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The risk element transmission theory research of multi-objective risk-time-cost trade-off☆

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ABSTRACT

Risk management project is an important aspect of general project risk element transmission theory. To solve the multi-objective time-cost trade-off problem considering the risk elements effectively, this paper establishes an analytical model for multi-objective risk-time-cost trade-off problem based on general project risk element transmission theory. We divide risk elements into discrete model and continuous model to be discussed separately, and the two models for multi-objective risk-time-cost trade-off problem are established by taking Markov dynamic PERT network into classical PERT network. Thus, we combine Radial Basis Function (RBF) neural network to solve the discrete model of the problem. Finally, a practical example illustrates the effectiveness of the algorithm.

1. Introduction

The time-cost trade-off problem (TCTP) is the most important content in the project management. Although many scholars did a lot of research on the problem and got many achievements, the TCTP considering the risk elements has not been solved effectively. In the TCTP, the objective is to determine the duration of each activity in order to achieve the minimum total costs of the project. Studies on TCTP have been done using various kinds of cost functions such as linear [1,2], discrete [3], convex [4,5], and concave [6] and so on. From the above we can see, they all concerned the complex time planning, but comparing with the real situation, there are two main problems. First, there are no model considering the risk elements on TCTP; Second, how to get the optimization about the risk-time-cost when the network is random.

In order to resolve the above problems, this paper improved the model in the paper [9] based on the general project risk element transmission theory [7,8] and get the multi-objective risk-time-cost trade-off problem(MRTCTP) model when risk elements considering in dynamic PERT network. Thus, this paper gets the relationship of the resource allocation and the risk elements by combining the RBF neural networks and putting the multi-objective risk-time-cost programming problem into the multi-objective cost, expectation, variance, risk-time probability programming.

The paper is organized as follows. Section 2 presents the analytical model of the multi-objective risk-time-cost trade-off problem (MRTCTP), and gives the definition of the Markov dynamic PERT network. Section 3 presents the neural network analytical algorithm of MRTCTP. An empirical example is presented in Section 4. Finally, Section 5 concludes the paper.

2. The model of multi-objective risk-time-cost trade-off problem

2.1. Raising multi-objective risk-time-cost trade-off problem

In this section, we combine multi-objective time-cost trade-off problem and raise the risk-time-cost trade-off problems considering risk element. In practical projects, We use CPM, PERT [10] network technology to get the project completion

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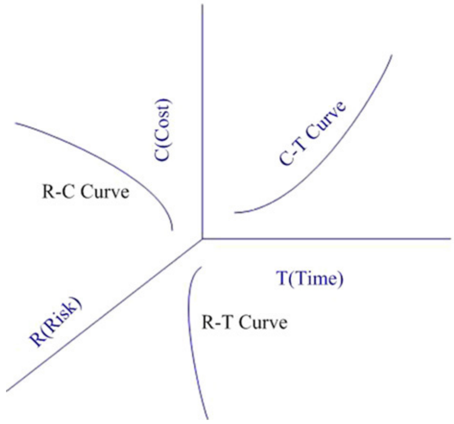

Fig. 1. The three-dimensional structural model of multi-objective risk-time-cost trade-off problem.

time and the corresponding costs. And there are the following balance problems: the cost of the project is not least or within expected expenditures when seeking to the shortest period. when the cost within expected expenditures, the time is not within expected time. The mathematical expression of the problem expresses as follows:

T is the completion time, T^* is the shortest completion time, the corresponding cost is C'. C is the total completion costs of project, C^* is the least completion costs, the corresponding completion time is T', then we can get:

When $C' \leq C$, T^* must be the best solution, that is the shortest completion time.

When $C' > C$, set $\Delta C = C' - C$, when ΔC becomes smaller, T^* becomes closer to the best solution.

In actual project, we often encounter the $C' > C$ situation. Therefore, our aim is to enable ΔC decreases, and getting the entire project satisfactory solution.

We can understand the risk-time-cost trade-off problems well with the introducing of the multi-objective time-cost trade-off problems. That is time-cost trade-off considering the influence of risk element. Here he risk element is random variables, its distribution can be discrete or continuous. We build the three-dimensional structural model of risk-time-cost trade-off problems referencing the three-dimensional structural model of general project risk element transmission theory. As shown in Fig. 1.

