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Sress and strain concentration factors for plate with small notch subjected to biaxial loading – Three dimensional finite element analysis

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Three dimension finite elements

Abstract The through-thickness variations of stress and strain concentration factors for plate with small central notch, circular notch or double U-notch, subjected to uniaxial and biaxial loading have been systematically analyzed by using three dimensional finite element method (3D FEM). It is found that the maximum stress and strain concentration factors occur on the mid plane of plate only in the case of thin plate. However, in the case of thick plates, the sites of these maximum values are found near the plate surface. Furthermore, this site is more close to the plate surface in the case of small notch radius and/or large plate thickness. The stress and strain concentration factors increase with decreasing the biaxial ratio at the plate interior, while, the opposite trend is found at the plate surface.

1. Introduction

Notches in structural components are places where stresses concentrate and, because of that, can trigger cracks leading to catastrophic failure or to a shortening of the assessed structural life. For very thin and very thick plates, corresponding to the so-called plane stress and plane strain state, respectively, the stress concentration factors (SCFs) are close to that given by Peterson [1,2]. But for the plate with intermediate thickness, the SCF shows a maximum as plate thickness increases [2]. The SCF in the interior of a plate with a hole or notch is significantly higher than the value on the free surface or the corresponding planar solution [3–8]. On the other hand, in engineering practice, SCF measurement is always made on the free surfaces. This may cause danger in engineering applications without proper consideration of the 3D stress distributions. With reference to U and V-notches, Li and Guo [9] also suggested that the assumption constant strain through the thickness can only be used in very thin or very thick plates and this kind of assumption was not suitable near the free

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surface. She and Guo [10] show that through thickness variation of SCF for elliptic holes subjected to tensile stress is very slight in the thin plate and increases with increasing the plate thickness. The corresponding peak value of SCF is located at the center of thickness for thinner plate but shifts to the free surface in thick plate under uniaxial loading [10]. Experimental evidence shows that for relatively thin plates the crack either originates at the corner, where the hole meets the free surface of the plate, or at the center of the plate. On the other hand for relatively thick plates the crack almost always originates in the vicinity of the corner [11]. Of course, cracks in general originate from small imperfections or discontinuities that may be present in the structure. Thus, one of the explanations for this phenomenon is that such discontinuities are most likely to be present in the vicinity of the free surface. Another possible contributing factor into this phenomenon is that in the corner or mid-plate region the stress level may actually be higher or lower than those predicted by the plane theory of elasticity [12,13]. Folias and Wang [14] show the stress concentration factor to be sensitive to the ratio of notch depth to plate thickness and to Poisson's ratio v.

1.1. Statement of the problem

Knowledge of the three-dimensional stress concentration factor is a prerequisite for an accurate design of structural components. The actual three-dimensional stress and strain fields near a curved boundary are very complex and there are only few analytical three dimensional solutions available in the literature for non-trivial geometries and particular boundary conditions [10–17]. To the best of the author's knowledge, this research is the first to study the effect of biaxial stress and notch radius on the 3-D stress/strain concentration factors for plate with small central notch. The previous methods of the analysis for uniaxial loading [10–17] will be adopted in the present work for biaxial loading. The main objective of the present work is to study the influence of the notch radius and plate thickness on the stress and strain concentration factors and their relations of elastic finite thickness plate containing a circular or double U-notch subjected to far ends uniform biaxial loading using 3D FEM.

2. Computational procedures and modeling

2.1. Mechanical properties and geometrical definition

The mechanical properties of the plate were as follows: modulus of elasticity, $E = 206$ GPa, and Poisson's ratio, $v = 0.3$. As illustrated in Fig. 1, the plate was 200 mm height, $2H$, 200 mm width, $2W$, and had a thickness, $2B$, ranged from 1 mm to 10 mm with step 1 mm. The notch depth (D) for double U-notch was 1 mm and notch root, $\rho = 0.25$ and 0.111 mm. The notch root for circular notch was 1 mm. The plate was loaded with a uniform axial stress in either uniaxial or biaxial loading. The plate was analyzed for uniaxial loading by applied stress in Y-axis direction, $\sigma_y = \sigma$, of 50 MPa uniformly applied at its ends. In biaxial loading case the plate was loaded by load in Y direction by σ and loaded in X direction by $\sigma_x = \lambda\sigma$ at plate ends as shown in Fig. 1. Although the problem is linear, i.e. two simulation with $\lambda = 0$ and 1 is sufficient

