Variation modeling of lean manufacturing performance using fuzzy logic
based quantitative lean index

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Abstract

The lean index is the sum of weighted scores of performance variables that describe the lean manufacturing characteristics of a system. Various quantitative lean index models have been advanced for assessing lean manufacturing performance. These models are represented by deterministic variables and do not consider variation in manufacturing systems. In this article variation is modeled in a quantitative fuzzy logic based lean index and compared with traditional deterministic modeling. By simulating the lean index model for a manufacturing case it is found that the latter tend to under or overestimate performance and the former provides a more robust lean assessment.

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Keywords: Lean manufacturing; performance measurement; variability analysis

1. Introduction

Lean manufacturing (LM) is the set of practices intended to attain perfection in the identification and elimination of waste through continuous improvement flowing the product at the pull of the customer [1]. These practices are encompassed in a broad range of tools and techniques: Just-in-Time, Total Quality Management, Total Productive Maintenance, Kaizen, Kanban, Poka Yoke, Statistical Process Control and many others. LM is a philosophy and so the practices are not concrete objects [2], but there are metrics or Key Performance Indicators (KPIs) that are used in tracking the success of lean initiatives. Each KPI assesses performance for one or more practices, and to assess overall system lean performance, the lean index (LI) is introduced as a metric [3].

With the lean index (LI), the many LM KPIs of a system can be compressed into one composite indicator. In the extant literature on LI models, qualitative [4-6] and quantitative LI [7-11] models have been established. The qualitative types rely on self-rated assessments and are susceptible to bias. The quantitative types, on the other hand, are more objective as they use data that are tracked directly with numbers. An attempt to incorporate both quantitative and qualitative LI modelling into a single lean assessment framework was made by Pakdil and Leonard [11]. In their work a Fuzzy Logic (FL) based LI was introduced for the qualitative aspects of LM while another LI was derived for the quantitative aspects. Fundamentally, a LI is either purely quantitative or exclusively qualitative.

Of the various LI models that have been advanced in research, the FL based LI has gained much attention in both qualitative and quantitative fields [3]. The qualitative FL based LI models have been well established and empirically validated in the literature, but the same cannot be reported for quantitative FL based LI models.

One reason for the popularity of FL modelling of the LI is that performance rating, against baseline or target performance, is arbitrary and imprecise, skewed towards personal judgement. Fuzzy models are based on Fuzzy set theory, which states that elements with un-sharp, non-crisp boundaries are defined by a class that has a continuum of grades of membership [12,13]. In addition, Fuzzy logic modelling is not stringent with model assumptions, and offers a simplistic yet comprehensive approach to lean performance assessment [3].

The FL based LI, like other LI models, has the ability to simultaneously assess multiple aspects of lean within the system while highlighting areas of weak lean performance. It has been applied as a benchmarking metric for internal operations [7].

A typical production system is a dynamic compilation of several heterogeneous sub-system functions and activities.
Variation is a given in manufacturing systems, and so needs to be considered in any model depicting the system. Much of the parameters used in lean assessment such as number of machine breakdowns, number of employee absenteeism, and number of employee suggestions that are implemented (Kaizen), are unpredictable. Other variables can be defined by a probability density function (pdf) often a normal distribution in terms of a mean, $\mu$, and its variance $\sigma$. The normal distribution best describes the natural occurrence of a class of lean assessment parameters such as defect rates and lead-time. Modeling LM metrics according to their stochastic nature have been established for cycle time [14,15]. Overall Equipment Effectiveness [16], throughput [17] and lead-time fulfillment [18]. It is therefore logical that the LI should also be considered as a stochastic measure. Within the extant literature on lean performance assessment and LI models, there is a paucity of research incorporating variation analysis and its effect in LM performance.

In the present study, a manufacturing case is set up for regular lean audits using the FL based quantitative LI model. Production data for a print packaging manufacturing case is imitated using a random number generator application. The FL based lean performance is then explained using: a) each daily assessment as a discrete case, b) the mean value for each month to represent average lean performance, and c) the statistical mean and variance for each month to define the range of lean performances. The former two approaches are the conventional ways of representing lean performance. The benefits of the third approach i.e. variation modelling of the LI, are afterwards discussed.

