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Analysis of Petri Nets by Stepwise Refinements

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If the firing of a transition in a Petri net is considered non instantaneous, it becomes possible to replace a transition in a net $\mathscr{P}$ by another net $\mathscr{P}'$. This allows to proceed the description and the analysis of a control structure by stepwise refinements. The necessary and sufficient conditions on $\mathscr{P}$ and $\mathscr{P}'$, for the resulting net to be bounded and live, are given.

1. INTRODUCTION

Petri nets are being used by researchers in various domains where the notions of choice and concurrency are essential. Among these domains are the analysis of production schemata [3], the description and realization of digital systems [8], the finding of the computation rate of activities in asynchronous concurrent systems [11], the formal verification of parallel programs [6]. Many other applications can be imagined [10].

Some interesting concepts such as boundedness, safeness and liveness have been defined on Petri nets [2], the analysis of a large Petri net is generally cumbersome or even impracticable.

Another problem results from the fact that the firing of a transition is generally supposed non divisible and instantaneous. In fact when a Petri net is used to represent the functioning of an actual system, actions or operations are associated with the firing of the transitions. These actions or operations not being instantaneous, it is thus necessary to decompose each of them into a sequence 'beginning of operation," "end of operation". The system has then to be depicted directly in detail and such a method does not allow stepwise refinements.

It seems more efficient to suppose that the firing of a transition is not instantaenous and that it is made up of two steps [11]. It is then possible to associate with a transition a complex operation that can later be depicted in detail by means of another Petri net.

The aim of this paper is to prove that such a methodology allows a description and an analysis of a system by stepwise refinements. That is to say, it is possible to build up complex Petri nets with desired properties (bounded and live for example). An informal proof has been given in [12] and a more detailed approach of this problem can be found in [13].

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In section 2 some definitions are given and section 3 considers the substitution of a transition by a Petri net verifying some properties and called a well-formed block. In Section 4, it is pointed out that it is possible to build up recursively safe and live Petri nets.

2. The Petri Net

The definition of the Petri net that we will use is essentially the same as that employed by M. Hack [4] and J. L. Peterson [9]. The notion of bag and the notations used will be mainly those of the appendix of the paper by J. L. Peterson. The set of all bags over a domain P will be noted P^ω and the number of occurrences of an element p in a bag B will be noted $B(p) = k \geqslant 0$.

2.1. *Definition of the Petri net*

A *Petri net* is a five-tuple defined by:

$$\mathscr{P} = (P, T, M_0, I, O)$$

where

— $P = \{p_1, p_2, \ldots, p_n\}$ is a non-empty finite set of *places*

— $T = \{t_1, t_2, \ldots, t_m\}$ is a non-empty finite set of *transitions*

— M_0 is the *initial marking*. It is a bag and $M_0 \in P^\omega$

— I and O are functions

$$I\colon T \to P^\omega$$

$$O\colon T \to P^\omega$$

such that

$$I_j = I(t_j) \text{ is the bag of } \textit{input places} \text{ for } t_j$$

$$O_j = O(t_j) \text{ is the bag of } \textit{output places} \text{ for } t_j$$

It must be noticed that $I(t_j)$ is a bag of P^ω but that $I_j(p_k)$ is the number of occurrences of the place p_k in the bag $I_j = I(t_j)$. This number corresponds to the weight of the arc connecting the place p_k to the transition t_j, or to the size of the arc bundle as defined by M. Hack [4].

2.2. *Execution rules for a Petri net*

(a) A *marking* M is a bag $M \in P^\omega$. $M(p)$ is the number of tokens contained by the place p for the marking M.

(b) A transition t of $\mathscr{P}$ is said to be *enabled* by the marking M iff:

$$I(t) \subseteq M$$

(c) A transition t of $\mathscr{P}$ is said to be *two-enabled* by a marking M iff:

$$\{I(t) + I(t)\} \subseteq M$$

(d) The *firing of a transition* t_i, enabled by a marking M_j, is made up of two uninterruptible steps:

. First $I_i(p_k)$ tokens are removed from each place p_k of P,
. Then $O_i(p_k)$ tokens are added to each place p_k of P.