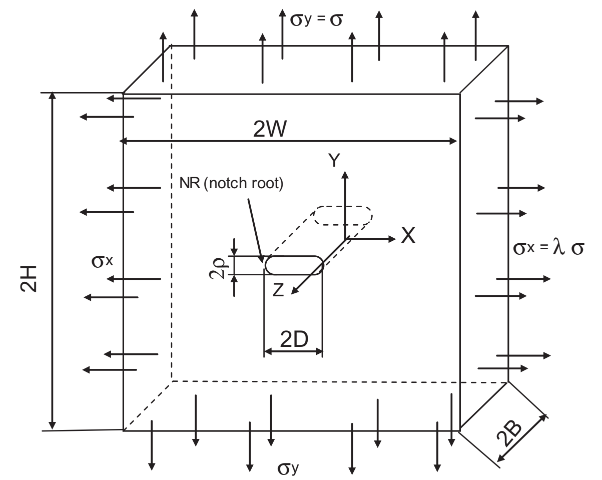

Figure 1 The geometry of the plate with an double U-shape notch under biaxial loading.

to determine the SCF at other values of λ by using the principle of superposition, the variation of SCF through the thickness is varied. Therefore, nine values of the biaxial ratio, λ, ranged from -1 to 1 with step 0.25, were chosen to show the effect of λ through the thickness. The plane x–y (plane $z = 0$) is the mid plane of plate and two plate surface are $z = B$ and $z = -B$, respectively.

2.2. Some analytical preliminaries

In this paper, the stress and strain concentration factors are denoted as K_σ and K_ε, respectively, and the stress and strain concentration factors at the mid point of the plate are $K_{\sigma mp}$ and $K_{\varepsilon mp}$, respectively, according to Yang et al. [17].

$$K_\sigma = \frac{\sigma_{NR}}{\sigma_{net}} \tag{1}$$

$$K_{\sigma mp} = K_\sigma \quad \text{at } z = 0$$

$$K_\varepsilon = \frac{\varepsilon_{NR}}{\varepsilon_{net}} \tag{2}$$

$$K_{\varepsilon mp} = K_\varepsilon \quad \text{at } z = 0$$

where

$$\sigma_{net} = \frac{\sigma_y W}{W - D} \tag{3}$$

$$\varepsilon_{net} = \frac{\sigma_y W}{E(W - D)} \tag{4}$$

Here, σ_{NR} and ε_{NR} are the longitudinal stress and strain in Y direction of plate at the notch root; σ_{net} and ε_{net} are the mean stress and strain of net section on the ligament, respectively. The K is meaning stress or strain concentration factor. The maximum values of K_σ and K_ε along the notch root are denoted as $K_{\sigma max}$ and $K_{\varepsilon max}$, respectively.

2.3. Finite element analysis

The general purpose finite element program ABAQUS was used [18]. A three-dimensional finite element model has been developed to account for geometric and material behavior of isotropic steel plate. The plate material is homogeneous, isotropic and elastic. The finite element meshes constructed with hexagonal structural mesh, C3D8 (8-node linear brick) elements, are used for steel plate under Standard/static analysis. Around from 20 to 120 planar layers are divided through the thickness of the plate varying with the plate thickness. Within each layer, the size of element decreases gradually with distance from the notch root decreasing. This means that the finite element (FE) meshes in the neighborhood of the notch are much denser. The ratio of the small size element to the notch root is kept constant equals 0.1.

3. Results and discussion

3.1. Uniaxial loading

Fig. 2 shows the influence of the notch root radius on the normalized stress concentration factor (the stress is normalized by each value of mid plane), $K_{\sigma}/K_{\sigma mp}$, due to uniaxial loading for different dimensionless plate thickness (B/D = 0.5, 2.5 and 5). The normalized SCF increases with decreasing the notch radius, ρ. The maximum value of K_{σ} occurs on the mid plane in thin plate only, see Fig. 2(a). However, for the thicker plates, the site of peak value of the K_{σ} is closed to the free surface of plate. This site is more closed to the free surface of the plate for small notch radius and/or large B/D. The present trend is in agreement with those already found in the literature [13,14,17]. For $D/\rho = 1$ and $B/D = 2.5$, the present work is identical to those obtained by Yang et al. [17], Fig. 2(b). Yang et al. [17] concluded that "the maximum stress and strain concentration factors do not always occur on the mid plane of the plate. They occur on the mid plane only in thin plates. The location of maximum concentration factor moves away from the mid plane of plate by increasing plate thickness". This means that, the distance between the site of crack initiation and the outer surface of the plate decreased with decreasing the notch radius and/or increasing the plate thickness.

