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Random Continuous Perturbations of Random *m*-Accretive Operators in Banach Spaces

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1. INTRODUCTION

In recent years the theory of nonlinear random equations involving operators of monotone type has been studied by many authors (e.g., Kannan and Salehi [8], Itoh [6], Kravvaritis [10], and references therein). This is due to the fact that there are many applications of this theory in the physical, biological, mechanical, and engineering sciences.

In this paper we consider questions of the existence of random solutions of the nonlinear random operator equation,

$$\eta(\omega) \in T(\omega)x + g(\omega)x,$$

where $T: \Omega \times D \to 2^X (D \subset X)$ is a multivalued random m-accretive operator and $g: \Omega \times \overline{D} \to X$ is a single-valued random continuous operator. In order to prove the measurability of solutions, we establish a lemma in the first section, depending on Aumann's measurable selection (see [14]). The second section contains our main results which are the stochastic generalization of Morales' [13], Kartsatos' [9], and He's [4] deterministic theorems.

Throughout this paper X stands for a real separable Banach space, X^* its dual, and $\langle x, f\rangle$ the pairing between $x \in X$ and $f \in X^*$. For any subset D of X, cl D (or $\overline{D}$), ∂D, and wcl D stand for the closure, the boundary, and the weak closure of the set D, respectively. The symbol $\overline{B}(o, b)$ stands for a closed ball of radius b about origin of X. We denote the collection

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of all natural numbers by N. We also denote strong and weak convergence by "$\to$" and "$\rightharpoonup$", respectively.

In what follows, the symbol (Ω, Σ, μ) denotes a complete σ-finite measure space. Let $F: \Omega \to 2^X \backslash \phi$ be a multifunction with closed values. Then $F(\cdot)$ is said to be measurable if it satisfies one of the following two equivalent conditions:

(i) for all $U \subset X$ open $F^-(U) = \{\omega \in \Omega: F(\omega) \cap U \neq \phi\} \in \Sigma$;

(ii) $\operatorname{Gr} F = \{(\omega, x) \in \Omega \times X: x \in F(\omega)\} \in \Sigma \times B(X)$, $B(X)$ = Borel σ-field of X. A measurable mapping $\xi: \Omega \to X$ is said to be a measurable selector of F if $\xi(\omega) \in F(\omega)$ for any $\omega \in \Omega$. We denote by $B(\Omega, X)$ the set of all measurable mappings $\xi: \Omega \to X$.

A multivalued operator $T: D(T) \subset X \to 2^X$ is called accretive if for every $x, y \in D(T)$, $u \in Tx$, $v \in Ty$, there exists $j \in J(x - y)$ such that

$$\langle u - v, j\rangle \geq 0,$$

where $J: X \to 2^X$ is the normalized duality mapping, i.e.,

$$Jx = \{j \in X^*: \langle x, j\rangle = \|x\|^2 = \|j\|^2\}.$$

An accretive operator T is called m-accretive if $I + \lambda T$ is onto X for every $\lambda > 0$, where I is the identity operator. If T is an m-accretive operator, the resolvent J_λ, and the Yosida approximation T_λ are defined by $J_\lambda = (I + \lambda T)^{-1}$ and $T_\lambda = \lambda^{-1}(I - J_\lambda)$, respectively. It is well known that (i) $J_\lambda: X \to D(T)$ is nonexpansive, (ii) $T_\lambda: X \to X$ is Lipschitz continuous and m-accretive, and (iii) $\|T_\lambda x\| \leq |Tx|$, where $|Tx| = \inf\{\|u\|: u \in Tx\}$. An operator $T: D(T) \subset X \to X$ is said to be compact if it is continuous on $D(T)$ and maps bounded subsets of $D(T)$ into relatively compact subsets of X. T is completely continuous if it is continuous on $D(T)$ from the weak topology of X to the strong topology of X.

For an operator $T: \Omega \times X \to 2^X$, we will write $T(\omega)x$ for the value of T at $(\omega, x) \in \Omega \times X$. An operator $T: \Omega \times X \to 2^X$ is said to be random if $T(\cdot)x$ is measurable for every $x \in X$. A random operator T is called m-accretive (continuous, compact, etc.) if $T(\omega)(\cdot)$ is m-accretive (continuous, compact, etc.) for every $\omega \in \Omega$.