If M_{j+1} is the marking such that:

$$M_{j+1} = M_j + O(t_i) - I(t_i),$$

then M_{j+1} is the marking obtained from M_j by the firing of t_i. We can write this:

$$M_j \xrightarrow{t_i} M_{j+1}$$

(e) Let σ be a finite sequence of transitions $t_i, t_{i+1}, \ldots, t_{i+k}$; σ is a *firing sequence* from M_i iff there exists markings $M_{i+1}, M_{i+2}, \ldots, M_{i+k}$ such that:

$$M_i \xrightarrow{t_i} M_{i+1},\ M_{i+1} \xrightarrow{t_{i+1}} M_{i+2}, \ldots, M_{i+k} \xrightarrow{t_{i+k}} M_{i+k+1}$$

We can write then:

$$M_i \xrightarrow{\sigma} M_{i+k+1}$$

(f) The *forward marking class* $\vec{M}_0$ is the set of markings that are reachable from M_0

$$M \in \vec{M}_0 \Leftrightarrow \exists\sigma; \qquad M_0 \xrightarrow{\sigma} M$$

2.3. *Consequence of the introduction of a two-step transition firing*

Let us consider an alphabet Σ and suppose that each transition of the Petri net $\mathscr{P}$ is labeled by a symbol of Σ. With each firing sequence σ of transitions, a string over Σ can be associated. The set of strings associated with all the firing sequences fireable between the initial marking M_0 and the final marking M_f is the computation sequence set of $\mathscr{P}$ between M_0 and M_f [9].

If the firings of the transitions are supposed instantaneous, a computation sequence set represents the behavior of a Petri net between two markings M_0 and M_f. If a firing time is associated with each transition, it is no longer true and there is no longer equivalence between a labeled Petri net and its computation sequence set.

2.4. *Concepts defined on Petri nets*

(a) Two transitions t_i and t_j of a Petri net $\mathscr{P}$ are *in conflict* iff there exists a marking M of $\vec{M}_0$ and a place p of P such that:

— t_i and t_j are enabled by M

— $I_i(p) + I_j(p) > M(p)$

(b) Two transitions t_i and t_j of a Petri net $\mathscr{P}$ are *parallel* iff there exists a marking M of $\vec{M}_0$ such that:

$$I(t_i) + I(t_j) \subseteq M$$

(c) A Petri net $\mathscr{P}$ is *bounded* iff there exists a positive integer $n_{\max}$ such that for every marking M of $\vec{M}_0$ and for every place p of P, $M(p) \leqslant n_{\max}$

(d) A Petri net $\mathscr{P}$ is *safe* iff it is bounded with $n_{\max} = 1$.

(e) A Petri net $\mathscr{P}$ is *live* iff for every transition t_i of T and for every marking M_j of $\vec{M}_0$ there exists a firing sequence σ_{ij} that can be fired from M_j and that contains t_i.

2.5. *Concept of block*

(a) Let us consider a Petri net with one and only one transition named *initial transition* (t_{ini}) and one and only one transition named *final transition* (t_{fin}). Such a Petri net is called a *block*.

(b) Let us consider the Petri net $\hat{\mathscr{P}}(\hat{P}, \hat{T}, \hat{M}_0, \hat{I}, \hat{O})$ obtained from a block $\mathscr{P}$ (P, T, M_0, I, O) by adding to it a place p_0 called the *idle place*, such that:

- p_0 has one and only one output transition and this transition is the initial transition t_{ini},
- p_0 has one and only one input transition and this transition is the final transition t_{fin},
- $\hat{M}_0 = M_0 + \{p_0\}$
  (M_0 is here considered as a bag defined over the domain $P \cup \{p_0\}$).

The Petri net $\hat{\mathscr{P}}$ is called the *associated Petri net* of the block $\mathscr{P}$.