3.2. Biaxial loading

Fig. 3 presents the variation of mid plane stress and strain concentration factors with λ for different dimensionless D/ρ with $B/D = 2.5$. For all values of D/ρ, the variation of either $K_{\sigma mp}$ or $K_{\varepsilon mp}$ with λ is linear where $K_{\sigma mp}$ and $K_{\varepsilon mp}$ decrease with increasing λ. This may due to increasing the compression stress at the notch root by increasing λ. In the case of uniaxial loading, i.e. $\lambda = 0$, $K_{\sigma mp}$ is close to that given by Peterson [1].

The variation of normalized SCF ($K_{\sigma}/K_{\sigma mp}$) through the plate thickness for circular notch under biaxial loading is shown in Fig. 4(a) and (b) for B/D equals 2.5 and 5, respectively. The peak value of normalized, $K_{\sigma}/K_{\sigma mp}$, increases by increasing the plate thickness. The effect of the plate thickness on the distribution of normalized SCF is similar to the case of uniaxial loading, i.e. the site of maximum normalized SCF. For λ equals 1 the normalized SCF has a constant value equal 1 through the plate thickness for $B/D = 2.5$ and 5. Fig. 5 illustrates the variation of normalized strain concentration factor, $K_{\varepsilon}/K_{\varepsilon mp}$, (the strain is normalized by each value of mid plane) versus z/B for different λ. he normalized strain concentration

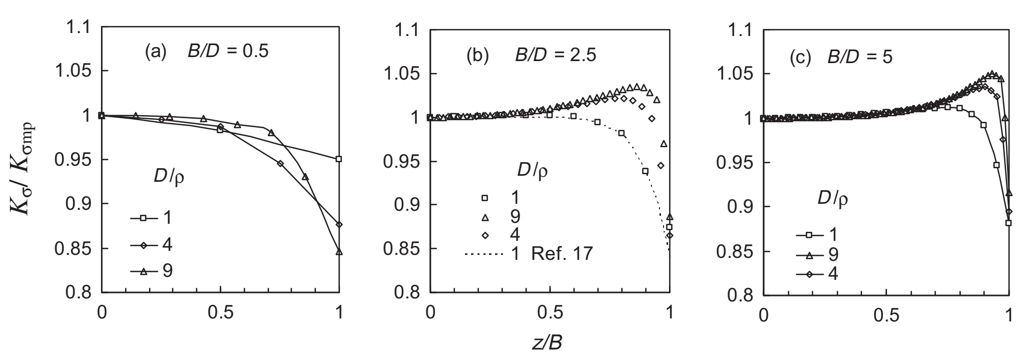

Figure 2 The through-thickness distributions of the normalized stress concentration factor along different notch root for thickness (a) $B/D = 0.5$, (b) $B/D = 2.5$ and (c) $B/D = 5$.

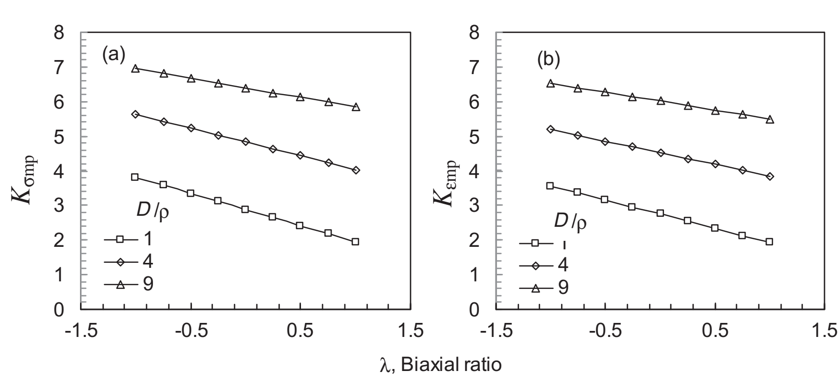

Figure 3 The (a) stress and (b) strain concentration factor at the mid point of the notch for different notch root.

