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BOOK REVIEWS

Elements of the Theory of Computation. By H. R. LEWIS and C. H. PAPADIMITRIOU. Prentice-Hall, Englewood Cliffs, New Jersey 1981. 466 pp.

Elements of the Theory of Computation is a valuable textbook, which has been much needed in most computer science curricula. Despite the existence of good advanced texts on computability, logic, analysis of algorithms, and formal languages and the recent advent of introductory discrete mathematics texts for computer science students, there has not been a comprehensive intermediate text in theoretical computer science. Lewis and Papadimitriou have provided such a text.

The background required is a semester of discrete mathematics, and some mathematical maturity; thus the material would be suitable for a two semester course for third year computer science or mathematics students. Judicious selection of topics would enable the presentation of a one semester course using this text, with the emphasis reflecting the preferences of the instructor.

Chapter 1, "Sets, Relations, and Languages", provides a brief but remarkably thorough overview of the basic required concepts of discrete mathematics: an introduction to logic and set theory, relations and functions, and formal languages. Chapter 2 introduces finite automata as abstractions of real computing devices. Deterministic and nondeterministic finite automata are described; it is shown that they accept the same set of languages, the "regular" languages. The chapter concludes by showing that non-regular languages exist.

Chapter 3 starts with the notion of a grammar and develops the natural correspondence between context-free languages, context-free grammars, and pushdown automata. Important properties of the context-free languages are derived; this leads into an excellent discussion of parsing techniques. Moving down the Chomsky hierarchy, Chapter 4 concerns Turing machines. The correspondence between accepting languages and computing functions is developed; then, from modest initial Turing machine computations, progressively more sophisticated ones are built. This culminates in the statement that all computable functions are computable by a Turing machine. Nondeterminism in Turing machines is introduced, and shown to leave unchanged the set of functions computed.

Starting with Church's thesis, Chapter 5 discusses the equivalence of Turing machines, unrestricted grammars, and partial recursive functions. In the process, the primitive recursive functions are defined, and shown not to include all computable functions. The chapter concludes with universal Turing machines. Chapter 6 shows that some propositions are undecidable and some functions are uncomputable. The halting problem and Post's correspondence problem, among others, are considered. The chapter does not attempt detailed coverage of this area, but rather is an intelligent selection of topics.

Chapter 7 proceeds to computational complexity, by introducing resource bounds on Turing machine computations. The complexity classes P and NP are defined, and their importance to everyday computation is emphasized. The NP-completeness of various combinatorial problems is proved. Chapters 8 and 9 change focus somewhat, to logic—specifically, the propositional calculus and the predicate calculus. Presentation of this material is fairly consistent from one text to another; notable in this presentation are the discussions of compactness and resolution theorem proving. In addition, parallels between unsolvability and NP-completeness are drawn.

The choice of topics appears well considered, and the logical progression from topic to topic is very good. One topic which is omitted is the design and analysis of algorithms; however, the existence of good introductory and advanced texts in this area compensates for this. The book is well written and precise. Numerous exercises serve to illustrate and expand on the material covered in the body of the text. Worked examples are not provided, however, and would have to be provided in accompanying lectures.

I see no serious drawbacks in this text, and as outlined here I see many strong points. In conclusion, I feel that the book is a valuable contribution to any computer science education.

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Geometrical Methods for the Theory of Linear Systems. Edited by C. I. Byrnes and C. F. Martin. D. Reidel, Dordrecht, 1980. $39.50.

This book is an edited compilation of tutorial lectures on applications of commutative algebra, algebraic geometry, and differential geometry to the theory of time-invariant linear systems. The original lectures were presented at a NATO Advanced Study Institute and AMS Summer Seminar in Applied Mathematics held at Harvard University in 1979.