Let $\hat{M}_i$ be a marking of the forward marking class of $\hat{\mathscr{P}}$ and M_i be the restriction of the bag $\hat{M}_i$ to the places of P:

$$\forall p \in P \qquad M_i(p) = \hat{M}_i(p) \qquad \text{and} \qquad M_i(p_0) = 0$$

The *set of bags* $\overline{M}_0$ will be the set of all the restrictions M_i of the markings $\hat{M}_i$ of the forward marking class of $\hat{\mathscr{P}}$.

(c) Then the following definitions can be given:

- The *block* $\mathscr{P}$ is said to be *bounded* iff the associated Petri net $\hat{\mathscr{P}}$ is bounded.
- The *block* $\mathscr{P}$ is said to be *safe* iff the associated Petri net $\hat{\mathscr{P}}$ is safe.
- The *block* $\mathscr{P}$ is said to be *live* iff the associated Petri net $\hat{\mathscr{P}}$ is live.

2.6. *Well-formed block*

A *block* $\mathscr{P}$ is said to be *well-formed* iff the associated Petri net $\hat{\mathscr{P}}$ is such that:

- $\hat{\mathscr{P}}$ is live
- $\hat{M}_0$ is the only marking of the forward marking class of $\hat{\mathscr{P}}$ such that the idle place is not empty
- The only transition enabled by $\hat{M}_0$ is the initial transition.

PROPERTY. *A well-formed block is necessarily a bounded block.*

Proof. The notion of "well-formed block" is essentially the same as the concept of "clean net" defined by M. Hack [5], therefore the proof is obvious.

Remark. A well-formed block is such that once activated (the initial transition fired) there always exists a firing sequence that is fireable and that contains the final transition. Furthermore after the firing of the final transition the marking of a well-formed block is necessarily the initial marking.

Intuitively it seems that such a block may be used to replace a transition in a Petri net. The following paragraph will give the necessary and sufficient condition for the resulting net to be bounded and live.

3. SUBSTITUTION OF A TRANSITION BY A BLOCK

3.1. DEFINITION. Let $\mathscr{P}$ be a Petri net (P, T, M_0, I, O) and t_i be a transition of T. Let $\mathscr{P}'$ be a block (P', T', M_0', I', O') with an initial transition t'_{ini} and a final transition t'_{fin}. It is supposed that $\mathscr{P}$ and $\mathscr{P}'$ are disjoint ($P \cap P' = \varnothing$ and $T \cap T' = \varnothing$). The result of the *substitution* of the transition t_i by the block $\mathscr{P}'$ is a Petri net $\mathscr{P}''$ $(P'', T'', M_0'', I'', O'')$ such that:

(a) $P'' = P \cup P'$

(b) $T'' = (T - \{t_i\}) \cup T'$

(c) $M_0'' = M_0 + M_0'$

(d) The functions I'' and O'' are defined in the following way:

. If $t_k \in T$ and $t_k \neq t_i$ then $\begin{cases} I''(t_k) = I(t_k) \\ O''(t_k) = O(t_k) \end{cases}$

. If $t'_k \in T'$ and if $t'_k \neq t'_{\text{ini}}$ and $t'_k \neq t'_{\text{fin}}$ then $\begin{cases} I''(t'_k) = I'(t'_k) \\ O''(t'_k) = O'(t'_k) \end{cases}$

. $I''(t'_{\text{ini}}) = I(t_i) + I'(t'_{\text{ini}})$

$O''(t'_{\text{ini}}) = O'(t'_{\text{ini}})$

. $I''(t'_{\text{fin}}) = I'(t'_{\text{fin}})$

$O''(t'_{\text{fin}}) = O(t_i) + O'(t'_{\text{fin}})$

Remark. The bags concerning the net $\mathscr{P}$ are elements of P^ω and those concerning the net $\mathscr{P}'$ are elements of P'^ω. As $P'' = P \cup P'$ all these bags can be considered as elements of P''^ω that allows to write the parts (c) and (d) of the Definition 3.1.