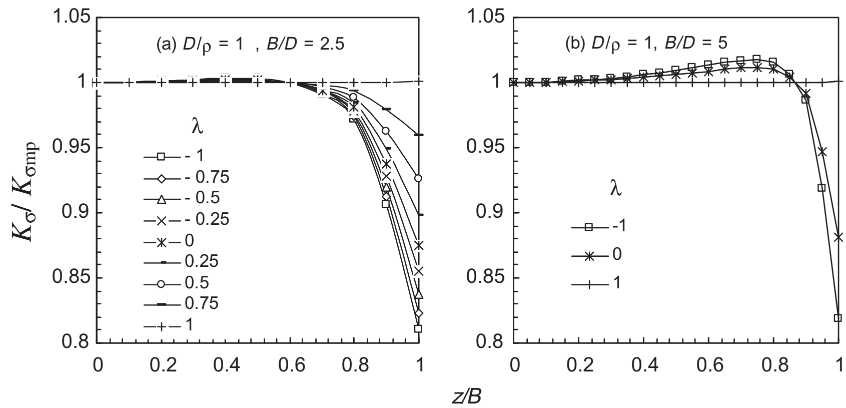

Figure 4 The through-thickness distributions of the stress concentration factor for thickness (a) $B/D = 2.5$ and (b) $B/D = 5$ along notch root for different biaxiality ratio λ.

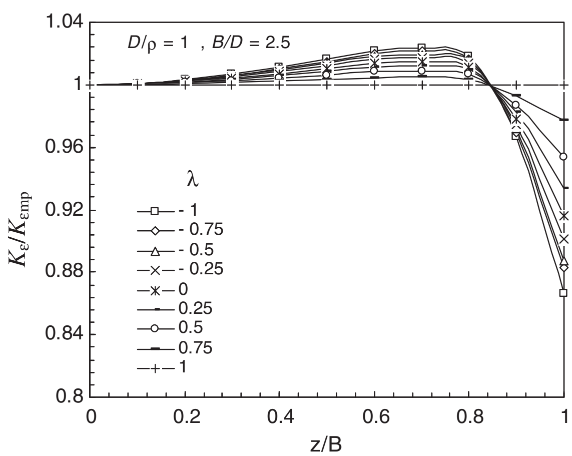

Figure 5 The through-thickness distributions of strain concentration factor along notch root for different, λ, biaxial ratio.

factor also has a constant value equal 1 for $\lambda = 1$. For others values of λ, the normalized strain concentration factor increases to reach to its peak value and then decreases by increasing the dimensionless z/B.

The previous curves representing the variation of either the normalized $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ versus z/B can be divided into three regions. In the first region $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ increased with increasing z/B to reach its peak value. In this region, the $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ increase with decreasing λ for the same z/B. In the second region, all values of $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ decrease with increasing z/B by different values of decrement to intersect at one point ($z/B \approx 0.82$ and $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp} = 1$). In the third region ($K < K_{mp}$), the previous decrement continued to get an opposite trend on the plate surface, i.e. the highest values of $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ for $\lambda = 1$ not -1 as found in the interior. Therefore, it can be concluded that the highest value of $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ at the plate interior is for $\lambda = -1$, while, $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ for $\lambda = 1$ have a constant value, i.e. equal 1, through the plate thickness and this value is the highest value at the plate surface.

Fig. 6 shows the effect of z/B and λ on $K_\sigma/K_{\sigma mp}$ and $K_\varepsilon/K_{\varepsilon mp}$ for double U-notch, i.e. $D/\rho = 4$ and 9. The K_σ and K_ε increase gradually to their peak values $K_{\sigma max}$ and $K_{\varepsilon max}$, respectively, then decrease gradually with increasing z/B. It is important to observe that the trend of ($K_\sigma/K_{\sigma mp} - z/B$) and ($K_\varepsilon/K_{\varepsilon mp} - z/B$) curves are almost the same. $K_\sigma/K_{\sigma mp}$ and $K_\varepsilon/K_{\varepsilon mp}$ are not constant for $\lambda = 1$ as found in the case of circular notch. It is worth to note that, for $\lambda = 1$ and $D/\rho \neq 1$, neither $K_\sigma/K_{\sigma mp}$ nor $K_\varepsilon/K_{\varepsilon mp}$ equal 1 and are constant through z/B.