From the figure we can see that risk time and cost are increasing mutual functions relations. According to the actual project, project resource allocations will affect the time and the cost. Allocations and the time are decreasing functions. Allocations and the cost are increasing functions. In this way we can take the risk impaction of time and cost into the allocation of resources impaction of time and cost. Eventually we can get the analytical model of multi-objective risk-time-cost trade-off problem combining the literature [9].

2.2. Definition of Markov dynamic PERT network

Dynamic PERT network can be described as a queuing system network. Each network will be built as a function of the corresponding service. From queuing theory, arrival rate of the service subjects to the parameters of a Poisson distribution. The desk assumption is that the number of systems in each service is limited or one and the service time is exponentially distributed. Thus, for each joint activities, it forms a queuing system ($M/M/..Model$), and Azaron et al. [9,11] take this dynamic PERT network into classical PERT network. Concrete steps are as follows :

Step 1. Compute the density function of the time spent in each service station.

Step 1.1. If there is one server in the service station settled in the ith node,then the density function of time spent in this ($M/M/1$) queuing system is $w_i(t) = (\mu_i - \lambda)e^{-(\mu_i-\lambda)t}$, $t > 0$, where λ and μ_i are the generation rate of new projects and the service rate of this queuing system, respectively. Therefore, the distribution of time spent in this service station would be exponential with parameter $\mu_i - \lambda$.

Step 1.2. If there are infinite number of servers in the service station settled in the ith node, then the time spent in this ($M/M/\infty$)queuing system would be exponentially distributed with parameter μ_i, because there is no queue.

Step 2. Transform the dynamic PERT network into an equivalent classical PERT network.

Step 2.1. Replace each node with a stochastic arc (activity) whose length is equal to the time spent in the particular service station.

Let us explain how to replace node k in the network of queues with a stochastic activity. Assume that $b_1, b_2, \ldots, b_m$ are the incoming arcs to this node and $d_1, d_2, \ldots, d_n$ are the outgoing arcs from it. Then, we substitute this node by activity k', k'', whose length is equal to the time spent in the corresponding queuing system. Furthermore, all arcs b_i for $i = 1, 2, \ldots, n$ end up with k' while all arcs d_j for $j = 1, 2, \ldots, m$ start from node k''.The indicated process is opposite of the absorption an edge, see [12] for more details.

Step 2.2. Eliminate all arcs with zero length.

*Step*3. Compute the distribution function of the longest path length in the classical PERT network with exponentially distributed activity durations obtained in Step 2.2. We use the method of Kulkarni and Adlakha [13] in this step, because this method is an analytical one, simple, easy to implement on a computer and computationally stable.

Azaron et al. has given definitions to the dynamic PERT network and see [9] for more details. On the basis of this, we build the multi-objective risk-time-cost trade-off problem model by transforming the dynamic PERT network into the classical PERT network.

2.3. The analytical model of multi-objective risk-time-cost trade-off problem

Let $G = (V, E)$ be the transformed classical PERT network with set of nodes $V = v_1, v_2, \ldots, v_m$ and set of activities $E = e_1, e_2, \ldots, e_n$. The source and sink nodes are denoted by s and y, respectively. We build the analytical model of MRTCTP combining [9], according to 2.1. Here are some important definition.

Definition 1. Risk-time probability, the probability that the project completion time does not exceed a given threshold, which is expected completion time, then

$$P(T \le t_0) = P_1(t_0). \tag{2.1}$$

Definition 2. Project $cost(D)$, the cost of activity $e \in E$ in the transformed classical PERT network is assumed to be a non-decreasing function $d_e(x_e)$ of the risk element x_e allocated to it. Therefore, the project $cost(D)$ would be equal to $D = \sum_{e \in E} d_e(x_e)$

Definition 3. The characteristic function of project completion time, expectation and variance as follows, respectively.

$$E(T) = \int_0^\infty (1 - p_1(t))\,\mathrm{d}t = \int_0^\infty tp_1'(t)\,\mathrm{d}t \tag{2.2}$$

$$D(T) = \int_0^\infty t^2 p_1'(t)\,\mathrm{d}t - \left[\int_0^\infty tp_1'(t)\,\mathrm{d}t\right]^2, \tag{2.3}$$

where $p_1'(t)$ is the density function of project completion time. $g_e(x_e)$ is the average cost function $e \in E$. When risk element is x_e, it equals the mean time spent in the service station. As explained in 2.2, if there is one server, the mean time spent would be equal to $\mu_e - \lambda$. if there are infinite number of servers in the corresponding service station, it is equal to μ_e. A_e is the maximum number of risk elements, B_e is the minimum number of risk elements.