Now, we prove a lemma which will be used in Section 2. It is the random version of the Leray–Schauder principle.

LEMMA. *Let $T: \Omega \times X \to X$ be a random compact operator and let $D_\lambda(\omega) = \{x \in X: T(\omega)x = \lambda x\}$ for every $\omega \in \Omega$ and $\lambda > 1$. If $D(\omega) = \bigcup_{\lambda > 1} D_\lambda(\omega)$ is bounded for every $\omega \in \Omega$, then T has a random fixed point.*

Proof. By the Leray–Schauder principle, we have

$$F(\omega) = \{x \in X: T(\omega)x = x\} \neq \phi.$$

for every $\omega \in \Omega$. Since T is continuous, the set $F(\omega)$ is closed. Applying Theorm 11 of Bharucha-Reid [1], we obtain that $F: \Omega - 2^x$ is measurable. From Kuratowski and Ryll-Nardzewski [12], it follows that there is a measurable selector $x: \Omega \to X$ of F and x is a random fixed point of T.

2. Main Results

We begin with extending the result of [13, Theorem 7] in the deterministic case to the random case as follows.

THEOREM 1. *Let X^* be a separable Banach space and let $T: \Omega \times D \to 2^x$ ($D \subset X$) be a random m-accretive operator for which J_λ is a random compact operator for every $\lambda > 0$. Further, let $g: \Omega \times \overline{D} \to X$ be a random operator which is continuous and bounded. Suppose that there exist positive constants b and r such that for every $x \in D$ with $\|x\| \geq b$ there exists $j \in Jx$ satisfying*

$$r\|x\| \leq \langle u + g(\omega)x, j\rangle, \qquad \omega \in \Omega \tag{1}$$

for $u \in T(\omega)x$. Then there exists a random solution $\xi \in B(\Omega, X)$ of the operator equation

$$\eta(\omega) \in T(\omega)\,\xi(\omega) + g(\omega)\,\xi(\omega) \qquad (\omega \in \Omega) \tag{2}$$

for each $\eta \in B(\Omega, X)$ with $\|\eta(\omega)\| \leq r$ ($\omega \in \Omega$).

Proof. Define the operator $S_n: \Omega \times D \to 2^x$ by $S_n(\omega)x = T(\omega)x + (1/n)x$. Then S_n is a random m-accretive operator for $n \in N$. From condition (1) of the theorem, we have for every $x \in D$ with $\|x\| \geq b$,

$$r\|x\| \leq \langle u_n + g(\omega)x, j\rangle, \qquad \omega \in \Omega,$$

where $u \in T(\omega)x$ and $u_n = u + (1/n)x \in S_n(\omega)x$. For given $\eta \in B(\Omega, X)$ with $\|\eta(\omega)\| \leq r(\omega \in \Omega)$, we know that

$$F_n(\omega) = \{u \in X: \eta(\omega) = u + g(\omega)[S_n(\omega)]^{-1}u\} \neq 0$$

for every $\omega \in \Omega$ by virtue of Theorem 7 of Morales [13]. Take a countable dense subset $\{x_m^*\}_{m\geq 1}$ of X^*. Then, we obtain

$$F_n(\omega) = \bigcap_{m\geq 1} \{u \in X: \langle \eta(\omega) - u - g(\omega)[S_n(\omega)]^{-1}u, x_m^*\rangle = 0\}.$$

It is easy to see that for every $m \geq 1$, $\langle \eta(\omega) - u - g(\omega)[S_n(\omega)]^{-1}u, x^*\rangle$ is measurable with respect to $\omega \in \Omega$ and continuous in u, respectively. Hence, from Lemma III-14 of Castaing and Valadier [3], we obtain that $\langle \eta(\omega) - u - g(\omega)[S_n(\omega)]^{-1}u, x_m^*\rangle$ $(n \in N)$ are jointly measurable with respect to (ω, u), and hence

$$\operatorname{Gr} F_n = \bigcap_{m \geq 1} \{(\omega, u) \in \Omega \times X: \langle \eta(\omega) - u - g(\omega)[S_n(\omega)]^{-1}u, x_m^*\rangle = 0\}$$
$$\in \Sigma \times B(X).$$