Fig. 7 illustrates the influence of the z/B and biaxial ratio on K_ε/K_σ for different values of D/ρ, ($D/\rho = 1$, 4, and 9). Generally, K_ε/K_σ was increasing by increasing λ. For all D/ρ, there is no effect of λ on K_ε/K_σ at the plate surface and $K_\varepsilon/K_\sigma = 1$. At mid plane, the difference between K_ε/K_σ value at $\lambda = 1$ and at $\lambda = -1$ decreases with increasing the value D/ρ and this difference decreased by increasing z/B.

3.3. Correlation of SCF

To correlate $K_{\sigma max}$ with λ, the variation of $K_{\sigma max}$ versus λ is drawn in Fig. 8 for different dimensionless D/ρ. For the same λ, $K_{\sigma max}$ increases by increasing D/ρ. For the same D/ρ, $K_{\sigma max}$ decreases with an increase in λ. On the other hand, 3D stress distribution around the site of $K_{\sigma max}$ is presented in the figure for different notch roots. This distribution describes the location of high stress located at the notch root of the plate.

The present numerical results of maximum value of SCF obeyed the following relation:

$$K_{\sigma the} = (1-\lambda) + \left(2 + 0.0436\lambda\sqrt{\frac{D}{\rho}}\right)\sqrt{\frac{D}{\rho}}$$

For the circular hole, i.e. $D/\rho = 1$ the above equation leads to:

$$K_{\sigma the} = 3 - 0.96\lambda$$

Which is in agreement with derived SCF by Peterson [1] using the superposition of single circular hole in an infinite thin element under biaxial in-plane stress, $K_{\sigma the} = 3 - \lambda$. And, if the plate is loaded by uniaxial load, i.e. $\lambda = 0$ the above correlation leads to:

$$K_{\sigma the} = 1 + 2\sqrt{\frac{D}{\rho}}$$

Also, it is identical to this obtained by Peterson [1].

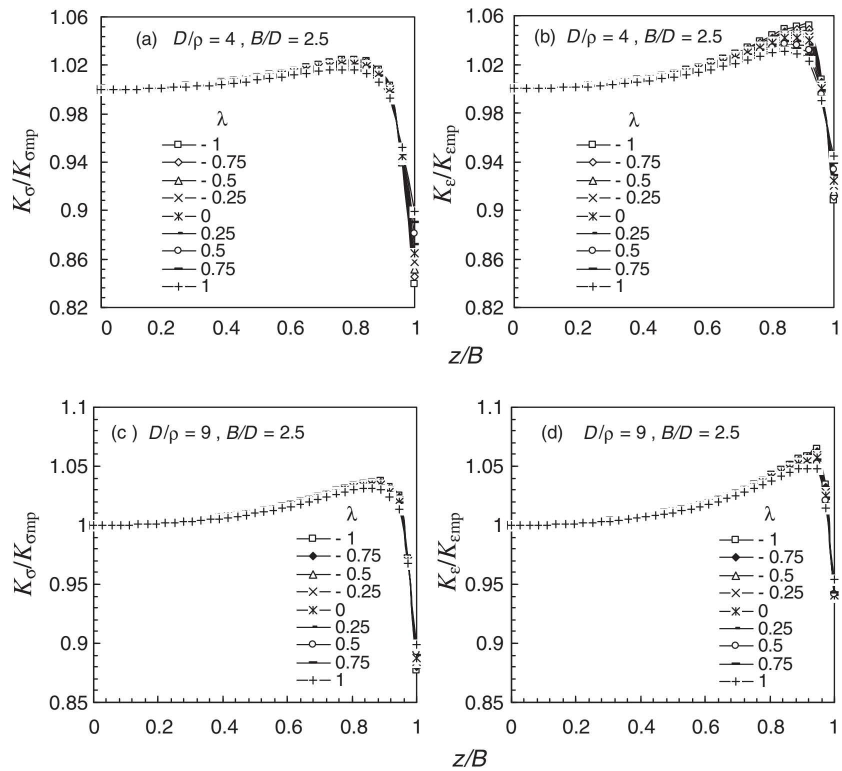

Figure 6 The through-thickness distributions of the (a and c) stress and (b and d) strain concentration factor along different notch root (a and b) $D/\rho = 4$, and (c and d) $D/\rho = 9$.