We can have the engineering data and experts prediction to estimate the $d_e(x_e)$ and $g_e(x_e)$. When a project to be a good result, the model can use the data to predict other similar period and the related costs, This is the main purpose we have established the model.

$\{X(t), t \ge 0\}$ is a finite-state absorbing continuous-time Markov process, it is concluded that this state is an absorbing one and obviously the other states are transient. We assume that the states are numbered $1, 2, \ldots, N$, where state 1 is the initial state, namely $X(t) = (O(s), \emptyset)$. State N is the absorbing state, namely $X(t) = (\emptyset, \emptyset)$. Let T represent the project completion time in the PERT network. Clearly, $T = \min(t > 0; X(t) = N/X(0) = 1)$. Thus, T is the time until $\{X(t), t \ge 0\}$ gets absorbed in the final state starting from state 1.

If $F(t) = P\{T \le t\}$, definite $P_i(t) = P(X(t) = N/X(0) = i)$, $i = 1, 2, \ldots, N$, then $F(t) = P_i(t)$. We can get the vector of equation left function, marked $P(t)$ then $P(t) = [P_1(t), P_2(t), \ldots, P_N(t)]^{\mathrm{T}}$.

$$\begin{aligned} &p'(t) = QP(t) \\ &P(0) = [0, 0, \ldots, 1]^{\mathrm{T}} \end{aligned} \tag{2.4}$$

where $P'(t)$ represents the derivation of the state vector $P(t)$ and Q is the infinitesimal generator matrix of the stochastic process $\{X(t), t \ge 0\}$. The infinitesimal generator matrix Q would be a function of the control vector $\mu = [\mu_e; e \in E]^{\mathrm{T}}$. Therefore, the non-linear dynamic model is

$$\begin{aligned} &p'(t) = Q(\mu)P(t) \\ &P_i(0) = 0; \quad \forall i = 1, 2, \ldots, N-1 \\ &P_N(t) = 1. \end{aligned} \tag{2.5}$$

Representing C as the set of nodes including $M/M/1$ service stations and D as the set of nodes including $M/M/\infty$ service stations in the original dynamic PERT network. Make the system stable, $\exists \xi$ following the establishment of relations

$$\begin{aligned} &\mu_e \ge \lambda + \xi, \quad e \in C \\ &\mu_e \ge \xi, \quad e \in D. \end{aligned} \tag{2.6}$$

Thus, when risk element subordinated to continuous state, multi-objective risk-time-cost trade-off model as follows.

$$\begin{aligned}
&\operatorname{Min} f_1(x) = \sum_{e\in E} d_e(X_e) \qquad (2.7)\\
&\operatorname{Min} f_2(x) = \int_0^\infty t p_1'(t)\,\mathrm{d}t\\
&\operatorname{Min} f_3(x) = \int_0^\infty t^2 p_1'(t)\,\mathrm{d}t - \left[\int_0^\infty t p_1'(t)\,\mathrm{d}t\right]^2\\
&\operatorname{Min} f_4(x) = P_1(t_0)\\
&\text{Subject to: } P'(t) = Q(\mu)P(t)\\
&P_i(0) = 0; \quad \forall i = 1, 2, \ldots, N-1\\
&P_N(t) = 1\\
&g_e(x_e) = \mu_e - \lambda, \quad e \in C\\
&g_e(x_e) = \mu_e, \quad e \in D\\
&\mu_e \ge \lambda + \xi, \quad e \in C\\
&\mu_e \ge \xi, \quad e \in D\\
&x_e \le A_e, \quad e \in E\\
&x_e \le B_e, \quad e \in E.
\end{aligned}$$

