III	5 - IV	5 – V	5 – VI	5 – VII	I – 2	II - 2	III- 2	IV - 2	III- 4	IV - 4	V - 4	VI – 5	VII – 5
3 – I	9.11	0.99	0.83	0.83							0.99	0.83	0.83					
3 – II	0.99	9.11	0.83	0.83						0.99		0.83	0.83					
3 – III	0.83	0.83	9.27	0.91	9.02								0.91					
3 – IV	0.83	0.83	0.91	12.27	1.07	9.02						1.00		1.07		1.07		
5 – III			8.84	0.88	9.01	0.72	0.51	0.51	0.51				0.88		0.72	0.72	0.51	0.51
5 – IV				11.84	0.72	12.01	0.51	0.51	0.51					0.72		0.89	0.51	0.51
5 – V					0.51	0.51	11.58	0.84	0.59								0.84	0.59
5 – VI					0.51	0.51	0.84	12.16	0.59									0.59
5 – VII					0.51	0.51	0.59	0.59	12.16								0.59	
I – 2	7.82	0.82								8.30	1.30	1.30	1.30					
II – 2	0.82	7.82								1.30	8.30	1.30	1.30					
III – 2	0.98	0.98	7.98	0.98	7.74					1.30	1.30	8.30	1.30	4.24				
IV – 2	0.98	0.98	0.98	10.98	0.89	10.74				1.30	1.30	1.30	11.30	0.89	4.24	0.74		
III – 4			7.74	0.89	7.91	0.91						5.74	0.89	8.12	1.12	1.12		
IV - 4				10.74	0.91	10.91							5.74	1.12	11.12	1.12		
V - 4				0.74	0.91	0.89	10.51						0.74	1.09	1.09	11.09		
VI – 5					1.09	1.09	1.09	11.09	1.09								11.09	1.09
VII – 5					1.09	1.09	1.09	1.09	11.09								1.09	11.09

**Table 20**Quadruplets of compatible routes for Praha Masarikovo (based on the simplified compatibility matrix, neglecting unused routes; i = incompatible, c = compatible).

		3-7-10-15	3-7-11-15	3-8-10-15	3-8-10-16	3-8-11-15	3-8-11-16	3-9-10-15	3-9-10-16	3-9-11-15	3-9-11-16	3-10-15-17	3-10-15-18	3-10-16-17	3-10-16-18	3-11-15-17	3-11-15-18	3-11-16-17	3-11-16-18
3 - I	1																		
3 - II	2																		
3 - III	3																		
3-IV	4																		
5 - III	5																		
5-IV	6																		
5-V	7																		
5-VI																			
5-VII	9																		
1-2																			
II - 2																			
III- 2																			
IV - 2																			
III- 4																			
IV - 4																			
V - 4	_	i	i	i		i		i		i									
VI-5		i	i	i	i	i	i	i	i	i	i								
VII-5	18	i	i	i	i	i	i	i	i	i	i	i		i		i		i	

**Table 21**Results applying the Potthoff method.

Нр	N	$n_{med}$	T <sub>(min)</sub>	$t_{med \ (min)}$	B <sub>(min)</sub>	$U_{20h}$	$\Sigma R_{ij \text{ (min)}}$	R <sub>(min)</sub>	(B+R)/T	α
1 2	240	2.29	1200	3.27	344	0.29	195	85	0.36	2.24
	539	2.29	1200	3.27	771	0.64	981	429	1.00	1.00

**Table 22** Results of the Corazza/Musso method.

Нр	$T_{(min)}$	$\Sigma p_{\nu} = U_{20\mathrm{h}}$	t (min)	N	$n_m$	$t_{m \text{ (min)}}$	$\Delta t_{1(\min)}$
1	1200	0.81	971	240	1.43	5.80	1.36
2	1200	1.03	1235	540	2.54	5.80	=-

bilities of movements described in the appendix A2 of Corazza and Musso, 1991, even the application of this method becomes easier and quite manageable.

- Moreover, considering the multiplicative coefficient  $\alpha$  saturating the node it is possible to observe:
- The second scenario (obtained multiplying all the actual  $n_i$  from the first scenario per  $\alpha$  = x = 2.24) corresponds to the saturation of the node using both the Potthoff approach (B + R = T) and the DB procedure with  $L_Z$  = 1 (x = 1); in particular, it should be noticed that  $N_Z$  in the DB results is different from N only because the first one is referred to the whole day (24 h and so 1440 min) and not only to the assumed operative time (T = 1200 min = 20 h).
- Tables 23 and 24 show that changing the assumed value for  $L_Z$  utilizing the DB procedure all the results will still coincide between them and also with the Potthoff's results (Table 21),

except of course for the maximum carrying capacity values (which depend from the extrapolation factor and from the queue length)

### 4.5. Final remarks

In conclusion the case studies not only illustrate in detail the three analysed procedures but they show how it could be possible to utilize a methodology or the other one on the basis of the aspects of capacity (saturation set of routes, priority assignment, overall results, queue theory parameters) that we want to explore or to highlight.

It may be quite interesting to extend the comparison to other synthetic or analytic methods or to try to correlate other parameters provided by the methodologies (such as delays and average waiting times) or, again, to try to explicate the recommended values of some indexes (for example, the recommended value of the coefficient of utilization or of the length of the queue).

The final aim of further researches could be to attempt to identify potential margins of improvement or to formulate a new approach to evaluate the use of stations in a synthetic mode, also considering the characteristics and the limits of existing models.

**Table 23**Results of the DB procedure, with Lz = 0.6.

Нр	N	$k_{DB}$	T <sub>(min)</sub>	<i>E</i> ( <i>t</i> ) <sub>(min)</sub>	B (min)	h	E(r) (min)	$P_b$	$L_z$	х	Nz
1	240	0.44	1200	3.27	344	0.29	8.16	195	0.60	1.94	559
2	539	0.44	1200	3.27	771	0.64	1.82	981	0.60	0.86	559

**Table 24** Results of the DB procedure, with Lz = 1.

Нр	N	$k_{DB}$	T <sub>(min)</sub>	$E(t)_{(min)}$	B (min)	h	$E(r)_{(\min)}$	$P_b$	$L_z$	х	$N_z$
1	240	0.44	1200	3.27	344	0.29	8.16	195	1	2.24	646
2	539	0.44	1200	3.27	771	0.64	1.82	981	1	1.00	646

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