EXAMPLE. In Figure 1 an example of substitution is given. The transition t_3 of the net $\mathscr{P}$ is substituted by the block $\mathscr{P}'$. The result of this substitution is the net $\mathscr{P}''$.

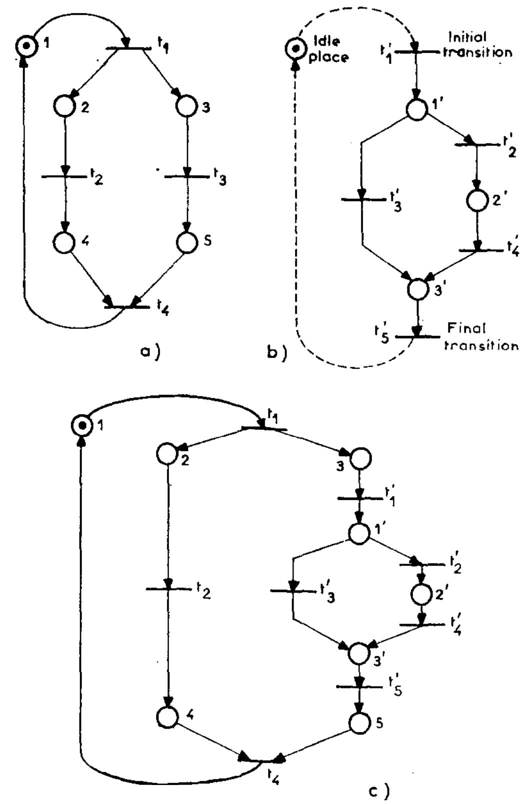

FIG. 1. Example of substitution. (a) Petri net $\mathscr{P}$. (b) Well-formed block $\mathscr{P}'$. (c) Petri net $\mathscr{P}''$.

The purpose of the substitution of transitions by blocks is to build up Petri nets that are bounded (or safe) and live. Thus only the case of the substitution by a well-formed block will be considered.

3.2. *Properties of the substitution of a transition by a well-formed block*

The markings of the Petri net $\mathscr{P}''$ are of the form:

$$M_i'' = M_j + M_k'$$

where M_j is a bag of P^ω but is not necessarily an element of $\vec{M}_0$, and M_k' is a bag of P'^ω but is not necessarily an element of $\vec{M}_0'$.

The following lemmas can then be given:

LEMMA 1. *Every marking $M_j + M_0'$ where M_j is an element of $\vec{M}_0$ and M_0' is the initial marking of $\mathscr{P}'$ is an element of $\vec{M}_0''$ (they are markings of the forward marking class of $\mathscr{P}''$).*

Proof. As $\mathscr{P}'$ is a well-formed block there exists at least one firing sequence σ' of transitions of $\mathscr{P}'$ beginning with t'_{ini} and finishing with t'_{fin} such that: $M'_0 \xrightarrow{\sigma'} M'_0$.

Thus any firing sequence σ of $\mathscr{P}$ can be transformed into a firing sequence σ'' of $\mathscr{P}''$ by replacing t_i by σ' whenever it appears in σ. It follows immediately that for any bag M_j of $\vec{M}_0$, $M_j + M'_0$ is a marking of $\vec{M}''_0$.

LEMMA 2. *If the transition t_i is two-enabled in $\mathscr{P}$ by no marking of $\vec{M}_0$ then the initial transition t'_{ini} of $\mathscr{P}'$, once fired in $\mathscr{P}''$ from a marking of the form $M_l + M'_0$ where $M_l \in \vec{M}_0$, cannot be fired again as long as the final transition t'_{fin} of $\mathscr{P}'$ has not also been fired.*

Proof. Let MI be the set of the markings of $\vec{M}_0$ enabling t_i, and MJ be the set of the bags of P^ω obtained from the markings of MI by subtracting from them the bag $I(t_i)$.

By construction of $\mathscr{P}''$ the firing of t'_{ini} will produce a marking of the form $M_j + M'_k$ where M_j is a bag of MJ and M'_k a bag of $\vec{M}'_0$. The only transitions enabled by these markings will be either transitions parallel to t_i in $\mathscr{P}$ or transitions of $\mathscr{P}'$. Thus as long as neither t'_{fin} nor t'_{ini} has been fired the markings of $\mathscr{P}''$ will be of the form $M_j + M'_k$.

