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ORIGINAL ARTICLE

The Application of the Secant's Equation to the Sewing Machine Needle

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Limitation of technological load to critical load

Abstract The sewing needle of the industrial sewing machines is an essential element. Its' target is to penetrate the sewn fabric layers by a penetrating axial compressive force, which, coincides with the sewing needle geometrical axis. Author wise, it will cause stress on the needle's cross section. Practically, there is always a shift – eccentricity- between the effective action line of the force and the sewing needle geometrical axis. In the present work a mathematical approach has been carried out to study the mathematical relationship between the eccentricity $\left(\frac{ec}{r^2}\right)$ and the penetration force (P_a), taking into consideration the critical load (P_{cr}) (Euler load) of the sewing needle. This relationship is named the "Secant formula", where it was computerized and graphed. It was found that; the limiting values for the $\left(\frac{ec}{r^2}\right)$ was 0.7 and for the ratio P_a/P_{cr} was 0.8 to make the needle to run in its design stress 538 MPa (steel). When the ratio P_a/P_{cr} was equal to unity, the sewing needle max stress σ_{max} changed from 101 MPa to 58 GPa i.e. the working penetration force P_a must be far enough from the critical load by about 20%. The required value of P_a must be equal or less than 0.8 P_{cr}. This work focused on the static case.

1. Introduction

The sewing machine needle is an important and vital machine member. The general objective of the sewing needle is to penetrate the sewn materials either single layered or multiple layered and to carry the sewing thread via the sewn fabrics for loop formation. During the penetrations of the sewing needle, a resisting force at the free end of the needle is built up, this subjects the needle to an axial compressive force. This force can lead to the needle buckling in elastic or plastic region of the needles material [steel]. In both cases the sewing needle may be bent. This will lead to the production of a miss stitch and a low sewing seam quality [1].

Hussien et al. (2009) developed the fabric hand tester to measure the penetration force of the sewing needle [2]. ElGholmy and Elhawary studied a formula for calculating the critical load of the needle used in the garment and apparels sewing technology [3]. In all these studies the subjection of the sewing needle to axial compressive forces act along the centroid axis of the needle was assured. But in real-life situation, often we come across the needle with an eccentric loading in such case, the line of action of the penetrating force does not pass through the centroid of the needle cross section.

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The mechanics of the columns and machine members subjected to axial compressive forces with all the associated phenomena have been studied extensively by several scientists [4–6].

Elhawary [7] has studied the mechanics of the needle used in the needle punching machine for the production of nonwoven fabric. The piercing load of such needle is an axial compressive force.

Sultan and Hearle [8] studied and evaluated experimentally the punching force machine. They found the average value of the punching force (working force) to be 6 N. The factor of stability of the needle ranges from 1.63 to 2.74.

Lemov [9] in this work has studied a predictive model for the penetration force of a woven fabric by a needle. Consequently he developed a formula for estimating the value of the penetration force that depends upon so many factors such as; weave pattern, fabric count and crimp. The predictive force was checked experimentally. The max measured value of the penetration force was 46 C–12 CN for plain weave (with different tightness and density) and for sateen 8/3 respectively. It was also reported that the needle during fabric penetration causes radial displacement of threads. From this point of view this displacement of the threads will react as an elastic reaction on the needle i.e. spring effect. This, of course, will affect the elastic stability of the needle itself during the sewing process.

Kawamura [10] stated that; in the sewing operation a needle could be deflected by factor as sewing fabric structure. Lui [11] has written that; the max load that a perfectly eccentrically loaded column can carry is the Euler load. In reality, however, because of material yielding, the Euler load is seldom reached and the maximum carrying-load-capacity of an eccentrically loaded column will fall far under the Euler force.

Thus the aim of this research work is to use the "Secant formula", to study the mathematical relationship between the eccentricity $\left(\frac{ec}{r^2}\right)$ and the penetration force (P_a), taking into consideration the critical load (Euler load) of the sewing needle.