The corresponding discrete state model as follows

$$\begin{aligned}
&\operatorname{Min} f_1(x) = \sum_{e\in E} d_e(X_e) \qquad (2.8)\\
&\operatorname{Min} f_2(x) = \sum_{k=0}^{K-1} k\Delta h(P_1(k+1) - P_1(k))\\
&\operatorname{Min} f_3(x) = \sum_{k=0}^{K-1} (k\Delta h)^2 (P_1(k+1) - P_1(k)) - \left[\sum_{k=0}^{K-1} (k\Delta h)(P_1(k+1) - P_1(k))\right]^2\\
&\operatorname{Min} f_4(x) = P_1(t_0/\Delta h)\\
&\text{Subject to: } P(k+1) = P(k) + Q(\mu)P(k)\Delta h, \quad k = 0, 1, \ldots, K-1\\
&P_i(0) = 0; \quad \forall i = 1, 2, \ldots, N-1\\
&P_N(k) = 1, \quad k = 0, 1, \ldots, K\\
&P_i(k) \le 1\\
&g_e(x_e) = \mu_e - \lambda, \quad e \in C\\
&g_e(x_e) = \mu_e, \quad e \in D\\
&\mu_e \ge \lambda + \xi, \quad e \in C\\
&\mu_e \ge \xi, \quad e \in D\\
&x_e \le A_e, \quad e \in E\\
&x_e \le B_e, \quad e \in E. \qquad (2.9)
\end{aligned}$$

According to the definition of integral thinking, we divide completion cost into K portions and set step is Δh. Then we can control accuracy of in order to make the discrete model solution closer to the continuous model.

3. The neural network analytical algorithm of multi-objective risk-time-cost trade-off problem

In the development of artificial neural networks (ANN) [14], it has formulated dozens of influential models. Such as, Perceptron (MLPs) [15], Grossberg adaptive theory [16,17], Fukushima cognitive neural network theory [18], Kohonen Self-Organizing Map [19], Hopfield feedback network model [20], BP networks [21], Boltzmann machine [23], Radial Basis Function (RBF) network [22], etc. With years of practice and development, artificial neural networks have been successfully applied to pattern recognition, control, optimization fields.

We apply RBF neural network to solve problem (8) on the basis of referencing the paper [24]. We introduce risk preference factor $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ and transform (8) into the following nonlinear programming problem

$$
\begin{aligned}
&\min Z(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) + \lambda_3 f_3(x) - \lambda_4 f_4(x)\\
&\text{subject to: } P(k+1) = P(k) + Q(\mu)P(k)\Delta h, \quad k = 0, 1, \dots, K-1 \qquad (3.1)\\
&P_i(0) = 0; \quad \forall i = 1, 2, \dots, N-1,\\
&P_N(k) = 1, \quad k = 0, 1, \dots, K,\\
&P_i(k) \le 1\\
&g_e(x_e) = \mu_e - \lambda, \quad e \in C\\
&g_e(x_e) = \mu_e, \quad e \subseteq D\\
&\mu_e \ge \lambda + \varepsilon, \quad e \in C\\
&\mu_e \ge \varepsilon, \quad e \in D\\
&x_e \le A_e, \quad e \in E\\
&x_e \ge B_e, \quad e \in E.
\end{aligned}
$$

This model will be simplified, when $x_e \in [B_e, A_e]$,$e \in C$, the arrival rate and service rate of queuing system both are greater than zero, $D \le 1$. Then (9) can be changed into the following form

$$
\begin{aligned}
&\min Z(x) = \lambda_1 f_1(x) + \lambda_2 f_2(x) + \lambda_3 f_3(x) - \lambda_4 f_4(x)\\
&\text{subject to: } P(k+1) = P(k) + Q(\mu)P(k)\Delta h, \quad k = 0, 1, \dots, K-1 \qquad (3.2)\\
&P_i(0) = 0; \quad \forall i = 1, 2, \dots, N-1,\\
&P_N(k) = 1, \quad k = 0, 1, \dots, K,\\
&g_e(x_e) = \mu_e - \lambda, \quad e \in C. \qquad (3.3)
\end{aligned}
$$