Let's assume that such a marking enables t'_{ini}. This would imply that there exists a marking M_j which enables t_i in $\mathscr{P}$ and thus that there exists at least one marking of $\vec{M}_0$ such that t_i is two-enabled, which is inconsistent with the hypothesis.

LEMMA 3. *Let MJ be the set of all the bags of P^ω obtained from the markings of $\vec{M}_0$ which enables t_i in $\mathscr{P}$, by subtracting from them the bag $I(t_i)$. If t_i is not two-enabled by any marking of $\vec{M}_0$ in $\mathscr{P}$ then the forward marking class $\vec{M}''_0$ of $\mathscr{P}''$ is made up of the following bags*:

(a) $(M_l + M'_0) \in \vec{M}''_0 \qquad \forall M_l,\ M_l \in \vec{M}_0$

(b) $(M_j + M'_k) \in \vec{M}''_0 \qquad \forall M_j,\ M_j \in MJ$

$\forall M'_k,\ M'_k \in \vec{M}'_0$ and $M'_k \neq M'_0$

(c) no other bag belongs to $\vec{M}''_0$.

Proof. Part (a) derives from Lemma 1 and part (b) from Lemma 2. Part (c) derives from the fact that the only transitions enabled by the markings of the form $M_j + M'_k$ are either transitions parallel to t_i in $\mathscr{P}$ or transitions of $\mathscr{P}'$.

THEOREM 1. *Let t_i be a transition of a Petri net $\mathscr{P}$ and assume that this transition is not two-enabled by any marking of the forward marking class of $\mathscr{P}$. Let $\mathscr{P}'$ be a well-formed block and $\mathscr{P}''$ the result of the substitution of t_i by $\mathscr{P}'$. The pairs of parallel transitions of $\mathscr{P}''$ are the following exclusively*:

(a) *every pair of parallel transitions of $\mathscr{P}$ that does not contain t_i,*

(b) *every pair of parallel transitions of the Petri net associated with the block $\mathscr{P}'$,*

(c) *every pair made up of a transition of $\mathscr{P}'$ and a transition of $\mathscr{P}$ that is parallel to t_i in the Petri net $\mathscr{P}$.*

This theorem derives straightforwardly from the preceding Lemma.

THEOREM 2. *Let t_i, $\mathscr{P}$, $\mathscr{P}'$ and $\mathscr{P}''$ be as defined in the Theorem 1. The pairs of transitions of $\mathscr{P}''$ which are in conflict are the following exclusively:*

(a) *every pair of transitions of $\mathscr{P}$ that are in conflict, the transition t_i being replaced by the initial transition t'_{ini} of $\mathscr{P}'$,*

(b) *every pair of transitions of $\mathscr{P}'$ that are in conflict in the Petri net associated with $\mathscr{P}'$.*

The proof is derived directly from the Lemma 3.

EXAMPLE. Consider again the example of the Figure 1 and let $\varnothing$ be the empty bag. The forward marking class $\vec{M}_0$ of $\mathscr{P}$ is made up of the following bags:

$$\{1\}, \{2, 3\}, \{3, 4\}, \{2, 5\}, \{4, 5\}$$

The set of bags $\overline{M}'_0$ is:

$$\varnothing, \{1'\}, \{2'\}, \{3'\}$$

The transition t_i is the transition t_3. It is enabled by the markings $\{2, 3\}$ and $\{3, 4\}$ in the Petri net $\mathscr{P}$. Thus the set MI is made up of the bags $\{2, 3\}$ and $\{3, 4\}$ and the set MJ is made up of the bags $\{2\}$ and $\{4\}$.

