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Guidance law with impact time and impact angle constraints

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Abstract A novel closed-form guidance law with impact time and impact angle constraints is proposed for salvo attack of anti-ship missiles, which employs missile's normal acceleration (not jerk) as the control command directly. Firstly, the impact time control problem is formulated as tracking the designated time-to-go (the difference between the designated impact time and the current flight time) for the actual time-to-go of missile, and the impact angle control problem is formulated as tracking the designated heading angle for the actual heading angle of missile. Secondly, a biased proportional navigation guidance (BPNG) law with designated heading angle constraint is constructed, and the actual time-to-go estimation for this BPNG is derived analytically by solving the system differential equations. Thirdly, by adding a feedback control to this constructed BPNG to eliminate the time-to-go error—the difference between the standard time-to-go and the actual time-to-go, a guidance law with adjustable coefficients to control the impact time and impact angle simultaneously is developed. Finally, simulation results demonstrate the performance and feasibility of the proposed approach.

1. Introduction

The impact angle and impact time are important constraints for missile's homing problem. There have been a lot of studies and applications on impact angle control for decades, from the biased proportional navigation guidance (BPNG) law with impact angle constraint[1,2] to optimal control guidance law with impact angle constraint,[3–6] from Lyapunov method[7] to the backstepping method,[8] too numerous to mention one by one. While the impact angle control is widely used to increase the lethality of warheads, the impact time control is employed to carry out a salvo attack for anti-ship missiles against close-in weapon system. The studies on the impact time control are relatively rare, because it is difficult for the missile to adjust its flight time by the normal force which only changes the velocity direction. Assuming that the heading angle is small, Jeon et al.[9] derived the closed-form solution based on the linear formulation. An impact time control guidance law is proposed which is a combination of the well-known proportional navigation guidance law with the navigation constant of 3 and the feedback of the impact time error. Based on Ref. [9], Sang and Tahk[10] proposed a guidance law switching logic for maintaining the seeker lock-on condition and a time-to-go calculation method for the missile with the limitations of maneuvering and seeker's field-of-view. Zhao and Zhou[11] proposed a time-cooperative guidance law using cooperative variables, and Zou et al.[12] proposed a decentralized time-cooperative guidance law using decentralized consensus algorithms. The

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impact time control can also be achieved by dynamic inverse method,[13] and the time-cooperative guidance is also achieved by leader–follower strategy.[14]

In contrast, the studies on the impact angle control and impact time control simultaneously are rare. Lee et al. expand their approach from Ref. [9] by including impact angle constraints to analysis, and then propose a guidance law to control both impact time and impact angle.[15] To improve the control precision of the guidance law of Ref. [15], when the terminal angle is large, Chen et al.[16] proposed a compensation method against the linearized error. Assuming the position of the target is known beforehand, Harl and Balakrishnan[17] presented a sliding mode based impact time and angle guidance law by introducing a line-of-sight (LOS) rate shaping process, in which the parameter must be tuned by hand or by off-line iterative routine. Huang et al.[18] transformed the missile's nonlinear kinematical model by using the heading angle as independent variable, and design a mid-course guidance law with impact angle and impact time constraints by using optimal control theory. The results in Refs. [17,18] may both contain singular solutions.

To date, the representative closed-form guidance law to achieve the impact time and impact angle control is only seen in Ref. [15], in which "jerk" is used as the control command (the missile's normal acceleration command must be produced by time integration of this command) and the calculation for the control command is very complex. Relative to the approach in Ref. [15], this paper proposes a new simple form of guidance law with impact time and impact angle constraints, which uses missile's normal acceleration as the control command directly. The proposed guidance law is composed of a constructed BPNG with impact angle constraint and a feedback control of the impact time error. Simulations show that the proposed guidance law has superior performance, especially for a moving or maneuvering target.

2. Problem formulation

Consider a two-dimensional homing scenario shown in Fig. 1 where the missile M has a constant speed V and the target T is stationary. R, q, θ and φ denote the range-to-go, the LOS angle, the heading angle and the lead angle in the inertial reference frame, respectively. The designated impact time and impact angle are represented as $T_{\rm d}$ and $\theta_{\rm d}$. The equations of this homing guidance problem are given by

$$\dot{R} = -V\cos\varphi, \quad R\dot{q} = V\sin\varphi, \quad \dot{\theta} = a_{\rm n}/V, \quad q = \theta + \varphi \tag{1}$$

where $a_{\rm n}$ is the missile's normal acceleration, i.e. the control command.