Invoking Aumann's selection theorem [14, Theorem 3], we obtain measurable selector u_n of F_n. Set $x_n(\omega) = [S_n(\omega)]^{-1}u_n(\omega)$. Now x_n also is measurable. We claim that $\{x_n(\omega)\}$ is bounded for every $\omega \in \Omega$. In fact, if not, there exists a subsequence $\{x_{n(k)}(\omega)\}$ of $\{x_n(\omega)\}$ such that $\|x_{n(k)}\| \geq b$. Thus, there exists $j \in Jx_{n(k)}(\omega)$ such that

$$\langle u_{n(k)}(\omega) + g(\omega)x_{n(k)}(\omega), j\rangle = \langle \eta(\omega), j\rangle,$$
$$r\|x_{n(k)}(\omega)\| + [1/n(k)]\,\|x_{n(k)}(\omega)\|^2 \leq \|\eta(\omega)\|\,\|x_{n(k)}(\omega)\|,$$

or $\|\eta(\omega)\| > r$. This is a contradiction. Consequently, $\{x_n(\omega)\}$ is bounded for every $\omega \in \Omega$. Recalling $S_n(\omega) = T(\omega) + (1/n)I$ and $x_n(\omega) = [S_n(\omega)]^{-1}u_n(\omega)$, it is easy to find

$$x_n(\omega) = J_1(\omega)\{\eta(\omega) + [(n-1)/n]x_n(\omega) - g(\omega)x_n(\omega)\}, \tag{3}$$

where $J_1 = (T + I)^{-1}$. By the compactness of J_1 and the boundedness of g and $\{x_n(\omega)\}$ in (3), we can see that $\{x_n(\omega)\}$ is precompact for each $\omega \in \Omega$. Let $G_n(\omega) = \operatorname{cl}\{x_i(\omega): i \geq n\}$, then $G_n(\omega)$ $(n \in N)$ is compact. Hence, by Theorem 4.1 of [5], $G(\omega) = \bigcup_{n=1}^{\infty} G_n(\omega)$ is measurable. Then, there exists a measurable selector ξ of G such that $\xi(\omega) \in G(\omega)$ for every $\omega \in \Omega$. So, to each $\omega \in \Omega$, we may extract a subsequence $\{x_{n(j)}(\omega)\}$ from $\{x_n(\omega)\}$ such that $\lim_{j\to\infty} x_{n(j)}(\omega) = \xi(\omega)$. With $n(j)$ replacing n in the equality (3) and letting $j \to \infty$, we obtain that $\xi(\omega) = J_1(\omega)[\eta(\omega) + \xi(\omega) - g(\omega)\xi(\omega)] \in D$ and $\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega)$ $(\omega \in \Omega)$ in virtue of the continuity of J_1 and g. Therefore, ξ is the random solution of Eq. (2). The proof is completed.

COROLLARY. *Let X^*, T, J, and g be as in Theorem 1. Suppose that there exists $b > 0$ and a function c: $R^+ \to R^+$ with $c(r) \to +\infty$ as $r \to +\infty$ such that for every $x \in D$ with $\|x\| \geq b$ there exists $j \in Jx$ satisfying*

$$c(\|x\|)\,\|x\| \leq \langle u + g(\omega)x, j\rangle, \qquad \omega \in \Omega$$

for $u \in T(\omega)x$. *Then for any* $\eta \in B(\Omega, X)$, *there exists* $\xi \in B(\Omega, X)$ *such that*

$$\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega)$$

for all $\omega \in \Omega$.

Remark. From Lemma 2.1 of Itoh [7], we know that if T is a continuous random accretive operator, the resolvent J_λ is a random operator for every $\lambda > 0$.

The following is a multivalued and stochastic generalization of He [4, Theorem 3].