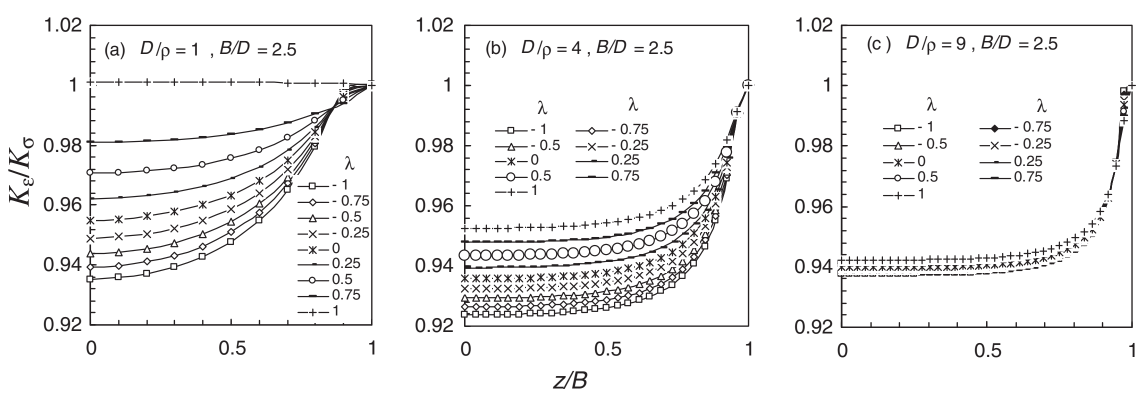

Figure 7 The K_ε/K_σ distribution along different notch root (a) $D/\rho = 1$, (b) $D/\rho = 4$, and (c) $D/\rho = 9$ for different biaxial ration.

3.4. Probable site of crack initiation

The theoretical results based on the elastic analysis [13] provide a convincing explanation of the phenomenon associated with the preferable crack formation region as for thin plate relatively plates $D/B > 1$ the maximum of stress is located on the mid-plane and the failure is expected to initiate from this location. At very law ratios D/B (relatively thick plates) the stress distribution across the thickness has a maximum in the vicinity of the free surface, and hence, the preferred crack initiation site of thick plates is likely in this region. Moreover, Hammouda et al. [19,20] concluded based on their experimental and numerical results that either the elastic analysis or elastic-plastic analysis is adequate to predict the site of crack initiation from elliptical oblique notches. Therefore, the sites of the maximum values of maximum tensile stress, σ_{yy}, or maximum von-Mises equivalent stress, σ_{eq}, or maximum tensile strain, ε_{yy}, were traced to predict the site of crack initiation, for different D/ρ and B/D. Fig. 9 depicts the site of crack initiation from notch roots of plate subjected to uniaxial loading with different D/ρ and B/D based on maximum σ_{yy}, von-Mises stress, and ε_{yy}. In the case of thin plate with $D/\rho = 1$ (circular notch), the predicted site of crack initiated is at the mid plan for all the different criteria. In the case of thin plate with $D/\rho = 4$ or 9, the predicted site of crack initiation depends on the adopted criterion. For the same plate thickness, the

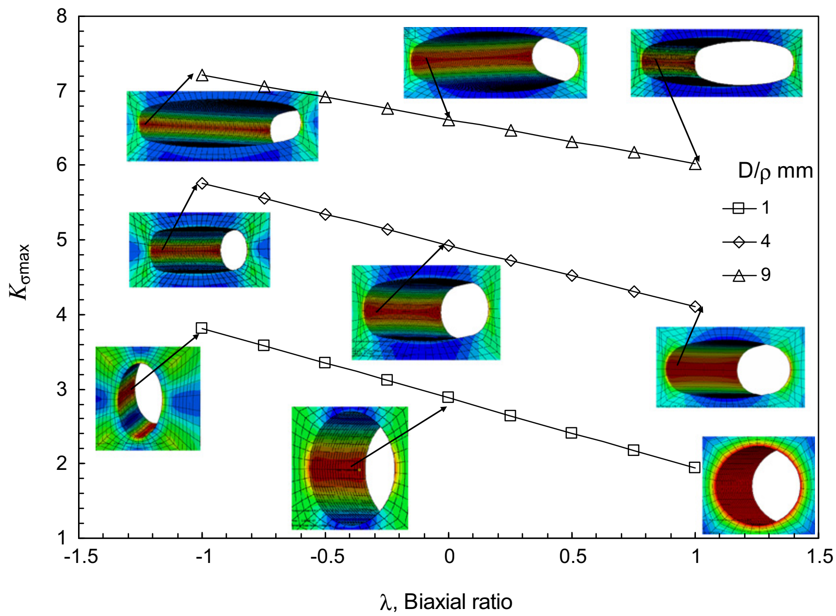

Figure 8 Relation between max stress concentration factor and biaxial ratio for different notch radius.