The designated time-to-go $\bar{t}_{\rm go}$ is the difference between the designated impact time and the current flight time t, so we have

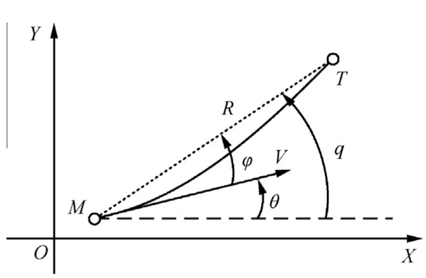


Fig. 1 Homing guidance geometry.

$$\bar{t}_{\rm go} = T_{\rm d} - t, \quad \dot{\bar{t}}_{\rm go} = -1 \tag{2}$$

The objective of simultaneously controlling the impact time and impact angle can be described as $t_{\rm go} \to \bar{t}_{\rm go}$, $\theta \to \theta_{\rm d}$ where $t_{\rm go}$ is the actual time-to-go of missile.

3. Biased proportional navigation guidance law with impact angle constraint

To obtain the analytical solution of time-to-go conveniently from guidance law with impact angle constraint, we construct a new form of BPNG law as follows:

$$a_{\rm BPNG} = NV\dot{q} - KV^2[\theta - Nq + (N-1)\theta_{\rm d}]/R \tag{3}$$

where the coefficients are chosen as $N \geqslant 3$, $K \geqslant 1$. Note that, when $N = 3$, $K = 1$, under the assumption of small angle, we have $a_{\rm BPNG} = 3V\dot{q} - V^2(\theta - 3q + 2\theta_{\rm d})/R = 3V\dot{q} + 3V^2 \times (q-\theta)/R + 2V^2(\theta - \theta_{\rm d})/R \approx 3V\dot{q} + 3V\dot{q} + 2V^2(\theta-\theta_{\rm d})/R = 6V\dot{q} + 2V^2(\theta - \theta_{\rm d})/R$.

This is in accordance with the results in Refs. [3,15].

Define

$$\alpha = \theta - Nq + (N-1)\theta_{\rm d} \tag{4}$$

Substituting $a_{\rm n} = a_{\rm BPNG}$ to Eq. (1) results in

$$\dot{R} = -V\cos\varphi \tag{5a}$$

$$\dot{\varphi} = -[(N-1)V\sin\varphi]/R + KV\alpha/R \tag{5b}$$

$$\dot{\alpha} = -KV\alpha/R \tag{5c}$$

Note that Eq. (5c) indicates that $\alpha \to 0$ and Eq. (5b) reveals that $\varphi \to 0$ when $\alpha \to 0$. From $\alpha = \theta - Nq + (N-1)\theta_{\rm d} = -N(q-\theta) - (N-1)(\theta - \theta_{\rm d}) = -N\varphi - (N-1)(\theta - \theta_{\rm d})$, we have $\theta \to \theta_{\rm d}$ finally. Thus, the guidance law (3) can achieve the desired impact angle requirement.

The time-to-go estimation of the guidance law (3) is derived in the follows.

Eliminating the time variable from Eqs. (5b) and (5c) yields

$$\frac{{\rm d}\varphi}{{\rm d}\alpha} = \frac{N-1}{K}\frac{\sin\varphi}{\alpha} - 1 \tag{6}$$

By using the approximation $\sin\varphi \approx \varphi$ (known from the above, this approximation is rational because φ goes to zero gradually during the homing guidance), the solution of the differential Eq. (6) is obtained

$$\varphi = C\alpha^{(N-1)/K} + \frac{K}{N-1-K}\alpha \tag{7}$$

where C is a constant, which is given from the initial condition

$$C = (\varphi(0) - \frac{K}{N-1-K}\alpha(0))/(\alpha(0))^{(N-1)/K} \tag{8}$$

Eliminating the time variable from Eqs. (5a) and (5c) with substitution by Eq. (7) yields

$$\frac{{\rm d}R}{R} = \frac{\cos\left(C\alpha^{(N-1)/K} + \frac{K}{N-1-K}\alpha\right)}{K\alpha}{\rm d}\alpha \tag{9}$$