When $F(k) = P(k+1) - P(k) - Q(\mu)P(k)\Delta h$, transform (10) by introducing Lagrange function can get

$$L(x, \beta_1, \beta_2) = \lambda_1 f_4'(x) + \lambda_2 f_2(x) + \lambda_3 f_3(x) - \lambda_4 f_4(x) + \beta_1 F(k) + \beta_2 (g_e(x_e) - (\mu_e - \lambda)),$$

where β_1, β_2 are Lagrange parameters, according to the nature of the Lagrange equation, $\exists \beta_1^*, \beta_2^*$ s.t. x^* we know the following relationship

$$
\begin{aligned}
&\frac{\partial L(x^*, \beta_1^*, \beta_2^*)}{\partial x} = \lambda_1 f_1'(x) + \lambda_2 f_2'(x) + \lambda_3 f_3'(x) - \lambda_4 f_4'(x) + \beta_1 F(k) + \beta_2 (g_e(x_e) - (\mu_e - \lambda)) = 0 \qquad (3.4)\\
&\frac{\partial L(x^*, \beta_1^*, \beta_2^*)}{\partial \beta_1} = F(k) = P(k+1) - P(k) - Q(\mu)P(k)\Delta h = 0\\
&\frac{\partial L(x^*, \beta_1^*, \beta_2^*)}{\partial \beta_1} = g_e(x_e) - (\mu_e - \lambda) = 0.
\end{aligned}
$$

In RBF neural network, select polymer-based function to Gaussian kernel, (12) can be transformed as follows

$$
\begin{aligned}
\min F(x) = {}& \lambda_1 \exp\left(-\frac{\sum_{i-1}^{n}(x_i f_1(x_i) - f_1(x))^2}{2r_{E^2}}\right) + \lambda_2 \exp\left(-\frac{\sum_{i-1}^{n}(x_i f_3(x_i) - E(x))^2}{2r_{E^2}}\right)\\
&+\lambda_3 \exp\left(-\frac{\sum_{i-1}^{n}(x_i f_3(x_i) - D(x))^2}{2r_{E^2}}\right) - \lambda_4 \exp\left(-\frac{\sum_{i-1}^{n}(x_i f_4(x_i) - f_4(x))^2}{2r_{E^2}}\right) + \beta_1 F(k) + \beta_2 (g_e(x_e) - (\mu_e - \lambda)). \qquad (3.5)
\end{aligned}
$$

The corresponding RBF neural network model shown in Fig. 2.

Where $G(.)$ is Gaussian function as follows

$$u_j = \exp\left[-\frac{(X - C_j)^{\mathrm{T}}(X - C_j)}{2\delta_j^2}\right], \quad j = 1, 2, \dots, N_h. \qquad (3.6)$$

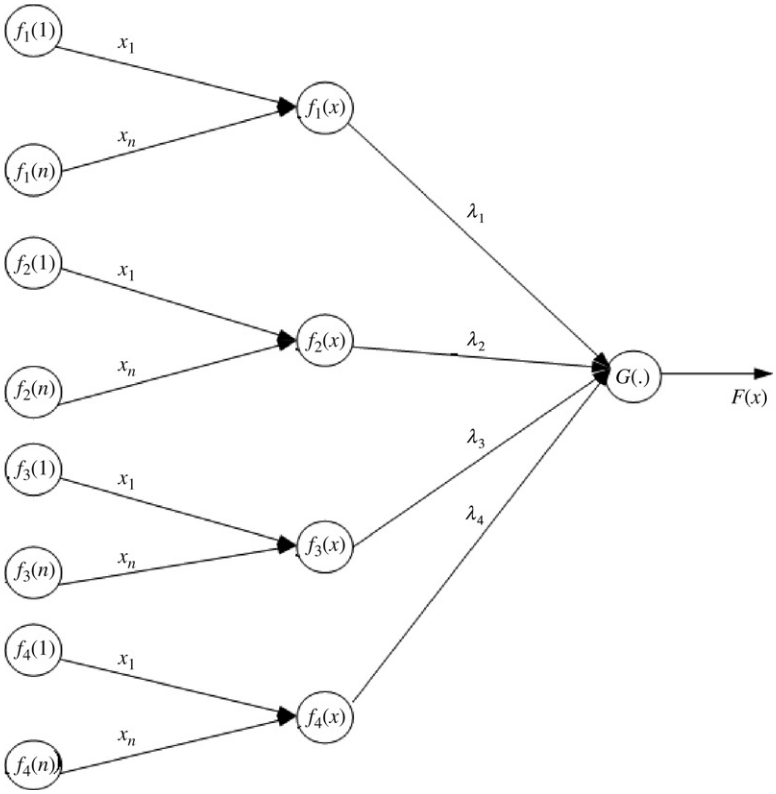

Fig. 2. The RBF neural network model of MRTCTP.