**Nomenclature**

- **LI**: lean index
- **LM**: lean manufacturing
- **KPI**: key performance indicator
- **SOP**: standard operating procedure
- **SMED**: single minute exchange of dies
- **FL**: fuzzy logic
- **TPM**: total productive maintenance
- **QM**: quality management
- **WIP**: works in process
- **$C_v$**: coefficient of variation
- **pdf**: probability density function

### 2. Description of Fuzzy Logic based Lean Index

#### 2.1. Basic concept of fuzzy logic

The two basic definitions of fuzzy set theory are:

**Definition 1**: Fuzzy set $A$ in a universe of discourse $X$ is characterized by,

$$ A = \{(x, \mu_A(x)) | x \in X, \mu_A(x) \in [0,1]\} $$

where $\mu_A(x)$ is the fuzzy membership function, which defines the degree to which $x$ belongs to $A$ and associates with each element $x$ in $X$, a real number in the interval $[0,1]$, [11].

**Definition 2**: A triangular fuzzy membership function in which ‘a’ and ‘b’ represent “target” and “baseline” (in this article) lean performance of each indicator, is defined by Eq. 2 [11].

$$\mu_A(x) = \begin{cases} 
1 - \frac{x-a}{b-a} & \text{if } x \leq a \\
\frac{x-a}{b-a} & \text{if } a < x < b \\
0 & \text{if } x \geq b
\end{cases} $$ (2)

When $X_i$ (the measured value of the performance variable) is worse than the baseline, $\mu_A(X_i)$ is assigned a value of “0”, and when $X_i$ is better than the target value, $\mu_A(X_i)$ is assigned a value of “1”.

Triangular and trapezoidal fuzzy sets are the most familiar types of FL operations, while complex membership functions, such as the Gaussian type, do not add significant advantages [19]. For these reasons, the LI model in the present study uses the triangular fuzzy set as described by Eq. 2.

The overall LI for the entire plant is taken as the average of all $\mu_A(x)$ [10,11] as defined by Eq. 3.

$$\sum_{i=1}^{n} \frac{\mu_A(X_i)}{n} \times 100$$ (3)

### 3. Methods and experimental set up

The proposed approach and methods are summarized in Figure 1 and best described using a manufacturing case (in this article it is a theoretical one).

#### 3.1. Defining the lean assessment scope and performance variables

##### 3.1.1. Lean assessment scope

The unit of analysis is the internal LM operations of a print packaging production plant having multiple product lines. The production system is a job-shop type, processing many different job orders on a daily basis. Lean management in its entirety is non-existent in the organization, however some lean practices are being implemented such as quick changeovers and cleanliness.

Manufacturing systems depend and interact with the external environment (suppliers, customers etc.), but for the intent of describing a perspective the internal operations alone are considered to be enough to be analysed.

![Fig. 1. Proposed methodology](image)

#### 3.1.2. Lean performance variables

Table 1 (column 2) describes the performance variables that were chosen for the analysis. Typically the LI has the
ability to combine an unlimited number of variables and should be made to assess all aspects of LM in the organization. This is necessary to prevent the deliberate exclusion of variables that the analyst may be biased against for example to conceal areas where there is evidently low performance. By emphasising on rate of change in LI, the whole assessment exercise is less prone to biased data. Additionally it may be better to represent a measurement variable with more than one metric. For example the measurement variable for TPM can be represented with two metrics: mean time between failure (MTBF) and mean time to repair (MTTR). However for the purpose of this article and the focus of the lean assessment on the internal operations of the plant, the few indicative KPIs in Table 1 are sufficient.

For metrics tracking WIP reduction and process related defects (QM), their definitions are straightforward and data collation is not complicated. Other metrics require proficiency when defining their measurement, because they are qualitative in nature: in such circumstances it may be convenient to use a surrogate metric. The use of surrogates is common in LM assessment [2] and can be used to define quantitative metrics for variables that are naturally qualitative in nature. In the present analysis surrogate metrics have been used to track Training and the prevalence of SOPs, and these have been defined in Table 1. In fact by using surrogate data, purely qualitative variables e.g. Management Commitment to LM practices can be included in the quantitative LI model to make lean assessment more comprehensive.