Integrating Eq. (9) then we have the relation between R and α:

$$\int_{R(0)}^{R}\frac{1}{R}{\rm d}R = \int_{\alpha(0)}^{\alpha}\frac{\cos\left(C\alpha^{(N-1)/K} + \frac{K}{N-1-K}\alpha\right)}{K\alpha}{\rm d}\alpha \tag{10}$$

By Taylor series expansion of Eq. (10) over $C\alpha^{(N-1)/K} + K\alpha/(N-1-K)$, i.e. φ, with higher order terms neglected, there is

$$R = D\alpha^{1/K} e^{h(\alpha)} \tag{11}$$

where

$$D = R(0)(\alpha(0))^{-1/K} e^{-h(\alpha(0))}$$

$$h(\alpha) = -\frac{C^2}{4(N-1)}\alpha^{2(N-1)/K} - \frac{KC}{(N-1)^2 - K^2}\alpha^{(N-1)/K+1} - \frac{K}{4(N-1-K)^2}\alpha^2$$

Substituting Eq. (11) into Eq. (5c) yields

$$D\alpha^{1/K-1} e^{h(\alpha)} d\alpha = -KV dt \tag{12}$$

The integration of Eq. (12) for $t \in [0, t_f]$ (t_f represents the final time) yields

$$\int_{\alpha(0)}^{\alpha(t_f)} D\alpha^{1/K-1} e^{h(\alpha)} d\alpha = -\int_0^{t_f} KV dt \tag{13}$$

Note that $\alpha(t_f) = 0$. By using $e^{h(\alpha)} \approx 1 + h(\alpha)$ (the accuracy about this approximation is discussed in Remark 1 in detail) with substitution of C and D, the estimation of t_f is obtained from Eq. (13) as

$$\hat{t}_f = R(0) e^{C_1(\varphi(0))^2 + C_2(\varphi(0)+\alpha(0))^2}[1 + C_3(\varphi(0))^2 + C_4\varphi(0)\alpha(0) + C_5(\alpha(0))^2]/V \tag{14}$$

where

$$C_1 = \frac{1}{4(N-1+K)}$$

$$C_2 = \frac{K}{4(N-1)(N-1+K)}$$

$$C_3 = -\frac{1}{4(N-1)(2N-1)}$$

$$C_4 = -\frac{K(3N-2+K)}{2(N-1)(2N-1)(N-1+K)(N+K)}$$

$$C_5 = -\frac{K^2}{4(N-1)(2N-1)(N-1-K)^2} + \frac{K^2}{[(N-1)^2 - K^2](N+K)(N-1-K)} - \frac{K}{4(N-1-K)^2(1+2K)}$$

$\hat{t}_f$ can be regarded as the time-to-go estimation of missile at the initial time, so the time-to-go estimation at the current time t can be denoted as

$$\hat{t}_{go} = R e^{C_1\varphi^2 + C_2(\varphi+\alpha)^2}(1 + C_3\varphi^2 + C_4\varphi\alpha + C_5\alpha^2)/V \tag{15}$$

Remark 1. The accuracy of the approximation $e^{h(\alpha)} \approx 1 + h(\alpha)$ depends on the absolute value of $h(\alpha)$. From the expression of $h(\alpha)$, we obtain

$$h(\alpha(0)) = -C_1(\varphi(0))^2 - C_2(\varphi(0)+\alpha(0))^2 = -\frac{1}{4(N-1+K)} \times (\varphi(0))^2 - \frac{K(N-1)}{4(N-1+K)}(q(0) - \theta_d)^2 \leqslant 0$$

$$\dot{h}(\alpha) = \frac{1}{2}\frac{V}{R}\left(C\alpha^{(N-1)/K} + \frac{K}{N-1-K}\alpha\right)^2 \geqslant 0$$

We find that $h(\alpha(0))$ is composed of two terms: the first denotes the initial heading error of missile; the second denotes the initial difference between initial LOS angle and the designated impact angle of missile. Notice that the absolute value of $h(\alpha)$ is maximal at the initial time, and later it goes to zero with time. Thus, the maximal error of this approximation locates at the initial time, and the error goes to zero with time.

4. Guidance law design with impact time and impact angle constraints

Now we find the solution to achieve the desired impact time requirement by adding a feedback control to the BPNG law (3).

The time-to-go estimation $\hat{t}_{go}$ of guidance law (3) is considered to be accurate, i.e. $\hat{t}_{go} = t_f - t$. When $\hat{t}_{go} = \bar{t}_{go}$, i.e. $t_f = T_d$, the guidance law (3) can satisfy the impact time and impact angle constraints simultaneously, and no additional feedback control is needed.

