Interacting Multiple Model Algorithm with the Unscented Particle Filter (UPF)

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Abstract: Combining interacting multiple model (IMM) and unscented particle filter (UPF), a new multiple model filtering algorithm is presented. Multiple models can be adapted to targets' high maneuvering. Particle filter can be used to deal with the nonlinear or non-Gaussian problems and the unscented Kalman filter (UKF) can improve the approximate accuracy. Compared with other interacting multiple model algorithms in the simulations, the results demonstrate the validity of the new filtering method.

Key words: interacting multiple model; UPF; UKF; nonlinear/non-Gaussian

引入 UPF 的交互式多模型的算法. 邓小龙, 谢剑英, 倪宏伟. 中国航空学报(英文版), 2005, 18(4): 366-371.

摘　要: 融合交互式多模型和 UPF(the unscented particle filter), 提出了一种新的多模型滤波算法。多模型结构能适应目标高度机动, 粒子滤波能处理非线性、非高斯问题, 而 UKF(the unscented Kalman filter) 可以提高估计精度。与其它交互式多模型算法进行了比较, 试验仿真结果证实了新滤波算法的有效性。

关键词: 交互式多模型; UPF; UKF; 非线性/非高斯
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In the fields of navigation, aviation and manoeuvring target tracking, *etc*, to precisely predict and estimate the motion information of targets, such as ships or planes, the exact models of targets should be designed. The interacting multiple model (IMM) algorithm selects multiple parallel running models and then automatically switches between the models according to the Markov transition probabilities matrix. For IMM obtains the tradeoff between the computational precision and cost, it is the most widely used multiple model estimation algorithm[1]. The keys for IMM are the selection of multiple models and filtering algorithm. Multiple models often contain a nonlinear maneuvering model, such as CT (coordinated turn), to cover targets' maneuvering. The filtering algorithm is often the Kalman filter (KF) or the extended Kalman filter (EKF). In Ref.[2], the EKF and an adaptive model set are combined. And the superiority of this method is also verified. However, on different cases, selecting responding suitable model set is a little hard. EKF simply linearizes the nonlinear functions to the first order by using the Taylor series expansions. Thus, it may degrade the nonlinear filtering performance. To reduce approximate errors by the EKF, Julier *et al* put forward the unscented Kalman filter (UKF)[3]. The UKF uses the deterministic sample points to completely capture the statistics of the Gaussian random variable accurately to the second order (Taylor expansions) for any nonlinear systems. But it does not apply to general non-Gaussian cases.

Recently, particle filter (PF) is presented to represent the required posterior density function

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(PDF) by a set of random sampled particles. Thus, it is not affected by the nonlinear and non-Gaussian limitations[4]. McGinnity *et al*[5] firstly combined the multiple model and particle filter. But they augmented the mode variable to the state vector and made the number of particles in every model be proportional to the mode probability. This has no control on the number of particles in each mode and leads to that the number of particles is very small in a low mode-probability model, which loses the essence of the multiple models. The multiple model particle filter in Ref. [6] has the fixed number of particles, but has no interactions among the particles in models and it is the static multiple mode algorithm.

In this paper, IMM and UPF (particle filter with the UKF proposal)[7] are firstly combined. The fixed multiple models are adopted and the UPF filtering method is used in each model. The number of particles in each mode is fixed and independent of the mode probability to fully represent the multiple models. Particles in every model are updated by UKF, and then are interacted. Lastly, particles are resampled to reduce the degeneracy. Thus, the developed new dynamical filtering algorithm is not affected by the nonlinear or non-Gaussian limitations and has high estimation accuracy. The experiments with maneuvering target tracking show that the new filtering method is valid.

1 Unscented Particle Filter (UPF)

1.1 Particle filter

Based on Bayesian filtering, particle filter represents the PDF by a set of random sample particles with associated weights. Since particle filter was firstly applied to practical engineering field[4], it has been one of the hottest issues. For example, particle filter is applied to fault diagnosis in Ref. [8] and the Gauss-Heimite particle filter is presented in Ref. [9], *etc*.