Performance sometimes tends to be biased towards operational scale and so lean metric needs to be standardized. For example WIP reduction has been standardized to quantity of job orders in the system at a specific time so that WIP and baseline is 10 minutes is achieved. One way is to use data from industry best-in-class and world-class manufacturers as a benchmark target. This is perfectly practical since seeking perfection is one of the main principles of LM. In the present analysis a mixture hypothetical baseline, hypothetical targets and best possible targets were used, since the manufacturing case is a theoretical one. In addition, part of the analysis in this article is concerned with the rate of change in lean performance (or LI) so the use of best-of-judgement baseline and target data is permissible since an absolute value for the LI is less emphasized. The chosen baseline and target for each performance variable is explained in column 3 of Table 1.

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Table 1. Lean performance variables and their metrics chosen for the case for the first month of assessment

<table>
<thead>
<tr>
<th>Lean performance criteria</th>
<th>Description of metric</th>
<th>Typical daily performance data</th>
<th>Selection of target and baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td>Change in factory efficiency</td>
<td>Normally distributed with a $\mu=0$ and $\sigma=0.01$</td>
<td>Target is 1.5% increase, while baseline is 0%</td>
</tr>
<tr>
<td>Cleanliness</td>
<td>Number of machine cleaning activities as a percentage of total number of machines</td>
<td>Normally distributed with a $\mu=30$ and $\sigma=10$</td>
<td>Target is to achieve 100% while baseline is 30%</td>
</tr>
<tr>
<td>SOP</td>
<td>Number of defects that could have been avoided if a SOP was used</td>
<td>Random between 6 and 12</td>
<td>Target is 0 while baseline is 10</td>
</tr>
<tr>
<td>Kaizen</td>
<td>Number of improvement suggestions per employee</td>
<td>Random between 0.05 and 0.2 suggestions per employee</td>
<td>Target is 1 suggestion per employee while baseline is 0.1 (i.e. 1 suggestion per 10 employees)</td>
</tr>
<tr>
<td>SMED</td>
<td>Total set-up time/total number of set-ups</td>
<td>Normally distributed with a $\mu=9$ and $\sigma=1$</td>
<td>Target is 1 minute and baseline is 10 minutes in accordance to single digit set-up time</td>
</tr>
<tr>
<td>TPM</td>
<td>Mean time between failure (total up time/number of breakdowns)</td>
<td>Up time is normally distributed with $\mu=290$ hours and $\sigma=5$. Number of breakdowns random between 3 and 10</td>
<td>Target is 1 machine breakdown per total recorded up time, while baseline is 8 breakdowns</td>
</tr>
<tr>
<td>Quality Management</td>
<td>Process related defect rate in %</td>
<td>Normally distributed with a $\mu=8$ and $\sigma=2$</td>
<td>Target is 0%, Baseline is 10%</td>
</tr>
<tr>
<td>WIP Reduction</td>
<td>Total closing process inventory as a percentage of total current work load</td>
<td>Normally distributed with a $\mu=11$ and $\sigma=1.5$</td>
<td>Target is 0%, Baseline is 10%</td>
</tr>
</tbody>
</table>


3.2. Data gathering

A period of data collation and assessment is chosen and fixed at the onset of the evaluation exercise. A period is...
chosen based on the organization’s discretion and could represent a shift, daily or weekly data. For the intent of this article, ninety assessments representing three months daily data were simulated using Microsoft Excel random number generator to depict gradual improvements in LM performance for the manufacturing case. The formulas used in the experiment to generate random numbers for the case are summarized in Table 2.

### Table 2. Formulas used to generate random numbers for the experiment

<table>
<thead>
<tr>
<th>Lean performance criteria</th>
<th>Values (x_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td>NORM.INV(RAND(),0,0.1)</td>
</tr>
<tr>
<td>Cleanliness</td>
<td>NORM.INV(RAND(),35,5)</td>
</tr>
<tr>
<td>SOP</td>
<td>NORM.INV(RAND(),35,5)</td>
</tr>
<tr>
<td>Kaizen</td>
<td>(RANDBETWEEN(5,20))÷100</td>
</tr>
<tr>
<td>SMED</td>
<td>NORM.INV(RAND(),9,1)</td>
</tr>
<tr>
<td>TPM</td>
<td>(NORM.INV(RAND();290,5))+(RANDBETWEEN(3,10))</td>
</tr>
<tr>
<td>Quality Management</td>
<td>NORM.INV(RAND();8,1)</td>
</tr>
<tr>
<td>WIP reduction</td>
<td>NORM.INV(RAND();11,1.5)</td>
</tr>
</tbody>
</table>

Randomly generated data (x_i values) for the first lean assessment is displayed in column 2 of Table 3. The interpretation of the values for the x_i is straightforward: plant efficiency (Training) decreased by 0.009%, defect rate (QM) was 8.57% and WIP was 12.13% of total job order.