THEOREM 2. *Let* $T\colon \Omega \times D \to 2^X (D \subset X)$ *be a random m-accretive operator for which* J_1 *is a random compact operator. Let* $g\colon \Omega \times \overline{D} \to X$ *be a random continuous operator and let* $(\omega, 0) \in D$ *for every* $\omega \in \Omega$. *Suppose that there exist positive constants b and r such that for every* $x \in D$ *with* $\|x\| \geq b$ *there exists* $j \in Jx$ *satisfying*

$$(|T(\omega)0| + r)\|x\| \leq \langle g(\omega)J_1(\omega)x, j\rangle \tag{4}$$

for all $\omega \in \Omega$. *Then for any* $\eta \in B(\Omega, X)$ *with* $\|\eta(\omega)\| \leq r$ *there exists* $\xi \in B(\Omega, X)$ *such that*

$$\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega) \tag{5}$$

for all $\omega \in \Omega$.

Proof. We consider the random approximation

$$T_1(\omega)x + g(\omega)J_1(\omega)x + (1/n)x = \eta(\omega), \qquad n \in N,$$

where $\eta \in B(\Omega, X)$, $\|\eta(\omega)\| \leq r(\omega \in \Omega)$. It is equivalent to

$$x = [n/(1 + n)][(I - g(\omega)]J_1(\omega)x + [n/(1 + n)]\eta(\omega).$$

Define the operator $A_n\colon \Omega \times X \to X$ by $A_n(\omega)x = [n/(1 + n)][I - g(\omega)]J_1(\omega)x + [n/(1 + n)]\eta(\omega)$. Clearly, A_n is a random compact operator for $n \in N$. Now, we can claim that the number b is an upper bound of the subset

$$D_n(\omega) = \{x \in X\colon A_n(\omega)x = \lambda x \text{ for some } \lambda > 1\}$$

for every $\omega \in \Omega$. Indeed, suppose $\|x\| \geq b$ with $A_n(\omega)x = \lambda x$ for some $\lambda > 1$. Then, by condition (4), there exists $j \in Jx$ such that

$$\begin{aligned}\|x\|^2 &= \langle x,j\rangle = \lambda^{-1}\langle A_n(\omega)x,j\rangle \\ &\times < [n/(1+n)](\langle J_1(\omega)x,j\rangle + \langle \eta(\omega),j\rangle - \langle g(\omega)J_1(\omega)x,j\rangle) \\ &\leq \|J_1(\omega)x\|\,\|x\| + \|\eta(\omega)\|\,\|x\| - (|T(\omega)0| + r)\,\|x\| \\ &\leq \|x\|^2 + \|J_1(\omega)0\|\,\|x\| - |T(\omega)0|\,\|x\| \\ &\leq \|x\|^2 + \|T_1(\omega)0\|\,\|x\| - |T(\omega)0|\,\|x\| \\ &\leq \|x\|^2\end{aligned}$$

which is a contradiction. By virtue of the lemma in the Section 1, we obtain that A_n has a random fixed point x_n for $n \in N$, i.e.,

$$x_n(\omega) = [n/(n+1)][I - g(\omega)]J_1(\omega)x_n(\omega) + [n/(n+1)]\eta(\omega) \qquad (n \in N). \tag{6}$$

As we proved in Theorem 1, $\{x_n(\omega)\}$ is also bounded for every $\omega \in \Omega$. Hence, for each $\omega \in \Omega$, $\{J_1(\omega)x_n(\omega)\}$ contains a convergent subsequence. Similar to the proof of Theorem 1, let $F_n(\omega) = \text{cl}\{J_1(\omega)x_i(\omega)\colon i \geq n\}$ and $F(\omega) = \bigcup_{n=1}^{\infty} F_n(\omega)$. So, $F_n(\omega)$ is nonvoid and measurable for every $n \in N$. Then, there exists a measurable selector ξ of F such that $\xi(\omega) \in F(\omega)$ for every $\omega \in \Omega$. It follows that there exists a subsequence $\{x_{n(k)}(\omega)\}$ of $\{x_n(\omega)\}$ such that $J_1(\omega)x_{n(k)}(\omega) \to \xi(\omega) \in \overline{D}$ for each $\omega \in \Omega$. Hence $g(\omega)J_1(\omega)x_{n(k)}(\omega) \to g(\omega)\xi(\omega) \in \overline{D}$ for each $\omega \in \Omega$ by the continuity of g. On the other hand, from the equality (6), $T_1 = I - J_1$, and $J_1 = (T + I)^{-1}$, we see that