Suppose $\boldsymbol{X}_k = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_k\}$ and $\boldsymbol{Z}_k = \{z_1, \cdots, z_k\}$ represent the state sequences and measurements up to time k respectively. Bayesian recursive filtering includes prediction and updating:

(1) Prediction

$$p(\boldsymbol{x}_k \mid \boldsymbol{Z}_{k-1}) = \int p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1}) \cdot p(\boldsymbol{x}_{k-1} \mid \boldsymbol{Z}_{k-1})\,\mathrm{d}\boldsymbol{x}_{k-1} \tag{1}$$

(2) Updating

$$p(\boldsymbol{x}_k \mid \boldsymbol{Z}_k) = \frac{p(z_k \mid \boldsymbol{x}_k)\,p(\boldsymbol{x}_k \mid \boldsymbol{Z}_{k-1})}{p(z_k \mid \boldsymbol{Z}_{k-1})} \tag{2}$$

The analytical solutions to above integrals are hard to be acquired. However, PDF can be approximated by the discrete randomly sampled particles. It is often not possible to sample directly from the PDF. One can approximate PDF by drawing from a known proposal distribution $q(\boldsymbol{X}_k \mid \boldsymbol{Z}_k)$, which is easy to sample. From the Large Number Theorem, the discrete particles randomly sampled from the proposal distribution would be convergent to the true distribution. The weight, $\boldsymbol{w}_k$, is defined as

$$\boldsymbol{w}_k = p(\boldsymbol{x}_k \mid \boldsymbol{Z}_k)\,p(\boldsymbol{Z}_k)/q(\boldsymbol{X}_k \mid \boldsymbol{Z}_k) \tag{3}$$

Thus, the weight can be evaluated in a recursive form[4]:

$$\boldsymbol{w}_k = \boldsymbol{w}_{k-1}\,p(z_k \mid \boldsymbol{x}_k)\,p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1})/ q(\boldsymbol{x}_k \mid \boldsymbol{X}_{k-1}, \boldsymbol{Z}_k) \tag{4}$$

After a few iterations of prediction, updating and evaluating weights, it may leads to the degeneracy, that is, all but one particle would probably have the negligible weights. To reduce the effect of degeneracy, Gordon *et al* introduced the resampling step[4]. The resampling step is to evaluate weights of the particles and resample the particles, which eliminates particles with small weights and multiplies particles with large weights. Thus, prediction, updating, evaluating weights and resampling particles constitute the general particle filter algorithm.

Particle filter relies on the importance sampling and thus requires the design of proposal distributions can approximate the PDF as well as possible. Several suboptimal proposal distributions are put forward, including the prior proposal[4], the EKF proposal[10], the UKF proposal[7], *etc*. When the proposal distribution equals to the prior distribution, that is, $q(\boldsymbol{x}_k \mid \boldsymbol{X}_{k-1}, \boldsymbol{Z}_k) = p(\boldsymbol{x}_k \mid \boldsymbol{x}_{k-1})$, it is named as prior proposal. The prior proposal has no consideration of the latest measurements and simplifies evaluating weights into evaluating likelihoods, $w_k = w_{k-1} p(z_k | x_k)$. The EKF proposal uses the EKF to update the states in proposal distribution and it has relative lage approximate errors. In Ref. [7], the UKF is used as proposal and the UPF algorithm with high estimation accuracy is developed and applied to state estimation.