When $\hat{t}_{go} \neq \bar{t}_{go}$, we add a feedback control to Eq. (3) to drive $\hat{t}_{go} \to \bar{t}_{go}$. Thus, we devise the closed-form guidance law as follows:

$$a_n = NV\dot{q} - KV^2\alpha/R + a_\varepsilon \tag{16}$$

where a_ε is the additional feedback control, designed to eliminate the error between the designated time-to-go and the time-to-go of BPNG, which is defined by

$$\varepsilon = \bar{t}_{go} - \hat{t}_{go} \tag{17}$$

From Eqs. (1) and (16), we have

$$\dot{\alpha} = -KV\alpha/R + a_\varepsilon/V \tag{18}$$

By observing Eq. (18), we choose the form of the feedback control as

$$a_\varepsilon = k_1 K\alpha\varepsilon \tag{19}$$

where k_1 is a positive constant, which results in a proper $\dot{\alpha}$ according to ε. Along with $\varepsilon \to 0$ and $a_\varepsilon \to 0$, Eq. (18) reduces to Eq. (5c), and the proposed guidance law (16) reduces to the BPNG law, and finally $\theta \to \theta_d$ is ensured. Thus, the proposed guidance law (16) can achieve the impact time and impact angle requirements simultaneously.

Remark 2. The feedback control law (19) maintains the dynamics of α as $\dot{\alpha} = -KV\alpha/R + k_1 K\varepsilon\alpha/V$. In general salvo attack scenarios, the designated impact time T_d is selected to ensure $\varepsilon > 0$. So the term of $k_1 K\,\varepsilon\alpha/V$ provides α a tendency of divergence if $\varepsilon \neq 0$.

Remark 3. When the limitation of missile's normal acceleration is given, the feasible interval of T_d can be calculated geographically by reference to Section 3 in Ref. [10] Using the proposed guidance law (16) and (19) as the original guidance law, the switching logic of Ref. [10] can also be used to maintain the seeker lock-on condition.

5. Simulation results and analysis

Let us first consider an engagement scenario in which the missile has a constant speed of 250 m/s and the target is a stationary ship. The initial position of the missile and the target are set to be (0, 0) km and (−10, 0.5) km. The missile's normal acceleration is limited within 5g. The parameters are taken as $N = 3$, $K = 1$, $k_1 = 7$. The missile is guided by the proposed guidance law (16) and (19). The simulation step is 0.01 s. To investigate the performance of the proposed law, three cases of simulations are carried out: (1) the designated impact time $T_{\rm d}$ is set to 50 s, designated impact angle $\theta_{\rm d}$ is set to 10°, and the initial heading angle θ_0 is taken as 0°, ±30°, and ±60° respectively; the simulation results are shown in Fig. 2; (2) the designated impact angle $\theta_{\rm d}$ is set to 10°, initial heading angle θ_0 is taken as 30°, and the designated impact time $T_{\rm d}$ is set to 45, 48, 50, and 55 s respectively; the simulation results are shown in Fig. 3; (3) the designated impact time $T_{\rm d}$ is set to 50 s, initial heading angle θ_0 is taken as 30°, and the designated impact angle $\theta_{\rm d}$ is set to 0°, ±10°, ±30° respectively; the simulation results are shown as Fig. 4. In Figs. 2–4, $\theta_{\rm f}$ represents the final heading angle and $t_{\rm f}$ the final time.

Simulation results show that the proposed guidance law can satisfy the requirements for multiple initial heading angles, multiple designated impact time and multiple designated

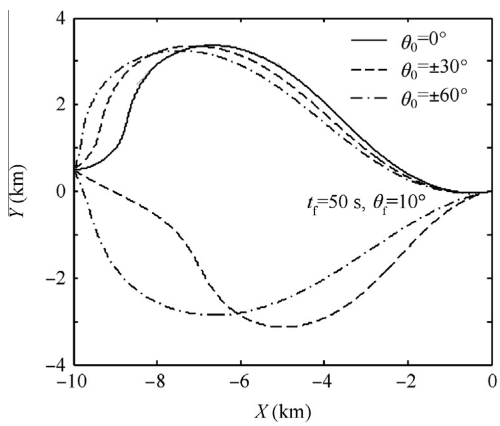


Fig. 2 Trajectories for multiple initial heading angles.