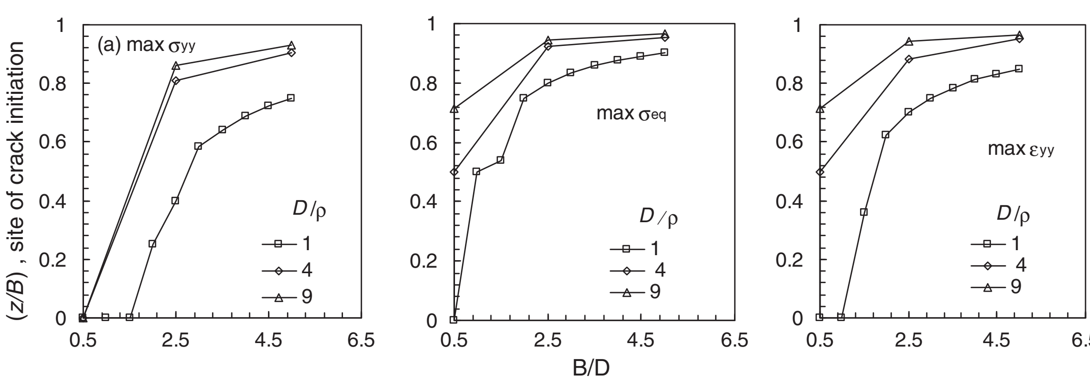

Figure 9 The site of crack initiation from notch root through the plate thickness under unaxial loading for different crack criteria (a) maximum σ_{yy}, (b) maximum σ_{eq} and (c) maximum $\varepsilon_{yy,}$ with deferent notch root $D/\rho = 1$, $D/\rho = 4$ and $D/\rho = 9$.

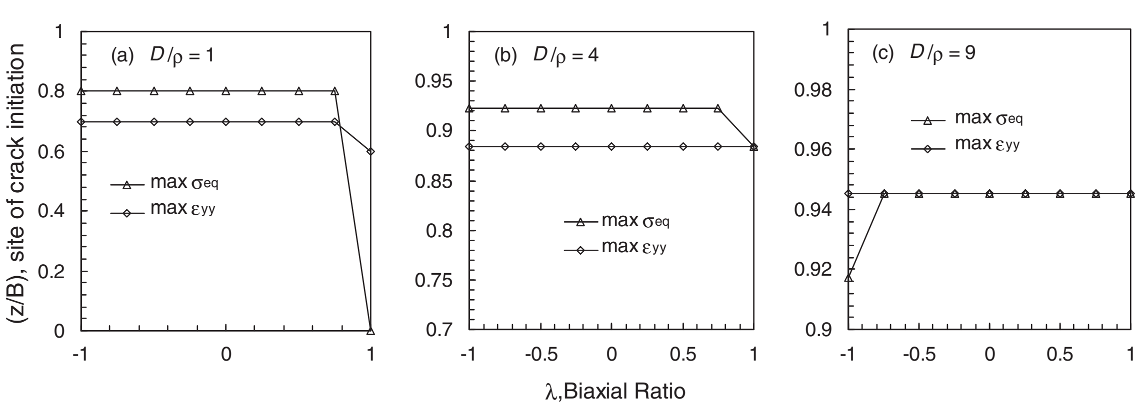

Figure 10 The site of crack initiation through the plate thickness under biaxial loading with different biaxial ratios (a) $D/\rho = 1$, (b) $D/\rho = 4$ and (c) $D/\rho = 9$.

distance between the site of crack initiation and the plate surface decreased by increasing D/ρ. For the same D/ρ, the distance between the site of crack initiation and the plate surface decreased by increasing the thickness of plate.

In the case of biaxial loading, the site of crack initiation for different λ, D/ρ and $B/D = 2.5$ was shown in Fig. 10. The site of crack initiation in the case of biaxial loading is almost the same of that in the case of uniaxial loading for the same plate thickness. This means that the site of crack initiation does not depend on λ.

4. Conclusions

Based on the present numerical results, the following conclusions can be drawn.