2. Mathematical approach

Referring to Fig. 1, the elastic line of the needle due to the technological eccentric load could follow:

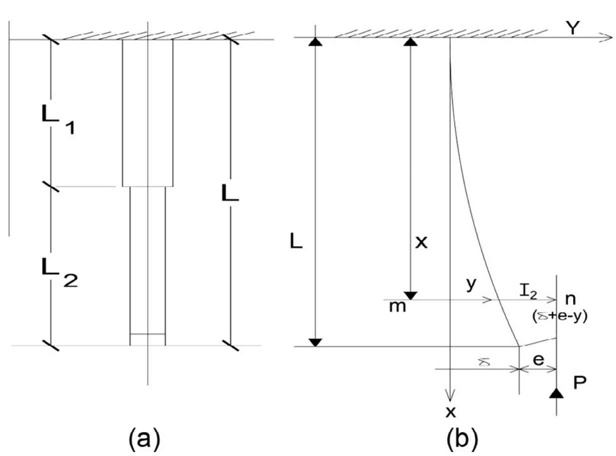


Fig. 1 An elastic line of a sewing needle.

$$\mathcal{Y} = (\delta + e)(1 - \cos \lambda X) \tag{1}$$

where

X – Instantaneous lateral displacement of the needle (vertical axis).

$\mathcal{Y}$ – The horizontal axis parallel to the sewing needle axis.

$$\lambda - \text{const} = \sqrt{P/EI} \tag{2}$$

P – Axial and actual axial compressive load on the sewing needle due to sewing technology.

E – Young's modulus.

I – Minimum inertia of the sewing needle cross section.

EI – Minimum bending stiffness of the needle.

e – Technological load-penetrating needle force eccentricity i.e. the shift between the needle geometrical axis and the technological force active line.

δ – The lateral displacement of the lower free end of the industrial sewing machine's needle.

The lateral movement of the sewing needle (lower end) could be calculated by:

$$\because\ \mathcal{Y} = (\delta + e)(1 - \cos \lambda X)$$

By putting $x = \ell$

$$\therefore\ \mathcal{Y} = (S + e)(1 - \cos \lambda \ell)$$

$$\therefore\ \mathcal{Y} = e\frac{(1 - \cos \lambda \ell)}{\cos \lambda \ell} \tag{3}$$

Return back to Eqs. (1) and (3), Then:

$$\mathcal{Y} = e\frac{(1 - \cos \lambda x)}{\cos \lambda \ell} \tag{4}$$

Eq. (4) represents the equation of the sewing needle elastic line (deflection curve), that equation can give the lateral displacement instantaneously for any distance at the sewing needle length.

The last formula (4) represents the equation of the needle elastic line (deflection curve). By using this equation the deflection at any cross section of the sewing machine's needle can be calculated.

In case of the needle of the industrial sewing machine, the value $\lambda\ell$ is small in comparison with unity and it is sufficiently accurate to take:

$$\cos \lambda \ell = 1 - \frac{1}{2}\lambda^2 \ell^2 \tag{5}$$

Using this quantity of $\cos \lambda\ell$ and neglecting the value $\lambda^2\ell^2/2$ in the denominator of expression (3), as being small in comparison with unity, We obtain:

$$\delta = \frac{e\lambda^2 \ell}{2} = \frac{P_e \ell^2}{2EI} \tag{6}$$

This represents the value of the lateral deflection at the needle's free end by the applied couple $P \cdot e$ due to load eccentricity e. Hence the use of the approximate expression (5) is equivalent to neglecting the effect of the deflection upon the magnitude of the bending moment and taking instead a constant moment equal to $P_a \cdot e$.

In case of relatively long sewing needles, the value $\lambda\ell$ is not small and δ is calculated by Eq. (3). In this way it was found that the deflection is no longer proportional to the load P_a (penetrating load). Instead, it increases more rapidly than P_a as shown from the values of this deflection as given in the second line of Table 1.

The maximum bending moment takes place at the upper end of the sewing needle [a built-in coupled] and its value is;

$$\begin{aligned} M_{\max} &= P_a(e+\delta) \\ &= P_a \cdot e \cdot sec\lambda\ell \end{aligned} \tag{7}$$

$sec\lambda\ell$ – is shown in Table 1.