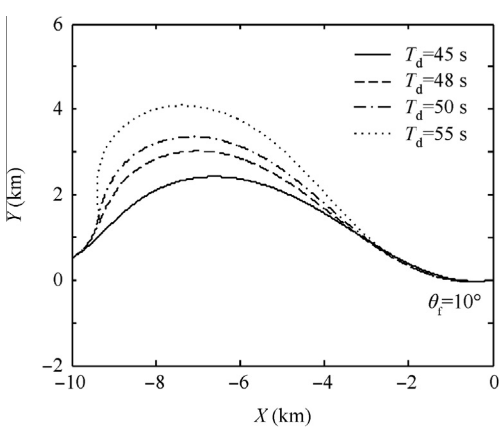


Fig. 3 Trajectories for multiple designated impact times.

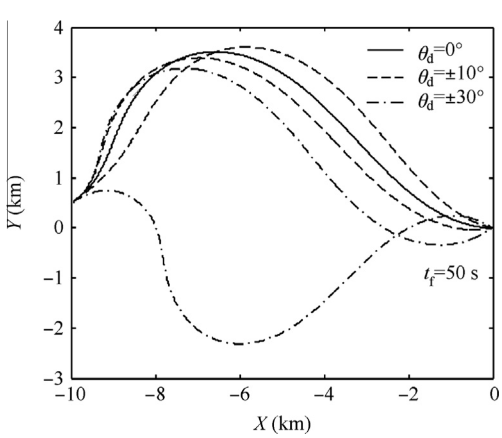
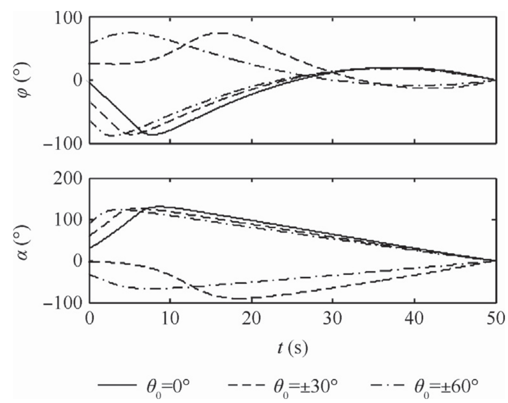
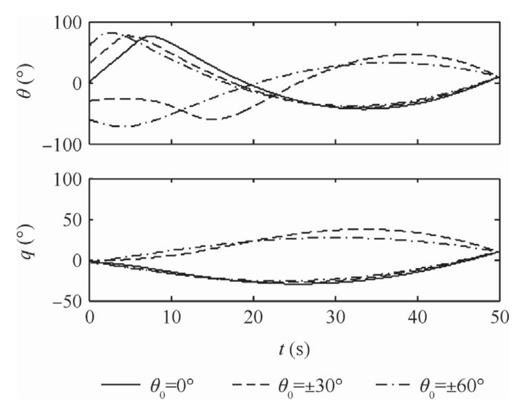


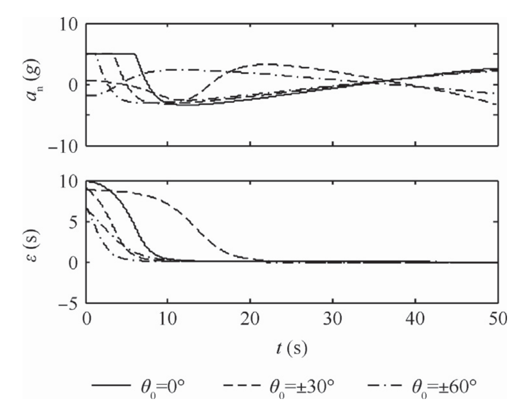
Fig. 4 Trajectories for multiple designated impact angles.



(a) Lead angle and angle α



(b) Heading angle and LOS angle



(c) Normal acceleration and time-to-go error

Fig. 5 Simulation results of the proposed guidance law for multiple initial heading angles.