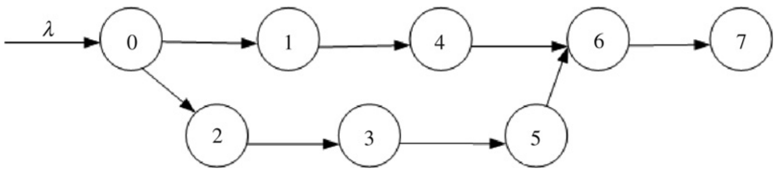

Fig. 3. Dynamic PERT network.

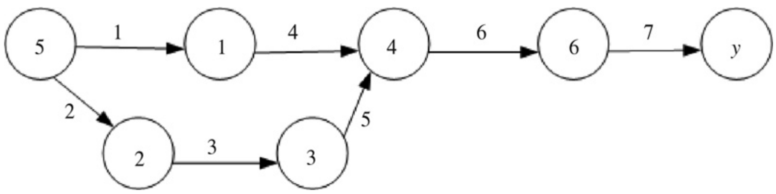

Fig. 4. Classical PERT network.

4. Practical analysis

In order to demonstrate the effectiveness of the algorithm, we also use the dynamic PERT network [9]. Assuming that the time of the network node followed exponential distribution and every activity followed the Poisson distribution that the parameter is λ and $\lambda = 10$. Its network diagram is shown in Fig. 3.

We assume that there is one service station in node 1, 2, 3, 6, 7 and two service stations in node 4, 5. According to the algorithm of 2.2, we transform the above figure into the classical PERT network, which is shown as Fig. 4.

Table 1 gives the risk-cost function and risk-time function of the classical PERT network activities, and the largest and smallest number of risk elements.

We set $\lambda_1 = 2, \lambda_2 = 2, \lambda_3 = 1, \lambda_4 = 1$. The longest trails of classic PERT network random process have 14 states, the set S includes the states. $S = [(1, 2), (1, 3), (1, 5), (1, 5^*), (2, 4), (2, 4^*), (3, 4), (3, 4^*), (4, 5), (4^*, 5), (4, 5^*), (6), (7), (\phi, \phi)]$.

Table 1
Risk-cost function and risk-time function.

e	$d_e(x_e)$	$g_e(x_e)$	A_e	B_e
1	$3x_1^2+2$	$0.7-0.1x_1$	5	1
2	$2x_2+1$	$1.5-0.1x_2$	6	1
3	x_3+2	$1-0.1x_3$	9	1
4	x_4	$1.5-0.3x_4$	4	1
5	$3x_5+4$	$0.9-0.1x_5$	5	1
6	x_6+3	$1.1-0.1x_6$	6	1
7	$4x_7+1$	$1-0.2x_7$	3	1

Table 2
Model results.

f_1	f_2	f_3	f_4	Δh	K
				0.8	10
30.897	4.576	2.254	0.382	0.4	20
30.904	4.614	2.602	0.363	0.27	30
30.885	4.634	2.824	0.371	0.2	40
$\lambda_1=2, \lambda_2=2, \lambda_3=1, \lambda_4=1$					
$F_{K=20}(x)=70.818$ $F_{K=30}(x)=70.327$ $F_{K=40}(x)=69.552$					

Q(μ) equal to the matrix of paper [9]. Set $K = 10, 20, 30, 40$, Δh is 0.8, 0.4, 0.27, 20 respectively. The results in following Table 2 show the effectiveness of the algorithm.

5. Conclusions

Based on the general project risk element transmission theory, this paper analyzes the multi-objective risk-time-cost trade-off problem by introducing the risk element, it improves the Markov dynamic PERT model, and finally get the analytical model of multi-objective risk-time-cost trade-off problem. By introducing neural network, and making an effective solution of the model, the effectiveness of the approach is illustrated with a practical example. By establishing the multi-objective risk-time-cost trade-off problem model, it not only has an important effect on the risk element transmission theory, but has an important significance for actual project. If we applied the model to the field of computers, the reliability model of computer network considering risk elements can be built. This should be a subject of future research and we expect further improvement of the risk element transmission theory.

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