1.2 UKF (the unscented Kalman filter)

To address some approximate issues of the EKF, the UKF is presented[3]. The UKF uses a minimal set of deterministically chosen sample points (sigma points) to completely capture the posterior mean and covariance of the Gaussian random variable accurately to the second-order (Taylor expansions) for any nonlinear system. Main steps of the UKF are as follows:

* Calculate sigma points

$$\chi_{k-1} = [\hat{\boldsymbol{x}}_{k-1} \quad \hat{\boldsymbol{x}}_{k-1} + \rho_{k-1} \quad \hat{\boldsymbol{x}}_{k-1} - \rho_{k-1}] \quad (5)$$

* Prediction

$$\chi_{k/k-1} = F(\chi_{k-1}) \quad (6)$$

$$\hat{\boldsymbol{x}}_{k/k-1} = \sum_{i=0}^{2n} (W_{mi} \chi_{i,k/k-1}) \quad (7)$$

$$\boldsymbol{P}_{k/k-1} = \sum_{i=0}^{2n} (W_{ci} [\chi_{i,k/k-1} - \hat{\boldsymbol{x}}_{k/k-1}][\bullet]^{\mathrm{T}}) + \boldsymbol{Q} \quad (8)$$

$$\eta_{k/k-1} = H(\chi_{k/k-1}) \quad (9)$$

$$\hat{\boldsymbol{z}}_{k/k-1} = \sum_{i=0}^{2n} (W_{mi} \eta_{i,k/k-1}) \quad (10)$$

* Updating

$$\boldsymbol{P}_{zz} = \sum_{i=0}^{2n} (W_{ci} [\eta_{i,k/k-1} - \hat{\boldsymbol{z}}_{k/k-1}][\bullet]^{\mathrm{T}}) + \boldsymbol{R} \quad (11)$$

$$\boldsymbol{P}_{xz} = \sum_{i=0}^{2n} (W_{ci} [\chi_{k/k-1} - \hat{\boldsymbol{x}}_{k/k-1}] \bullet [\eta_{i,k/k-1} - \hat{\boldsymbol{z}}_{k/k-1}]^{\mathrm{T}}) \quad (12)$$

$$\boldsymbol{K}_k = \boldsymbol{P}_{xz} P_{zz}^{-1} \quad (13)$$

$$\bar{\boldsymbol{x}}_k = \boldsymbol{x}_{k/k-1} + \boldsymbol{K}_k (z_k - \hat{\boldsymbol{z}}_{k/k-1}) \quad (14)$$

$$\boldsymbol{P}_k = \boldsymbol{P}_k^- - \boldsymbol{K}_k \boldsymbol{P}_{zz} \boldsymbol{K}_k^{\mathrm{T}} \quad (15)$$

where $F(\bullet)$ and $H(\bullet)$ denotes the transition function and measurement function respectively. $\rho = \sqrt{(n+\lambda)\boldsymbol{P}_x}$, W_{mi} is the weight coefficient in calculating the mean and W_{ci} is the covariance weight coefficient. $\{\chi_i, i = 0, \cdots, 2n\}$ are sigma points, n is the state dimension of $\boldsymbol{x}$, $\boldsymbol{P}_x$ is the covariance matrix of $\boldsymbol{x}$, and λ is scaling factor. The values of these parameters are set referring to Ref. [7]. $\boldsymbol{Q}$ and $\boldsymbol{R}$ is the covariance of process noise, $\boldsymbol{v}(k)$, and measurement noise, $\boldsymbol{r}(k)$, respectively.

In this paper, particle filter with the UKF proposal is introduced into IMM algorithm.

2 IMM with the UPF

2.1 Interacting multiple model (IMM)

IMM is an adaptive estimation method. It designs a set of models, each of which matches one specific mode of the system. Suppose the mode j in the IMM has the following state transition equation and measurement equation:

$$\boldsymbol{x}_j(k+1) = \Phi_j(k)\boldsymbol{x}_j(k) + \boldsymbol{G}_j(k)\boldsymbol{v}_j(k) \quad (16)$$

$$\boldsymbol{z}(k+1) = \boldsymbol{h}_j(k+1)\boldsymbol{x}_j(k+1) + \boldsymbol{r}_j(k+1) \quad (17)$$

where $\Phi_j(k)$ is the state transition matrix, and $\boldsymbol{h}_j(k+1)$ is the measurement matrix. Suppose the Markov mode transition probability matrix is $\boldsymbol{T} = \{\pi_{ij}, \ i, j = 1, 2, \cdots, m\}$ and the mode probability matrix is $\boldsymbol{U} = \{u_i^n(k), \ i = 1, 2, \cdots, m, \ n = 1, 2, \cdots, N\}$, where m is the number of modes. IMM has four main algorithmic steps: interaction, filtering, updating and combination.