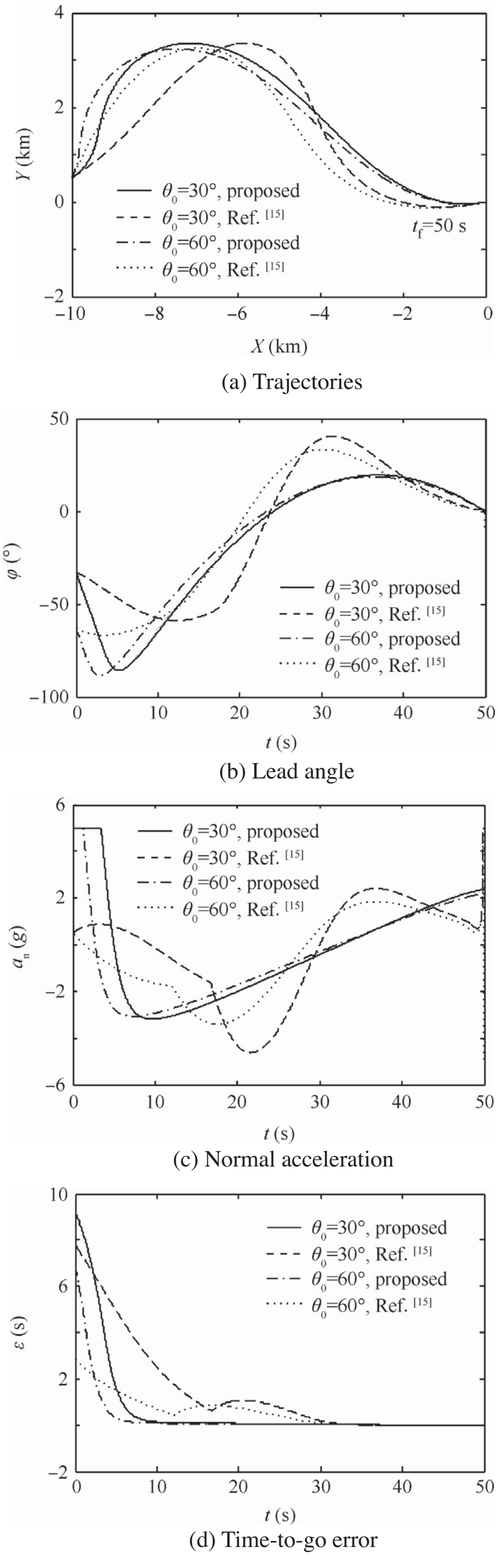


Fig. 6 Simulation results of the proposed guidance law and the guidance law in Ref. [15].

impact angles of missiles. Fig. 5 shows the concerned results of the first set of simulation correspondingly. Known from Fig. 5(c), the guidance command reaches maximal at the early stage, which drives the missile to turn slightly away from the target (reflected by the significant changes of φ and α in Fig. 5(a) at the early stage) to adjust the time-to-go. Along with the time-to-go error going to zero, the proposed guidance law becomes the BPNG law with impact angle constraint, and θ and q tend to the designated angle 10° (as shown in Fig. 5(b)). Both the impact time control and the impact angle control are achieved finally.

Fig. 6 illustrates the comparing results between the proposed guidance law and the guidance law in Ref. [15] in the first case of the initial heading angle 30° and 60°. From Fig. 6(d), we know that the time-to-go error with the proposed guidance law vanishes faster than that with Ref. [15]. To achieve the impact time control, missile is needed to maneouver during the homing guidance. From Fig. 6, we can visualize the proposed guidance law as large-maneuvering first and straight-flight second, while the guidance law in Ref. [15] as straight-flight first and large-maneuvering second (normal acceleration command even diverging finally). In actually application, the former is very valuable for missiles to implement the impact.

Then let us consider the salvo attack scenario for multi-missiles. Suppose that three missiles cooperatively attack a single target which is located at $(0, 0)$ km, the speed of each missile is 250 m/s, the designated impact time is 50 s, and the other initial parameters are shown as Table 1. The results for this simulation are shown in Fig. 7, which indicate that the proposed guidance law can be applied to the cooperative attack for multi-missiles.

Maneuvering of the target ship will bring about some error in impact time and impact angle, and even cause failure for the impact in the cooperative guidance field. To test the feasibility of the proposed law for the case of target maneuvering, the first missile in Table 1 is taken as an example. Suppose that the target speed is 20 m/s with different initial heading angles

Table 1 Initial parameters in simulation.