The first section of this book (comprising roughly one quarter of its length) is a joint exposition by four of the contributors (C. Byrnes, M. Hazewinkel, C. Martin and Y. Rouchaleau) on the mathematical background necessary for understanding the remainder of the lectures. This section includes a short outline of some problems of classical algebraic geometry (plane curves, Riemann surfaces, and invariant theory), followed by brief descriptions of some basic material in the theory of modules over Noetherian rings, differentiable manifolds, algebraic varieties, and linear algebra over rings. Each of the remaining lectures treats one or more application of commutative algebra and/or geometry to problems in linear systems theory. Some of the applications discussed include feedback systems, parametric families of systems, Riccati equations, and linear systems defined over rings.

To get some idea of why algebraic geometry is applicable to the study of time-invariant linear systems, consider the description of such a system in state space form, say $x(t) = Ax(t) + Bu(t)$, $x(0) = 0$ and $y(t) = Cx(t)$, where $x(t) \in \mathbf{R}^n$,

$u(t) \in \mathbf{R}^m$ and $y(t) \in \mathbf{R}^p$. The corresponding frequency domain description is $\hat{y}(s) = G(s)\hat{u}(s)$, where $\hat{u}$ and $\hat{y}$ are the Laplace transforms of u and y, and $G(s) = C(Is - A)^{-1}B$ is the transfer function matrix of the system. $G(s)$ is a $p \times m$ matrix of proper rational functions and can be expressed as $N(s)D(s)^{-1}$ where $N(s)$ and $D(s)$ are right coprime polynomial matrices which are $p \times m$ and $m \times m$ respectively. $N_1(s)D_1(s)^{-1}$ is another right coprime factorization of $G(s)$ if and only if there exists an $m \times m$ unimodular matrix $R(s)$ such that $N_1(s) = N(s)R(s)$ and $D_1(s) = D(s)R(s)$. In turn, this implies that the $(p + m) \times m$ matrices $\begin{bmatrix} N(s) \\ D(s) \end{bmatrix}$ and $\begin{bmatrix} N_1(s) \\ D_1(s) \end{bmatrix}$ have the same column span for each $s \in C$. Furthermore, it can be shown[1] that the coprimeness condition implies that this column span is m-dimensional for each s. Thus, the transfer function $G(s)$ induces a well-defined mapping ϕ: $C \to G^m(C^{p+m})$ where $G^m(C^{p+m})$ is the set of all m-dimensional subspaces of C^{p+m} (called the Grassmann manifold of m-planes in C^{p+m}), and $\phi(s)$ is the column span of $\begin{bmatrix} N(s) \\ D(s) \end{bmatrix}$ for any right coprime factorization $G(s) = N(s)D(s)^{-1}$. If the domain of ϕ is extended to include the point at infinity, then ϕ maps the Riemann sphere into $G^m(C^{p+m})$, and is in fact a morphism of algebraic varieties. Thus, information about the properties of the linear system can be obtained by using algebraic geometry to study the mapping ϕ. The lectures of C. Byrnes include a discussion of the problem of pole placement by output feedback from this perspective.

In my view, the applications of geometry discussed in these lectures are quite varied. Furthermore, in most of the system theory problems considered, the geometrical framework offers insights which are not readily available by other methods. This is definitely not a case where heavy mathematical machinery is applied to problems which could be solved using simpler tools.

This book seems to be directed toward systems theorists who are interested in learning what geometry has to offer to linear systems theory. The introductory section on mathematical background makes the book readable by someone with limited background in differential and algebraic geometry. However, such a reader is likely to be dissatisfied with the brevity with which the long list of mathematical preliminaries is treated. I do feel that the introductory section contains sufficient material for such a reader to obtain an understanding of the basic ideas in the lectures on applications which follow, and to determine whether s/he wishes to invest the time necessary to learn the larger amount of geometry necessary to do research in this area. I also think this book would be quite useful as companion reading to a lecture course for advanced graduate students on geometric methods for systems theory. However, I do not think this book is well suited for mathematicians interested in learning about systems theory since a working knowledge of standard linear systems theory is a requirement for understanding this book.

REFERENCES

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