2.2 IMM with the UPF

van der Merwe R. *et al* firstly put forward UPF in Ref. [7]. They effectively combined high approxiamte accuracy of UKF and optimizing function of particle filter, and applied it to one-dimensional state estimation. To improve the estimation accuracy of the IMM, the UKF algorithm is introduced and the nonlinear maneuvering model, CT model, and constant velocity (CV) model are used in this paper. The fixed multiple models are relatively easy to be implemented and they are automatically softly switched according to Markov transition probability matrix. They can nearly cover the non-manoeuvering and maneuvering of targets.

Moreover, for the dimensions of the state composed by target's position and velocity, *etc* are not high, the UKF introduced into IMM would be not affected by the dimension of the state.

In this paper, UPF is firstly and successfully introduced into IMM. Firstly, particles in each model are randomly sampled from the prior. After that, they are interacted and updated by model-matched UKF instead of EKF. Then they are resampled to be optimized. For the residual resampling has relatively high computational efficiency and low calculated covariance[11], it is adopted. Lastly, particles are combined. These steps are recursively done to propagate and update particles. Thus, a dynamical adaptive interacting multiple model algorithm is developed, which has high estimation accuracy and is not affected by nonlinear or non-Gaussian problems. Prediction and estimation accuracy are effectively improved through the fixed multiple models and the UPF filtering algorithm.

One recursive cycle of the new multiple model algorithm is shown as Fig. 1. The IMM with the UPF has the following main steps:

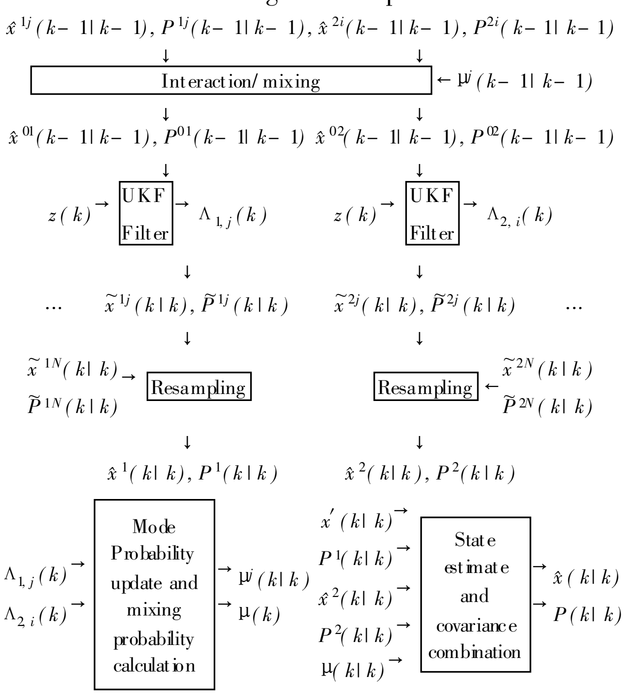

Fig. 1 One recursive cycle of IMM with the UPF (i, j is the sequence number of particles)

1) Randomly sample the particles. At time step k, particles in each mode are randomly sampled according to the mean and covariance of the state. Let the state value and covariance of sampled N particle from the m modes be $\{x_i^n(k|k), P_i^n(k|k), i=1, \cdots, m, n=1, \cdots, N\}$.