The maximum stress in the sewing needle is $\sigma_{\max}$where

$$\sigma_{\max} = \frac{M_{\max}}{I_{\min}} c \tag{8}$$

where

$I_{\min} = I'$ – minimum cross-sectional inertia of the sewing needle.

c – The distance of extreme fiber of the compressed side of the needle from the neutral axis.

$$\begin{aligned} \therefore\ \sigma_{\max} &= \frac{P_a e}{I'/c} sec\lambda\ell \\ &= \frac{Pec}{I'} sec\lambda\ell \end{aligned} \tag{9}$$

But the compressive force P_a can create a tensile stress with value:

$$\sigma_{max} = \frac{P_a}{A_{\min}} = \frac{P_a}{A'}$$

Therefore the maximum stress $\sigma_{\max}$ is;

$$\begin{aligned} \sigma_{\max} &= \frac{P}{A'} + \frac{Pec}{I'} sec\lambda\ell \\ &= \frac{P_a}{A'}\left[1 + \frac{S_{ec}A'}{I'} sec\lambda\ell\right] \\ &= \frac{P}{A'}\left[1 + \frac{S_{ec}}{(I'\backslash A')} sec\lambda\ell\right] \end{aligned} \tag{10}$$

where

I' – minimum sewing needle's inertia of cross section,

$A_{\min} = A'$ – minimum sewing needle's cross section.

Assume $(I'/A') = r_1^2$ where r is the minimum radius of gyration about neutral axis of the needle cross section, then the Secant formula for the sewing needle is,

Table 1 Value of Eq. (3) parameters for large displacement.

$\lambda\ell$	0.1	0.5	1.0	1.5	$\pi/2$
S_{acc}	0.005e	0.139e	0.851e	13.1e	∞
Approx. S	0.005e	0.139e	0.840e	17.8e	∞
$sec\lambda\ell$	1.005	1.140	1.867	13.2	∞
$P_a/P_{cr} = 1/m$	0.004	0.101	0.405	0.911	1
Stability factor m = P_{cr}/P_a	250	10	2.20	1.1	1

$\lambda = \sqrt{\frac{P_a}{(EI)_{min}}}$ – const.

ℓ – Total length of the needle (ms).

δ_{acc} – Accurate lateral movement of the needle free and due to couple (P.e), in $\delta_{acc} = \frac{e(1-\cos\lambda x)}{\cos\lambda\ell}$.

δ_{app} – approximated $= (P_a \cdot e) \cdot \frac{\ell^2}{2EI}$

$$\sigma_{\max_{tot}} = \frac{p}{A'}\left[1 + \frac{ec}{r^2}\left(S_{ec}\frac{\ell}{r}\sqrt{\frac{P}{EA'}}\right)\right] \tag{11}$$

where

$\sigma_{\max}$ – Max stress in the sewing needle due to penetration force and the bending moment (max).

P_a – Penetration force – actual force P_a – axial resisting force of the sewing needle.

A' – Minimum cross-sectional area of the sewing needle.

e – Eccentricity of the compressive penetrating force.

c – The distance from the sewing needle neutral axis to the needle's outer fibers (surface).

r – Radius of gyration, $r^2 = I'/A'$, I' – minimum sewing needle inertia of its cross section.

ℓ – Total needle length.

$\left(\frac{\ell}{r}\right)$ – Slenderness ratio.

EA' – Axial rigidity or stiffness.

The following part of Secant's formula;

$$\left[1 + \frac{e \cdot c}{r^2}\left(S_{ec}\frac{\ell}{r}\sqrt{\frac{P}{EA'}}\right)\right] \tag{12}$$

Could be called:

Secant's factor, It can help in calculation $\sigma_{\max_{tot}}$ by multiplying it by (P/A') as tensile stress.

The secant formula (11) for the sewing needle can be rewritten as follows:

$$P_c = \frac{\pi}{4\ell^2} \times EI \tag{13}$$

$$\sigma_{\max} = \frac{P}{A'}\left[1 + \frac{ec}{r^2}\left(S_{ec}\frac{\ell}{r}\sqrt{\frac{P_a}{EA'}}\right)\right] \tag{14}$$

From Eq. (13):

$$E = P_c \times \frac{4\ell^2}{\pi^2} \times \frac{1}{I}$$

$$\therefore\ EA' = \frac{P_c \times 4\ell^2}{\pi^2 \times I} \times A'$$

$$\therefore\ sec\frac{\ell}{r}\sqrt{P_a/EA'}$$

$$\begin{aligned} \therefore\ sec\frac{\ell}{r} \times \sqrt{\frac{P_a}{EA'}} &= sec\frac{\ell}{r} \times \sqrt{\frac{P_a}{\frac{P_c \times 4\ell^2}{\pi^2 \times I} \times A'}} \\ &= sec\frac{\ell}{r} \times \sqrt{\frac{P_a}{P_c} \times \frac{\pi^2}{4\ell^2} \times \frac{I}{A'}} \end{aligned}$$

$$\begin{aligned} sec\frac{\ell}{r} \times \frac{\pi}{2\ell} \times \sqrt{\frac{P_a}{P_c} \times \frac{I}{A'}} \\ &= sec\frac{\pi}{2r} \times \sqrt{\frac{P_a}{P_c}} \times R \\ &= sec\frac{\pi}{2r} \times r \times \sqrt{\frac{P_a}{P_{cr}}} \\ &= sec\frac{\pi}{2} \times \sqrt{\frac{P_a}{P_{cr}}} \end{aligned} \tag{15}$$

where $\sqrt{I/A'} = r$ – radius of gyration of the sewing needle cross section.