$$T_1(\omega)x_{n(k)}(\omega) = \eta(\omega) - g(\omega)J_1(\omega)x_{n(k)}(\omega) + \{[1/n(k)]\}x_{n(k)}(\omega) \tag{7}$$

and

$$T_1(\omega)x_{n(k)}(\omega) \in T(\omega)J_1(\omega)x_{n(k)}(\omega). \tag{8}$$

Since $\{x_{n(k)}(\omega)\}$ is bounded, $T_1(\omega)x_{n(k)}(\omega) \to \eta(\omega) - g(\omega)\xi(\omega)$ as $k \to \infty$ from (7). Finally, by the closedness of T and (8), we obtain

$$\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega) \qquad (\omega \in \Omega);$$

i.e., ξ is the random solution of Eq. (5).

The deterministic case corresponding to the following theorem was obtained by Morales [13, Theorem 3].

THEOREM 3. *Let* $T\colon \Omega \times D \to 2^X (D \subset X)$ *be a random m-accretive operator for which* J_λ *is a random compact operator for every* $\lambda > 0$. *Let*

g: $\Omega \times \overline{D} \to X$ be a random operator which is continuous and bounded. Suppose that there exist positive constants a and b such that

$$a\|x\| + \|g(\omega)x\| \leq |T(\omega)x + g(\omega)x|, \qquad \omega \in \Omega \tag{9}$$

for all $x \in D$ with $\|x\| \geq b$. Then for any $\eta \in B(\Omega, X)$, there exists $\xi \in B(\Omega, X)$ such that

$$\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega)$$

for all $\omega \in \Omega$.

Proof. Let $\eta \in B(\Omega, X)$ and select $n_o \in N$ such that $(1/n) < a$ for $n \geq n_o$. Then there exists $b_1(\omega)$ ($\omega \in \Omega$) such that $b_1(\omega) \geq b$ and $[a - (1/n)]b_1(\omega) > \|\eta(\omega)\|$ for $n \geq n_o$. Define the operator A_n: $\Omega \times X \to X$ by given $A_n(\omega)x = \eta(\omega) - g(\omega)(T(\omega) + (1/n)I)^{-1}x$. Obviously, A_n is a random compact operator for $n \in N$. Now, we claim that for every $\omega \in \Omega$

$$D_n(\omega) = \{x \in X\colon A_n(\omega)x = \lambda x \text{ for some } \lambda > 1\}$$

is bounded. Let $x \in D_n(\omega)$. Then

$$\lambda x + g(\omega)(T(\omega) + (1/n)I)^{-1}x = \eta(\omega)$$

for some $\lambda > 1$. Suppose $\|u\| \geq b_1(\omega)$ with $u = (T(\omega) + [(1/n)I])^{-1}x$ and $v \in T(\omega)u$, from condition (9) we have that

$$\begin{aligned}\|\eta(\omega)\| &\geq \lambda^{-1}\|\eta(\omega)\| = \|v + \lambda^{-1}g(\omega)u + (1/n)u\| \\ &\geq \|v + g(\omega)u\| + \lambda^{-1}\|g(\omega)u\| - \|g(\omega)u\| - (1/n)\|u\| \\ &\geq [a - (1/n)]\,\|u\| \geq [a - (1/n)]b_1(\omega),\end{aligned}$$

which is a contradiction. Therefore $\|u\| < b_1(\omega)$. It follows from the boundedness of g that there exists $M(\omega) > 0$ such that $\|g(\omega)u\| \leq M(\omega)$ and, hence,

$$\|x\| \leq \lambda\,\|x\| \leq \eta(\omega)\| + \|g(\omega)u\| \leq \|\eta(\omega)\| + M(\omega).$$

Thus, by our lemma we know that A_n has a random fixed point $x_n(\omega)$ for $n \geq n_o$. Let $u_n(\omega) = (T(\omega) + (1/n)I)^{-1}x_n(\omega)$. Clearly, u_n is measurable and bounded for $n \geq n_o$. Then, as we proved in Theorem 1, there exists $\xi \in B(\Omega, X)$ such that

$$\eta(\omega) \in T(\omega)\xi(\omega) + g(\omega)\xi(\omega)$$

for all $\omega \in \Omega$. This completes the proof.