Missiles in group	Initial position (km)	Initial heading angle (°)	Designated impact angle (°)
M_1	(−10, 0.5)	30	10
M_2	(−6, 6)	15	−30
M_3	(−3, −10)	20	60

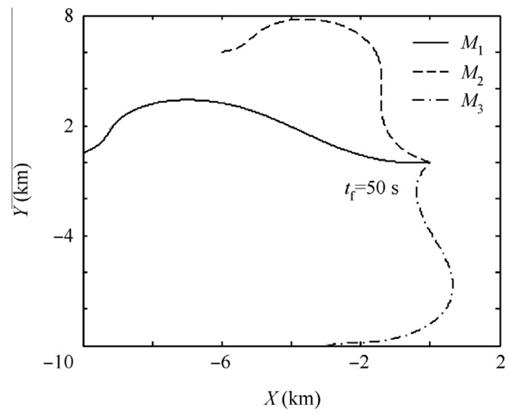


Fig. 7 Trajectories of cooperative attack for multi-missiles.

Table 2 Simulation results with target speed 20 m/s and normal acceleration 0 m/s^2.

Guidance law	(R_f, T_f, θ_f)				
	$\lambda_0 = 0°$	$\lambda_0 = 60°$	$\lambda_0 = -60°$	$\lambda_0 = 120°$	$\lambda_0 = -120°$
Proposed approach	(2.2 m, 53.8 s, 8.8°)	(0.4 m, 50.6 s, 15.1°)	(0.3 m, 53.4 s, 3.6°)	(0.4 m, 50.0 s, 4.6°)	(16.3 m, 47.4 s, 7.6°)
Ref.[15]	(2.6 m, 52.2 s, 7.6°)	(1.5 m, 50.8 s, 19.2°)	(1.3 m, 51.7 s, −3.3°)	(85.8 m, 49.9 s, 14.1°)	(19.9 m, 50.0 s, 15.0°)

Table 3 Simulation results with target speed 20 m/s and normal acceleration ± 0.7 m/s^2.

Guidance law	(R_f, T_f, θ_f)			
	$\lambda_0 = 120°, \tau = 0.7$ m/s^2	$\lambda_0 = 120°, \tau = -0.7$ m/s^2	$\lambda_0 = -120°, \tau = 0.7$ m/s^2	$\lambda_0 = -120°, \tau = -0.7$ m/s^2
Proposed approach	(19.4 m, 47.5 s, −3.8°)	(2.2 m, 50.4 s, 11.0°)	(2.0 m, 53.5 s, 7.4°)	(19.8 m, 47.3 s, 4.2°)
Ref.[15]	(106.6 m, 49.7 s, 35.5°)	(1.7 m, 51.2 s, 11.3°)	(2.4 m, 51.9 s, 4.2°)	(59.9 m, 49.9 s, 54.5°)

(denoted as λ_0). We use (R_f, T_f, θ_f) to describe the miss distance, impact time and impact angle. Notice that the designated impact time and impact angle are 50 s and 10° respectively. The simulations are stopped if $R < 2.5$ m or $\dot{R} > 0$ m/s. When the target has no normal acceleration, the results are summarized in Table 2. For the worse case of the initial heading angle of $\pm 120°$, when normal acceleration of the target (denoted as τ) is set to ± 0.7 m/s^2, the results of this maneuvering case are summarized in Table 3.

Extensive simulations demonstrate that the miss distance of the proposed approach is smaller than Ref. [15], especially when target moves against the incoming missile direction. Small miss distance is the most important in the homing. As for the accuracy of the impact time and impact angle, we find that the proposed approach has smaller impact angle control error while Ref. 15 has smaller impact time control error in most cases.

6. Conclusions

To improve the performance of impact time and impact angle control, this paper proposes a novel closed-form guidance law with impact time and impact angle constraints. Compared with the previous representative guidance law, the proposed guidance law in this paper has a simple form and takes missile's normal acceleration as the control command directly and thus simplifies the calculation complexity, which leads to an easy implementation; while the previous one takes "jerk" as the control command which must be integrated to obtain the missile's normal acceleration command, and the formula of the previous one is very complex, which leads to a difficult implementation. Extensive simulations of various engagements demonstrate that the proposed guidance law provides satisfactory performance against stationary or slightly maneuvering targets.

The proposed guidance law is designed with the consideration of stationary target, although the examples show its validity to the slow or small-maneuver targets. In future study, the design and analysis of guidance law with impact time and impact angle constraints against the moving or maneuvering targets should be considered.

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