P_a – actual load or penetration load on the sewing needle during the sewing process.

∴ The Secant formula for the sewing needle is:

$$\sigma_{\max} = \frac{P}{A'}\left[1 + \frac{ec}{r^2}\left(sec\frac{\pi}{2}\right)\sqrt{\frac{P_a}{P_{cr}}}\right] \tag{16}$$

3. Results and discussion

From the last mathematical models, it was found that the secant formula (13) has two views: Eqs. (14) and (16). To graph Eq. (16) we shall assume that the value $\frac{ec}{r^2}$ varies from: 0 (Pure Euler), 0.1, 0.2, 0.4, 0.6, 0.8 and 1.0. While for each of these values the ratio (P_a/P_{cr}) – the ratio between the actual penetrating force and the critical bucking load of the industrial sewing needle will vary as:0.2, 0.4, 0.6, 0.8 and 1 (max-load carrying ability of an shifted-loaded column will fall far under the Euler force (critical load). Thus, the maximum load with a perfect elastic sewing needle with a shifted line of perpetrating force can withstand is the critical load.

Fig. 2 shows the relationship between the sewing needle maximum stress $\sigma_{\max}$ and the dimensionless quantity $\frac{ec}{r^2}$ for different ratios of P_a/P_{cr}.

It is shown that the increase of the ratio $\frac{ec}{r^2}$ will lead to the increase of the sewing needle max stress $\sigma_{\max}$. It is clear that the increase of the ratio P_a/P_{cr} will increase dramatically the sewing needle maximum stress $\sigma_{\max}$.

From Fig. 2 it could be concluded that, the increase of the ratio $\frac{ec}{r^2}$ from 0 to 1.0 will increase $\sigma_{\max}$ from 101 to 235 MPa for $P_a/P_{cr} = 0.2$ while $\sigma_{\max}$ increases from 101 to 712 MPa for $P_a/P_{cr} = 0.8$.

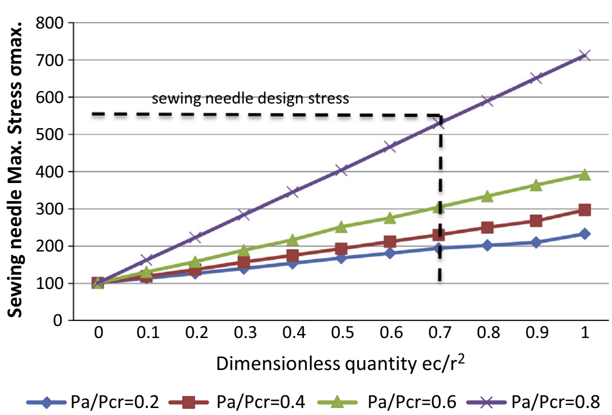
For the ratio $P_a/P_{cr} = 1$, the sewing needle maximum stress $\sigma_{\max}$ increases from 101 MPa to 57.970 GPa when ratio $\frac{ec}{r^2}$ changes from 0 to 1.0. For the ratio $P_a/P_{cr} = 1$, it was very high. Care attention for the calculations because the secant value was infinity. Therefore, we selected $\frac{P_a}{P_{cr}} = 0.9$ to avoid the value $\frac{P_a}{P_{cr}} = 1$ where it is a nearest value for unity.

Assuming the sewing needle's steel has a Data Base as follows:

- Type of steel SAE 2330 (drawn at 200 °C).
- Yield stress = 1344 MPa.
- Ultimate stress = 1523 MPa.
- Hardness BHN = 425.

In Fig. 2 a horizontal dotted line was drawn to determine the max allowable design stress of the sewing needle. The limiting ratio $\frac{ec}{r^2}$ (dimensionless quantity) is determined by dropping a vertical dotted line. The practical limit value $\left(\frac{ec}{r^2}\right)$ is 0.7, from which it is easy to calculate the max allowable value of the sewing needle's perpetration force, more eccentricity means more needles stresses, Also, the ratio of P_a/P_{cr} is limited by 0.8 i.e. the sewing needle's penetration force must not exceed 80% of the critical load capacity.