Finally, we give a stochastic generalization of Theorem 6 of Kartsatos [9] as follows.

THEOREM 4. *Let X, X^* both be separable uniformly convex Banach spaces. Let $T: \Omega \times D \to X$ $(D \subset X)$ be a random m-accretive operator for which J_λ is a random operator for all $\lambda > 0$. $g: \Omega \times \overline{B}(o, b) \to X$ is a completely continuous random operator and that $(\omega, o) \in \Omega \times D$ for every $\omega \in \Omega$. Suppose further that there exists $r > 0$ such that for every $x \in \partial B(o, b)$ there exists $j \in Jx$ satisfying*

$$(\|T(\omega)o\| + r)b \leq \langle g(\omega)x, j\rangle, \qquad \omega \in \Omega. \tag{10}$$

Then for any $\eta \in B(\Omega, X)$ with $\|\eta(\omega)\| \leq r$ there exists $\xi \in B(\Omega, X)$ such that

$$\eta(\omega) = T(\omega)\xi(\omega) + g(\omega)\xi(\omega) \tag{11}$$

for all $\omega \in \Omega$.

Proof. Let $\eta \in B(\Omega, X)$ with $\|\eta(\omega)\| \leq r(\omega \in \Omega)$, and let $a > 0$. Define the operator S_n: $\Omega \times X \to X$ by $S_n(\omega)x = T_{1/n}(\omega)x + ax$. Then S_n is a continuous random m-accretive operator for $n \in N$ and $\|S_n(\omega)o\| \leq \|T(\omega)o\|$ $(\omega \in \Omega)$. From Theorem 6 of Kartsatos [9] and condition (10) of the theorem, we have

$$F_n(\omega) = \{\|x\| \leq b\colon \eta(\omega) = S_n(\omega)x + g(\omega)x\} \neq \phi$$

for every $\omega \in \Omega$. Pick a countable dense subset $\{x_m^*\}_{m\geq 1}$ of X^*; then

$$F_n(\omega) = \bigcap_{m\geq 1} \{\|x\| \leq b\colon \langle \eta(\omega) - S_n(\omega)x - g(\omega)x, x_m^*\rangle.$$

Because of our hypotheses, for every $m \geq 1$, $\langle \eta(\omega) - S_n(\omega)x - g(\omega)x, x_m^*\rangle$is measurable with respect to $\omega \in \Omega$ and $\langle \eta(\omega) - S_n(\omega)x - g(\omega)x, x_m^*\rangle$ is continuous in u. By Lemma III-14 of Castaing and Valadier [3], we know that $\langle \eta(\omega) - S_n(\omega)x - g(\omega)x, x_m^*\rangle$ is jointly measurable. Hence,

$$\operatorname{Gr} F_n = \bigcap_{m\geq 1} \{(\omega, x) \in \Omega \times X\colon \langle \eta(\omega) - S_n(\omega)x - g(\omega)x, x_m^*\rangle = 0,$$
$$\|x\| \leq b\} \in \Sigma \times B(x).$$

Applying Aumann's selection theorem [14, Theorem 3], we obtain a measurable selector x_n of F_n with $\|x_n(\omega)\| \leq b(\omega \in \Omega)$, i.e.,

$$T(\omega)J_{1/n}(\omega)x_n(\omega) + g(\omega)x_n(\omega) + ax_n(\omega) = \eta(\omega), \qquad \omega \in \Omega, n \in N. \tag{12}$$