### 3.3 Computation and mapping of LI

By applying the x_i, best case (a) and the worst case (b) values into Eq. 2 the µ_A(x_i) are compute for each lean performance criteria, column 5 of Table 3. The overall LI is computed from Eq. 3 and is displayed in the last row of Table 3. The values for the µ_A(x_i) and overall LI have been computed for one lean assessment, Table 3, and for multiple assessments, Table 4.

### 4. Analysis of results

#### 4.1 Representation of lean performance using single lean audits

Lean performances can be inferred for each assessment according to values displayed in Table 4. Lean performances according to Training, SOP and SMED respectively were 0%, 10% and 4.5% respectively for the first lean assessment, while overall lean performance was 6.1%. A score of 0% for Training is because the recorded x_i value was below the baseline. The physical meaning to these scores is that overall LM performance needs to improve by about 93.9% for the plant to be ideally or perfectly lean. The low scores are an indication that there is considerable need for improvement in all areas as of lean as well as the overall system.

#### 4.2 Representation of lean performance using periodic average

Another conventional way of describing lean performance is to base performance on average values over a given period. Lean performance based on monthly average values (µ_A) for the manufacturing case is displayed in Table 5.
Fig. 2. Lean performances for training, QM and the overall system, covering a ninety-day period.

Table 5. Monthly average lean performances and their standard deviations.

<table>
<thead>
<tr>
<th>Lean performance criteria</th>
<th>1st month</th>
<th>2nd month</th>
<th>3rd month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Training</td>
<td>3.0</td>
<td>3.7</td>
<td>13.4</td>
</tr>
<tr>
<td>Cleanliness</td>
<td>7.4</td>
<td>10.8</td>
<td>25.3</td>
</tr>
<tr>
<td>SOP</td>
<td>6.6</td>
<td>5.3</td>
<td>26.9</td>
</tr>
<tr>
<td>Kaizen</td>
<td>4.7</td>
<td>3.8</td>
<td>27.5</td>
</tr>
<tr>
<td>SMED</td>
<td>12.5</td>
<td>9.3</td>
<td>21.1</td>
</tr>
<tr>
<td>TPM</td>
<td>5.4</td>
<td>7.7</td>
<td>17.4</td>
</tr>
<tr>
<td>QM</td>
<td>17.5</td>
<td>9.8</td>
<td>40.1</td>
</tr>
<tr>
<td>WIP reduction</td>
<td>1.1</td>
<td>3.2</td>
<td>7.4</td>
</tr>
<tr>
<td>Overall LI</td>
<td>9.2</td>
<td>3.8</td>
<td>22.7</td>
</tr>
</tbody>
</table>

The general acceptance of representing performance with average values allows the information to be communicated and understood easily. However, the generally accepted rule may not be the ideal case. The average lean performance score for training in the 1st month was 3%. However, from Table 4 there are lean performances as low as 0% while a high lean performance of 12.9% was recorded on the 25th assessment. For the overall LI, the average for the first month was 9.2% but 14.4% was that was recorded for the 15th lean audit and 5.5% was achieved for the 30th audit. If average lean performances are to be used, it means that these two extreme but important values will be concealed from the assessment and will not be explained.

4.3. Representation of lean performance using mean and standard deviation

If enough data has been collected, lean performance can be described using statistical parameters namely mean (μ) and variance or standard deviation (σ). To improve confidence in the statistical summary, it is important to choose an appropriate time span that will contain enough data points. Since data collation and lean assessment are done on a continuous basis and it is expected that LM performance will alter with time, an appropriate time span should correspond with every noticeable change in lean performances for the system. In the present case this is equivalent to monthly data as observed in Figure 2.