The following Fig. 3 shows the relation between different factor of safeties ranging from (2.5) to (1.25) and the design stress, assuming factor of safety (F.S) for static case 2.5, then the design stress $[\sigma] = 1344/2.5 = 538$ MPa.



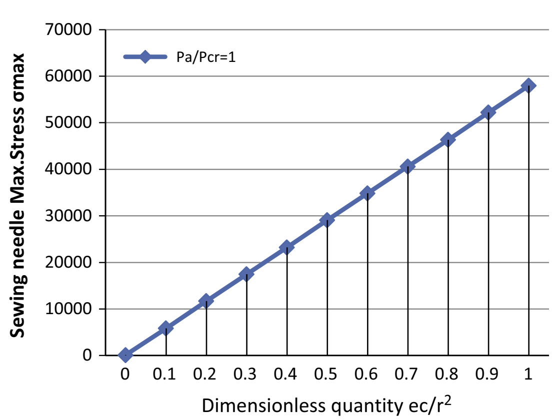


Fig. 2 Sewing needle's maximum stress $\sigma_{\max}$ vs dimensionless value $\frac{ec}{r^2}$ For different ratios of P_a/P_r.

4. Conclusions and recommendations

(1) The secant formula for the industrial sewing machines needle is:

$$\sigma_{max_{tot}} = \frac{P}{A'}\left[1 + \frac{e.c}{r^2}\left(S_{ec}\frac{\pi}{2}\sqrt{\frac{P_a}{P_{cr}}}\right)\right]$$

where$\sigma_{\max_{tot}}$ – total maximum stress in the sewing needle, due to; technological penetrating force and the maximum bending moment, P – actual axial compressive penetrating force on the sewing needle free end, A' – minimum cross – sectional area of the sewing needle, e

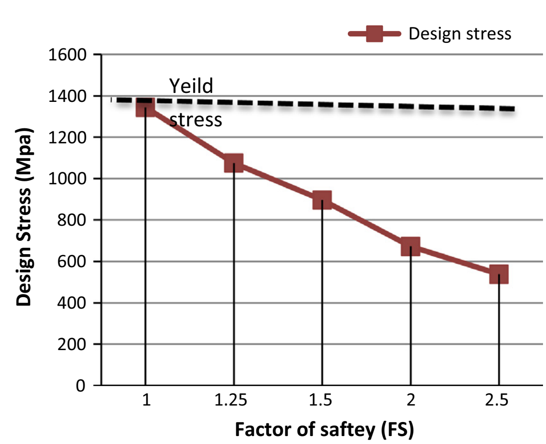


Fig. 3 Design stress and factor of safety.

– eccentricity of the penetrating force, c – distance from the sewing needle centroid of cross – sectional – neutral axis – to outer surface, r – radius of gyration, ℓ – total needle length, (ℓ/r) – slenderness ratio and $EA'^{\backslash}$ – axial rigidity (minimum).

(2) The sewing needle's Secant factor is:

$$S.F = \left[1 + \frac{e \cdot c}{r^2}\left(S_{ec}\frac{\pi}{2}\sqrt{\frac{P_a}{P_{\mathrm{cr}}}}\right)\right]$$

(3) The increase of the dimensionless value $\frac{ec}{r^2}$ from 0 to unity will increase the sewing needle's design stress from 101 to 712 MPa for ratio $\frac{P_a}{P_{\mathrm{cr}}} = 0.8$ i.e. the increase of the sewing needle's penetration force, will increase the stress on the needle during sewing process.

(4) The sewing needle's design stress $[\sigma]$ is 538 MPa that limits the ratio $\frac{ec}{r^2}$ to 0.7.

(5) The lowest stress was obtained when the sewing force coincides with the needle axis i.e. eccentricity $e = 0$ i.e. $\sigma_{\max} = 101$ MPa.

(6) When the load ratio $\frac{P_a}{P_{\mathrm{cr}}}$ approaches unity the max stress approaches 58 GPa i.e. changes from 101 MPa to 58 GPa; The max allowable value of P_a is 80% of P_{cr}.

(7) Our study for the sewing needle's buckling has been carried out for a static status. It is recommended to study the dynamic status in the future.

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