1. For circular notch with biaxial ratio equals 1, either $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ is constant through the plate thickness and equals 1. However, for double U-notch with $\lambda = 1$, $K_\sigma/K_{\sigma mp}$ and $K_\varepsilon/K_{\varepsilon mp}$ are not constant through the plate thickness.
2. For biaxial ratio equals -1, the highest value of $K_\sigma/K_{\sigma mp}$ or $K_\varepsilon/K_{\varepsilon mp}$ is at the plate interior, while, these maximum values are at the plate surface for $\lambda = 1$.
3. The maximum stress concentration factor for the circular and double U-notch is correlated as a function of notch geometry and biaxial ratio.
4. The distance between the site of crack initiation through the thickness and the outer surface of the plate decreased with decreasing the notch radius and/or increasing the plate thickness.
5. The site of crack initiation does not depend on biaxial ratio.

References

[1] Peterson RE. Stress concentration factors. New York: John Wiley and Sons; 1997.
[2] Feng Li, Yuting He, Fan Chaohua, Li Hongpeng, Zhang Hengxi. Investigation on three-dimensional stress concentration of LY12-CZ plate with two equal circular holes under tension. Mater Sci Eng A 2008;483–484:474–6.
[3] Hammouda MMI, Abdel-Alhady AA, Fayed AS, Sallam, HEM. Notch elastic affected zone of oblique elliptical edge notches, MEATIP4. Fourth Assiut University Int. Conf., December 12–14, CD-Room, Paper No. MMP23; 2006.
[4] Nakamura T, Parks DM. Three-dimensional crack front fields in a thin ductile plate. J Mech Phys Solids 1990;38:787–812.
[5] Wang X. Elastic T-stress solutions for semi-elliptical surface cracks in finite thickness plates. Eng Fract Mech 2003:731–56.
[6] Li ZH, Guo WL, Kuang ZB. Three-dimensional elastic stress fields near notches in finite thickness plates. Int J Solids Struct 2000;37:7617–31.
[7] Yang Z, Guo WL, Xu X. Three-dimensional elastic stress fields near notches in finite thickness bimaterial plates. J Mech Strength 2003;25:90–4.
[8] She CM, Guo WL. Numerical investigations of maximum stress concentration at elliptic holes in finite thickness piezoelectric plates. Int. J Fatigue 2006;28:438–45.
[9] Li ZH, Guo WL. Three-dimensional elastic stress fields ahead of blunt V-notches in finite thickness plates. Int J Fract 2001;107: 53–71.
[10] She CM, Guo WL. Three-dimensional stress concentrations at elliptic holes in elastic isotropic plates subjected to tensile stress. Int J Fatigue 2007;29:330–5.
[11] Broek D. Elementary engineering fracture mechanics. Plates with holes. Groningen Nordhoff 1974:361–70 [chapter 4].
[12] Kirsch G Die. Theorie der Elastizitat and die Bediirfnisse der Festigkeitslehre. Z Vereines Deutscher Ing 1898;42:797–807.
[13] Kotousov A, Lazzarin P, Berto F, Harding S. Effect of the thickness on elastic deformation and quasi-brittle fracture of plate components. Eng Fract Mech 2010;77:1665–81.
[14] Folias ES, Wang JJ. On the three-dimensional stress field around a circular hole in a plate of arbitrary thickness. Comput Mech 1990;6:379–91.
[15] Kotousov A, Wang CH. Three-dimensional stress constraint in an elastic plate with a notch. Int J Solids Struct 2002;39:4311–26.
[16] Peishi Yu, Guo WL, Chongmin She, Junhua Zhao. The influence of Poisson's ratio on thickness-dependent stress concentration at elliptic holes in elastic plates. Int J Fatigue 2008;30:165–71.
[17] Zheng Yang, Chang-Boo Kim, Chongdu Cho, Hyeon Gyu Beom. The concentration of stress and strain in finite thickness elastic plate containing a circular hole. Int J Solids Struct 2008;45: 713–31.
[18] ABAQUS user's manual version 6.3. Pawtucket, RI: Hibbitt, Karlsson and Sorensen Inc.; 2002.
[19] Hammouda MMI, Abdel-Alhady AA, Fayed AS, Sallam HEM. Notch elastic affected zone of oblique elliptical edge notches. In: The 4th Assiut university international conference on mechanical engineering advanced technology for industrial production, December 12–14; 2006.
[20] Hammouda MMI, Abdel-Alhady AA, Fayed AS, Sallam HEM. Cyclic crack tip deformation of short cracks emanating from elliptical oblique notches. In: The 4th Assiut university international conference on mechanical engineering advanced technology for industrial production, December, 12–14; 2006.

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