The μ and σ values for each assessment month are summarized in Table 5. The statistical description has accounted for all lean performances in the month for each performance criteria and the overall LI. In the 1st month QM had an average lean performance score of 17.5% and a standard deviation of 9.8. The standard deviation for Cleanliness was 10.8 and for the overall LI it was 3.7 for the 1st assessment month. By accounting for variation this way, all lean performances for a given period can be summarised with just two parameters.

On its own one cannot judge if the variances are high (unacceptable) or low (acceptable), but by using the coefficient of variation (c_v) measure, it can be deduced if the variation is within acceptable limits.

\[ c_v = \frac{\sigma}{\mu} \]  

(4)

Using the overall LI as an example, for the 2nd assessment month the variation (c_v) increased to 3.9 compared to that of the 1st month, which was 3.8. Using Eq. 4 to compute c_v shows that the variation for the 2nd month (c_v of 0.172) was actually better than that of the 1st month (c_v of 0.413), alongside the marked improvement in overall lean performance from a mean value of 9.2% to 22.7%. Interestingly also the variation for the overall LI is fairly constant over the three-month period. As expected when multiple inputs of varying degrees of variation are converged in an average fashion like the LI, the variation for the aggregate (overall LI) takes a fairly constant value.

A histogram plot is a good tool for graphical representation of variation, Figure 3. This also supports Visual Management of the LI.

5. Discussion

Conventional methods depict lean performances with deterministic variables using values based on a single lean assessment or on average values for a given period of assessments. The benefits and weaknesses of these two approaches have been highlighted in the previous section.

Process variation is anti-lean and needs to be considered in lean assessments. By using two statistical parameters namely mean and standard deviation of a group of lean assessments, the variation within the system can be accurately accounted for. It has been shown in the previous section that this method of representation interprets the real happenings in the system,
making it more robust than the conventional methods.

The variation modelling of the LI also enables the computation of coefficients of variation, $c_v$, such that the degree of variation can be classified as high (bad) or low (good). $c_v$, that is greater than 1 for example can be considered high and unacceptable, even for lean performances that have a high mean value, say 80%. A truly lean organization will continuously strive for higher lean performances and minimal variations in performance variables.

One key benefit of modelling variables with statistical parameters, using mean and standard deviation, is that the variables (input and output) can be defined by their pdf, and therefore enable Monte Carlo type simulation analysis. With Monte Carlo Simulation (MCS) the capabilities of LI are greatly enhanced to perform:

- Scenario analysis. By using MCS methods, more accurate “what ifs” experiments can be performed using the LI model when deterministic variables are used to model the LI. For example if the number of machine breakdowns is reduced by 50%, MCS can be used to compute the likely overall LI (or overall lean performance) based on this change alone.

- Probability analysis. This allows the LI to be used for predictive purposes. For example if there are no new lean initiatives after the 3rd assessment period (month) and no new lean improvements were undertaken, then lean performances can be predicted to maintain the given values depicted for the 3rd month in Table 5 of Section 4.2.

- Correlation analysis. This allows the LI to be used to determine the cause-effect relationships between input variables and between the inputs and overall LI. For example if machine breakdown (TPM) is reduced does percentage defects (QM) also reduce?

There are limitations to modelling using variation analysis. Confidence in statistical analysis improves with the sampling size, and this is a function of the study population: the larger the sample size, the higher the confidence in the statistics. In reality it may be expedient to assess lean performances for short periods say a week, and statistical data based on a limited number of observations cannot accurately represent assessment. LM is long term in nature and because lean improvements take time to set in, it serves no purpose to describe the lean performance using few assessments.

6. Conclusion

This article has established the limitations of conventional deterministic methods of representing lean performance and shown variability modelling to be of a better technique. This was demonstrated using Fuzzy Logic based quantitative Lean Index applied to a manufacturing case with simulated data. Variation modelling of the lean Index makes the lean index a good candidate for Monte Carlo Simulation, thereby enhancing the capabilities of the LI. In this article a normal probability distribution or a random number between specific ranges were used to represent the input variables. In reality the input variables can take on multiple forms and different pdfs, and so the assumed approach of stochastic modelling of the Lean Index needs to be empirically validated in a real life case study.

References