2) Mixing. Mix the corresponding particles from all the modes.

$$x_j^n(k \mid k) = \sum_{i=1}^{m} x_i^n(k \mid k) u_{i|j}(k \mid k) \tag{18}$$

$$P_j^n(k \mid k) = \sum_{i=1}^{m} u_{i|j}(k \mid k)\{P_j^n(k \mid k) + [x_i^n(k \mid k) - x_i^n(k \mid k)][.]^{\mathrm{T}}\} \tag{19}$$

where $u_{i|j}^n(k|k) = \frac{1}{\bar{c}_j^n}\pi_{ij} u_j^n(k)$,

$$\bar{c}_j^n = \sum_{i=1}^{m} \pi_{ij} u_j^n(k),\ j = 1, \cdots, m,\ n = 1, \cdots, N$$

3) Model-matched filtering with the UKF. The particles with $\{x_j^n(k|k), P_j^n(k|k), n=1, \cdots, N\}$ are filtered through the jth mode with the UKF. The UKF is used to update the states at time step $k+1$, including the state $x_j^n(k+1|k+1)$ and its covariance $P_j^n(k+1|k+1)$, the likelihood $\Lambda_j^n(k+1)$ and the importance weights of the particles, $w_j^n(k+1)$ *etc*.

4) Resampling step. Evaluate the importance weights of the particles in each mode to resample particles, which produce a new set of discrete optimized particles with the same weights.

5) Mode probability update.

$$u_j^n(k+1) = \frac{1}{c^n}\Lambda_j^n(k+1)\bar{c}_j^n \tag{20}$$

where $c^n = \sum_{j=1}^{m} \Lambda_j^n(k+1)\bar{c}_j^n$.

6) Combination. Combine the corresponding particles in modes and then sum all the particles with the weights to obtain the mean and covariance of the state, $\{x(k+1|k+1), P(k|k)\}$, at $k+1$ time step.

$$x^n(k+1 \mid k+1) = \sum_{j=1}^{m} x_j^n(k+1 \mid k+1) \cdot u_j^n(k+1) \tag{21}$$

$$x(k+1 \mid k+1) = \sum_{n=1}^{N} x^n(k+1 \mid k+1)/N \tag{22}$$

3 Simulations

Suppose the plane is flying in the plane of the coordinate X and Y, the observer can only measure the position values of the target at the sampling interval $T = 1$ s. The tracking scenario is set as follows: From 0 to 60 s, the target starts to fly in level constant velocity -0.3 km/s at the initial position (40.2, 40.2) km. From 61 s to 85 s, the target makes the left turn with a centripetal acceleration of 30 m/s^2. From 86 s to 135 s, the target moves with constant velocity. From 136 s to 150 s, the target makes a right turn with a centripetal acceleration of 40 m/s^2. Finally, the target resumes the constant velocity motion until 195 s.

The IMM includes two models: the first model is CT model, and the second model is CV model. The state vector is $[\xi, \dot{\xi}, \eta, \dot{\eta}, w]^{\mathrm{T}}$, where ξ and η are the position value in the coordinates X and Y respectively, and $\dot{\xi}$ and $\dot{\eta}$ are the velocity values, and w is the turning velocity. And it has

$$\boldsymbol{T} = [\pi_{ij}] = \begin{bmatrix} 0.98 & 0.02 \\ 0.02 & 0.98 \end{bmatrix}, \boldsymbol{h}(k+1) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad u_1(0) = u_2(0) = 0.5,$$

$\boldsymbol{Q}_{\mathrm{CV}} = (0.004)^2 \boldsymbol{I}_3$, $\boldsymbol{Q}_{\mathrm{CT}} = (0.001)^2 \boldsymbol{I}_3$, $\boldsymbol{R}_{\mathrm{CV}} = \boldsymbol{R}_{\mathrm{CT}} = \sigma_R^2 = 0.01 \boldsymbol{I}_2$ km. $\boldsymbol{I}_m$ denotes $m \times m$ identity matrix. In the simulations, the number of particles in each model is $N = 200$, and the times of Monte Carlo runs is 100. The initial state value and the covariance are obtained from two point filtering method. Based on IMM, the general IMM algorithm with the EKF (IMM), particle filter with the EKF proposal (IMM-PF) and particle filter with the UKF proposal (IMM-UPF) are compared in this paper. Fig. 2 and Fig. 3 show the root mean square (RMS) position errors of the filtering methods in the coordinate X and Y respectively. Fig. 4 and Fig. 5 show the root mean square (RMS) velocity errors of the filtering methods in the coordinate X and Y respectively.