The forward marking class $\vec{M}''_0$ of $\mathscr{P}''$ will be formed of the following sets of bags:

— on one hand the set:

$$\{1\}, \{2, 3\}, \{3, 4\}, \{2, 5\}, \{4, 5\}$$

corresponding to the part (a) of the Lemma 3,

— on the other hand the sets:

$$\{2, 1'\}, \{2, 2'\}, \{2, 3'\} \qquad \{4, 1'\}, \{4, 2'\}, \{4, 3'\}$$

corresponding to the part (b) of Lemma 3.

The only pair of parallel transitions of $\mathscr{P}$ is the pair (t_2, t_3) (enabled by the marking $\{2, 3\}$). No pair of parallel transition exists in $\mathscr{P}'$, thus the pairs of parallel transitions of $\mathscr{P}''$ are:

$$(t_2, t'_1), \quad (t_2, t'_2), \quad (t_2, t'_3), \quad (t_2, t'_4) \quad \text{and} \quad (t_2, t'_5).$$

There are no pairs of transitions in conflict in $\mathscr{P}$ and the only pair in conflict in $\mathscr{P}'$ is (t'_2, t'_3). Thus the only pair of transitions in conflict in $\mathscr{P}''$ is (t'_2, t'_3).

THEOREM 3. *Let t_i be a transition of a Petri net $\mathscr{P}$ that is not two-enabled by any marking of the forward marking class of $\mathscr{P}$. Let $\mathscr{P}'$ be a well-formed block and $\mathscr{P}''$ the result of the substitution of t_i by $\mathscr{P}'$. The three following results can be given:*

(a) *The Petri net $\mathscr{P}''$ is bounded iff the Petri net $\mathscr{P}$ is bounded.*

(b) *The Petri net $\mathscr{P}''$ is safe iff the Petri net $\mathscr{P}$ and the block $\mathscr{P}'$ are safe.*

(c) *The Petri net $\mathscr{P}''$ is live iff the Petri net $\mathscr{P}$ is live.*

Proof. Parts (a) and (b) of Theorem 3 are derived directly from the Lemma 3 taking into account that a necessary and sufficient condition for a Petri net to be bounded is that its forward marking class is finite. In the case of part (a) it is also necessary to utilize Lemma 1.

The sufficient condition of part (c) is derived from the fact that any firing sequence σ fireable in the Petri net $\mathscr{P}$ can be transformed into a firing sequence in $\mathscr{P}''$ by replacing, each time it appears, the transition t_i by a sequence σ' of transitions of $\mathscr{P}'$ fireable in $\mathscr{P}'$ from the marking M_0' and finishing by t'_{fin}.

In order to prove the necessary condition it should be noticed that if two transitions t_1 and t_2 are parallel for a marking M_1, then the two sequences $t_1; t_2$ and $t_2; t_1$ will both be fireable from M_1 and if:

$$M_1 \xrightarrow{t_1;t_2} M_2$$

then

$$M_1 \xrightarrow{t_2;t_1} M_2$$

Let σ'' be a sequence of transitions fireable in the Petri net $\mathscr{P}''$ and containing t'_{ini} and t'_{fin}. Every transition of $\mathscr{P}$ appearing in σ'' between t'_{ini} and t'_{fin} is necessarily parallel to every transition of $\mathscr{P}'$ in $\mathscr{P}''$ (according to Lemma 3). By commutations of pairs of parallel transitions in $\mathscr{P}'$, it is always possible to transform σ'' in such a way that no transition of $\mathscr{P}$ appears between t'_{ini} and t'_{fin}. By replacing the sequence of transitions of $\mathscr{P}'$ beginning with t'_{ini} and finishing with t'_{fin} it is then possible to obtain a sequence σ fireable in the net $\mathscr{P}$. It follows that if the Petri net $\mathscr{P}''$ is live, then the Petri net $\mathscr{P}$ is necessarily live.