Since g is compact, there exists a subsequence $\{x_{n(k)}(\omega)\}$ of $\{x_n(\omega)\}$ such that $\{g(\omega)x_{n(k)}(\omega)\}$ is convergent. Setting $u_{n(k)}(\omega) = J_{1/n(k)}(\omega)x_{n(k)}(\omega)$ and recalling the definition of resolvent $J_{1/n(k)}$, we find that

$$x_{n(k)}(\omega) = u_{n(k)}(\omega) + [1/n(k)]T(\omega)u_{n(k)}(\omega)\} \tag{13}$$

is also bounded. So, we deduce that

$$\lim(x_{n(k)}(\omega) - u_{n(k)}(\omega)) = 0 \qquad (\omega \in \Omega) \tag{14}$$

from (13). Combining (12) with (14), we know that $\{T(\omega)u_{n(k)}(\omega) + au_{n(k)}(\omega)\}$ is a convergent sequence. Since $T + aI$ is strong accretive, $\{u_{n(k)}(\omega)\}$ converges to a $u(\omega)$ $(\omega \in \Omega)$ and $x_{n(k)} \to u(\omega)$ as $k \to \infty$. Let $G_n(\omega) = \mathrm{cl}\{x_i(\omega): i \geq n\}$. Then $G_n(\omega)$ is a nonvoid and compact set for every $\omega \in \Omega$ $(n \in N)$. By Theorem 4.1 of [5], $G(\omega) = \bigcap_{n=1}^{\infty} G_n(\omega)$ is measurable. Hence, there exists a measurable selector ξ of G such that $\xi(\omega) \in G(\omega)$ for every $\omega \in \Omega$. For a fixed $\omega \in \Omega$, we may extract a subsequence $\{x_{n(k)}(\omega)\}$ of $\{x_n(\omega)\}$ such that $\lim x_{n(k)}(\omega) = \xi(\omega)$. From (12), (14), the closedness of T, and the continuity of g, and letting $j \to \infty$, we obtain

$$T(\omega)\xi(\omega) + g(\omega)\xi(\omega) + a\xi(\omega) = \eta(\omega) \qquad (\omega \in \Omega),$$

where $\|\xi(\omega)\| \leq b$ $(\omega \in \Omega)$. Taking a positive sequence $\{a_n\}$ with $a_n \to 0$ $(n \to \infty)$, the equation

$$T(\omega)x + g(\omega)x + a_n x = \eta(\omega) \tag{15}$$

has a random solution ξ_n $(n \in N)$ with $\|\xi_n(\omega)\| \leq b$ $(\omega \in \Omega)$ by virtue of the previous conclusion. For each $n \in N$, we define a mapping $\Gamma_n\colon \Omega \to B(0, b)$ by given $\Gamma_n(\omega) = \mathrm{wcl}\{\xi_i(\omega): i \geq n\}$. By the reflexivity of X, it must be nonvoid. Since $B(0, b)$ is a metrizable separable space in the weak topology, the mapping Γ_n is weakly measurable [3, p. 67]. Then the mapping $\Gamma\colon \Omega \to \mathrm{wcl}\, B(0, b)$ defined by

$$\Gamma(\omega) = \bigcap_{n=1}^{\infty} \Gamma_n(\omega)$$

is also weakly measurable [3, Proposition III.4]. Invoking Aumann's selection theorem, we obtain a measurable selector ξ of Γ. Therefore, to each $\omega \in \Omega$, there exists a subsequence $\{\xi_{n(j)}(\omega)\}$ of $\{\xi_n(\omega)\}$ such that $\xi_{n(j)}(\omega) \to \xi(\omega)$ as $j \to \infty$. Recall that $\xi_{n(j)}(\omega)$ is the random solution of Eq. (15), i.e.,

$$\begin{aligned} &T(\omega)\xi_{n(j)}(\omega) + g(\omega)\xi_{n(j)}(\omega) + a_{n(j)}\xi_{n(j)}(\omega) \\ &= \eta(\omega) \qquad \omega \in \Omega, j \in N. \end{aligned}$$

Taking note of the m-accretiveness of T and the complete continuity of g and letting $j \to \infty$ and applying [9, Lemma 1], we conclude that $\xi(\omega)$ is a random solution of Eq. (11). The proof is complete.

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