The average RMS position, velocity error and complexity from various filtering methods are shown as Table 1. As seen from the figures and table, although IMM-UPF has a little higher complexity, it has the minimum RMS position and velocity errors. Especially when the target starts to manoeuvre, IMM-UPF can quickly respond to it. But IMM obviously lags behind.

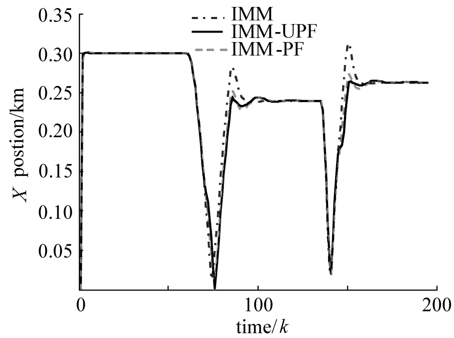

Fig. 2 RMS position errors in the coordinate X

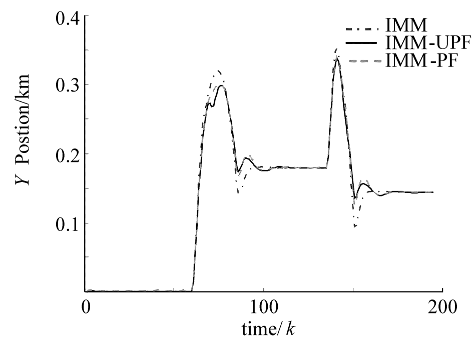

Fig. 3 RMS position errors in the coordinate Y

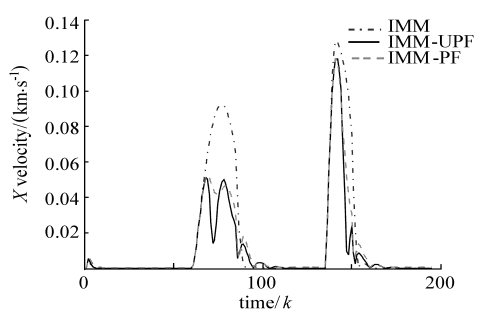

Fig. 4 RMS velocity errors in the coordinate X

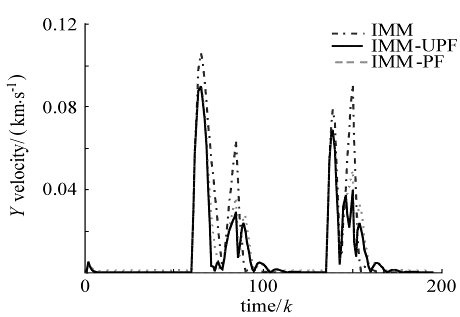

Fig. 5 RMS velocity errors in the coordinate Y

Table 1 RMS position and velocity errors in the coordinate X and Y respectively and complexity from various filtering methods (n is the dimension of the state, N is the number of particles)

		IMM	IMM-PF	IMM-UPF
Complexity		$O(n^3)$	$O(Nn^3)$	$O(Nn^4)$
Position RMSE	X	0.24943	0.24564	0.2453
	Y	0.01702	0.01225	0.01029
Velocity RMSE	X	0.13102	0.13054	0.12937
	Y	0.01187	0.01042	0.00874

4 Conclusions

IMM is the most widely used multiple model algorithm. Particle filter is one of the hottest filtering methods to deal with nonlinear, non-Gaussian problems. And UKF often has higher estimation accuracy than EKF. The IMM, particle filter, UKF and other techniques including the resampling step, etc are effectively combined to develop a new multiple model filtering method. The experiments with the manoeuvring target tracking confirm that the new algorithm is valid.

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