Remark. It may be asked whether the restriction concerning the transition t_i (It cannot be two-enabled by any marking of the forward marking class of $\mathscr{P}$) is necessary. The example given by Figure 2, where the transition t_2 is substituted by the well-formed block $\mathscr{P}'$, shows that this restriction is necessary for the case (c) of the Theorem 3. In fact the transition t_2 is two-enabled by the marking $\{2, 2\}$, and the Petri net $\mathscr{P}''$ is not live because the sequence:

$$t_1\ ;\quad t'_{\text{ini}}\ ;\quad t'_2\ ;\quad t'_{\text{ini}}\ ;\quad t'_3\ ;\quad t'_{\text{fin}}$$

fireable from the initial marking $\{1\}$ of $\mathscr{P}''$ produces the marking $\{2', 5', 3\}$ for which no transition of $\mathscr{P}''$ is enabled.

3.3. *Description and analysis of a Petri net by stepwise refinements*

Theorems 1, 2 and 3 show that if the detailed description of the operation symbolized by the transition t_i is a well-formed block, and if the transition t_i is two-enabled by no marking of $\mathscr{P}$, then it is not necessary to do the analysis of the global Petri net $\mathscr{P}''$ because all its properties can be deduced directly from those of the Petri nets $\mathscr{P}$ and $\mathscr{P}'$. This proves that it is possible to describe a system with a Petri net and to analize it by stepwise refinements. In the case of a parallel system such a procedure is not possible if the repre-

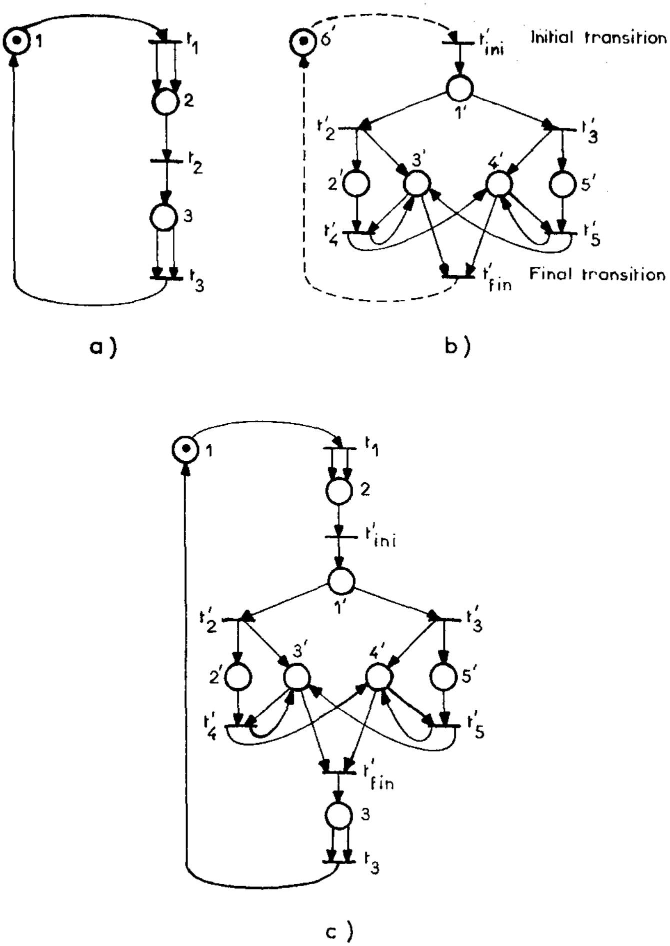

FIG. 2. Example of the substitution of a transition that can be two-enabled. (a) Petri net $\mathscr{P}$. (b) Well-formed block $\mathscr{P}'$. (c) Resulting Petri net $\mathscr{P}''$.

sentation model utilized is a state machine. In fact the state machine model requires the knowledge of the global state of the system when in the case of a Petri net only partial or local states are utilized.

4. *D*-PETRI NETS

Theorems 1, 2 and 3 form, in a certain sense, a generalization of the results of J. Bruno and S. M. Altman [1]. For example consider the Petri nets of Figure 3. They can be called "sequence block", "if-then-else block", "do-while block", "fork-join block". Although they are not minimal and they introduce the parallelism, they are similar to the "*D*-chart constructs" [7]. They can thus be called the *D-blocks* and it can be easily proved that they are safe well-formed blocks.

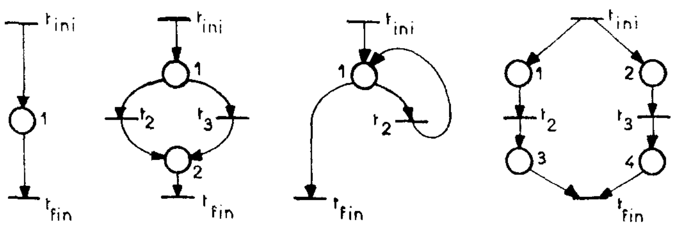

FIG. 3. *D*-blocks. (a) "Sequence" block; (b) "if-then-else" block; (c) "do-while" block; (d) "fork-join" block.

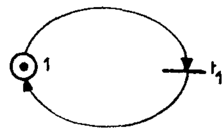

FIG. 4. Elementary Petri net.

Consider now the Petri-net of Figure 4. This Petri net, called the *elementary Petri net*, is safe and live.

The *D-Petri nets* can then be defined recursively the following way:

DEFINITION. (1) The elementary Petri net is a *D*-Petri net.

(2) Any Petri net, obtained from a *D*-Petri net by substitution of a transition by a *D*-block, is a *D*-Petri net.

As in a safe Petri net no transition can be two-enabled, the following property can be given:

PROPERTY. *Any D-Petri net is live and safe.*

5. CONCLUSION

The analysis of a Petri net is currently quite cumbersome. When pratical applications are concerned it is necessary to avoid the analysis of large complex Petri nets and thus it seems convenient to proceed by stepwise refinements. It has been shown in this paper that such procedures are possible when it is assumed that the firings of the transitions are not instantaneous. Then complex operations can be associated with the transitions of a Petri net. A gross abstract description of the system can be done. The level of representation can thus be modified progressively by replacing transitions by blocks. If the initial Petri net has the desired properties and if only well-formed blocks are utilized, the correct behavior of the final, complex Petri net is ensured and it is not necessary to analyze it. Such a procedure is an extension of some of the principles of the structured programming to the description and the validation-oriented analysis of parallel systems.

REFERENCES

1. J. BRUNO AND S. M. ALTMAN, A theory of asynchronous control networks, *IEEE Trans. Comput.* **C-20** (1971), 629–638.
2. F. COMMONER, A. W. HOLT, S. EVEN, AND A. PNUELI, Marked directed graphs, *J. Comput. System Sci.* **5** (1971), 511–523.
3. M. HACK, "Analysis of Production Schemata by Petri Nets," Master of Science Thesis, M.I.T., 1972.
4. M. HACK, "Decidability Questions for Petri Nets," Ph.D. Thesis, M.I.T., 1975.
5. M. HACK, "Petri Net Languages," TR 159, M.I.T., Laboratory for Computer Science, March 1976.
6. R. M. KELLER, Formal verification of parallel programs, *Comm. ACM* **19** (1976), 371–384.
7. S. R. KOSARAJU, Analysis of structured programs, *J. Comput. System Sci.* **9** (1974), 232–255.
8. S. S. PATIL AND J. B. DENNIS, The description and realization of digital systems, *RAIRO* (Février 1973), 55–69.
9. J. L. PETERSON, Computation sequence sets, *J. Comput. System Sci.* **13** (1976), 1–24.
10. C. A. PETRI, Interpretation of net theory, International Symposium on Mathematical Foundations of Computer Sciences at Marianske Lazne, 1975.
11. C. RAMCHANDANI, Analysis of asynchronous systems by Petri nets, Ph.D. Thesis, M.I.T., 1973.
12. R. VALETTE AND R. PRAJOUX, A model for parallel control systems and communication systems, Conference on Information Sciences and Systems, The Johns Hopkins University, 1976.
13. R. VALETTE, Sur la description, l'analyse et la validation des systèmes de commande parallèles, Thèse de Doctorat d'État, Université Paul Sabatier de Toulouse, 1976.