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Abundant new exact solutions for the $(3+1)$-dimensional Jimbo–Miwa equation

Zitian Li [a,*,1], Zhengde Dai [b]

[a] College of Mathematics and Information Science, Qujing Normal University, Qujing Yunnan 655011, PR China
[b] School of Mathematics and Statistics, Yunnan University, Kunming 650091, PR China

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ABSTRACT

In this paper, by using the generalized Riccati equation mapping method, and picking up its new solutions authors obtain abundant new exact solutions including kink solutions, periodic form solutions, soliton-like solutions and rational solutions to a $(3+1)$-dimensional Jimbo–Miwa equation, respectively.

1. Introduction

Computerized symbolic computation has sped up the investigation of high-dimensional nonlinear evolution equations. With the development of computer science, recently, direct searching for exact solitary wave and soliton-like solutions to NEEs in high-dimensional has attracted much attention. Numerous methods have been proposed to obtain explicit solutions of NEEs, such as the inverse scattering method [1,10], the tanh-sech method [2,11,12], the extended tanh method [3], the homogeneous balance method [4,13], and so on.

In Ref. [5], Zhu developed the extended tanh function method by introducing a generalized Riccati equation mapping method and its new solutions. Along this way, in this paper we plan to study a $(3+1)$-dimensional Jimbo–Miwa equation [6],

$$u_{xxxy} + 3u_x u_{xy} + 3u_y u_{xx} + 2u_{yt} - 3u_{xz} = 0. \tag{1.1}$$

It is known that this model is not Painlevé integrable. For many years, many workers have researched it and certain explicit solutions are obtained [6–9]. In this work, using the method in [5], we obtain rich new families of its special solutions including soliton-like solutions, cross kink-wave solutions, periodic form solutions and rational solutions.

* Corresponding author.
E-mail address: lizitian88@163.com (Z. Li).

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2. New types traveling wave solutions and periodic wave solutions

In this section we describe the generalized Riccati equation method for finding special solutions of the $(3+1)$-dimensional Jimbo–Miwa equation

$$u_{xxxy}+3u_xu_{xy}+3u_yu_{xx}+2u_{yt}-3u_{xz}=0. \tag{2.1}$$

For the basic idea of the generalized Riccati equation mapping method, we seek the following formal travelling wave solutions

$$u=u(x,y,z,t)=U(\xi),\qquad \xi=kx+ly+mz+\omega t, \tag{2.2}$$

where k, l, m and ω are arbitrary constants.

Substituting (2.2) into (2.1) gives rise to ODE

$$k^3lU^{(4)}+6k^2lU''U'+(2l\omega-3km)U''=0. \tag{2.3}$$

Integrating (2.3) once, we have

$$k^3lU'''+3k^2l(U')^2+(2l\omega-3km)U'=C, \tag{2.4}$$

where C is an integration constant.

We next introduce a new independent variable φ as follows:

$$u(x,y,z,t)=U(\xi)=\sum_{i=0}^{m}a_i\varphi^i, \tag{2.5}$$

where a_i and b_j are constants to be determined later, m is fixed by balancing the highest-order linear term with the nonlinear term in (2.3). φ expresses the solution of the following generalized Raccati equation:

$$\varphi'(\xi)=r+p\varphi(\xi)+q\varphi^2(\xi), \tag{2.6}$$

where r, p and q are all variable real constants.

By balancing the order between U''' and $(U')^2$ in (2.4), we have $m=1$. Then, we can suppose (2.5) in the form

$$u(x,y,z,t)=U(\xi)=a_0+a_1\varphi. \tag{2.7}$$

Substituting (2.7) with (2.6) into (2.4) yields a set of algebraic equations for φ^i. Setting the coefficients of φ^i to zero yields

$$\begin{aligned}
&k^3l\big(p^2+2qr\big)r+3k^2lr^2a_1+(2l\omega-3km)r=C,\\
&\big(p^3+8pqr\big)k^3l+6k^2lpra_1+(2l\omega-3km)p=0,\\
&\big(7p^2q+8q^2r\big)k^3l+3k^2l\big(p^2+2qr\big)a_1+(2l\omega-3km)q=0,\\
&12k^3lpq^2+6k^2lpqa_1=0,\\
&6k^3lq^3+3k^2lq^2a_1=0.
\end{aligned}$$

Solving these algebraic equations, we obtain

$$a_1=-2kq,\qquad \omega=\frac{4k^3lqr+3km-p^2k^2l}{2l},\qquad C=0, \tag{2.8}$$

and a_0, k, l, m, r, p and q are arbitrary constants.

Based on the solutions of (2.6), selecting different values of r, p and q we can obtain new types solutions for (2.1).

Case I. When $p^2-4qr>0$ and $pq\neq 0$ (or $qr\neq 0$), the soliton and soliton-like solutions of Eq. (2.1) are

$$\begin{aligned}
&u_1=a_0+k\left[p+\sqrt{p^2-4qr}\tanh\left(\frac{\sqrt{p^2-4qr}}{2}\xi\right)\right],\\
&u_2=a_0+k\left[p+\sqrt{p^2-4qr}\coth\left(\frac{\sqrt{p^2-4qr}}{2}\xi\right)\right],\\
&u_3=a_0+k\big[p+\sqrt{p^2-4qr}\big(\tanh\big(\sqrt{p^2-4qr}\xi\big)\pm i\,\mathrm{sech}\big(\sqrt{p^2-4qr}\xi\big)\big)\big],\\
&u_4=a_0+k\big[p+\sqrt{p^2-4qr}\big(\coth\big(\sqrt{p^2-4qr}\xi\big)\pm \mathrm{csch}\big(\sqrt{p^2-4qr}\xi\big)\big)\big],\\
&u_5=a_0+\frac{k}{2}\left[2p+\sqrt{p^2-4qr}\left(\tanh\left(\frac{\sqrt{p^2-4qr}}{4}\xi\right)\pm\coth\left(\frac{\sqrt{p^2-4qr}}{4}\xi\right)\right)\right],
\end{aligned}$$

$$u_6 = a_0 - k\left[-p + \frac{\sqrt{(A^2+B^2)(p^2-4qr)} - A\sqrt{p^2-4qr}\cosh(\sqrt{p^2-4qr}\xi)}{A\sinh(\sqrt{p^2-4qr}\xi)+B}\right],$$

$$u_7 = a_0 - k\left[-p - \frac{\sqrt{(B^2-A^2)(p^2-4qr)} + A\sqrt{p^2-4qr}\cosh(\sqrt{p^2-4qr}\xi)}{A\sinh(\sqrt{p^2-4qr}\xi)+B}\right],$$

$$u_8 = a_0 - \frac{4kqr\cosh(\frac{\sqrt{p^2-4qr}}{2}\xi)}{\sqrt{p^2-4qr}\sinh(\frac{\sqrt{p^2-4qr}}{2}\xi) - p\cosh(\frac{\sqrt{p^2-4qr}}{2}\xi)},$$

$$u_9 = a_0 + \frac{4kqr\sinh(\frac{\sqrt{p^2-4qr}}{2}\xi)}{p\sinh(\frac{\sqrt{p^2-4qr}}{2}\xi) - \sqrt{p^2-4qr}\cosh(\frac{\sqrt{p^2-4qr}}{2}\xi)},$$

$$u_{10} = a_0 - \frac{4kqr\cosh(\frac{\sqrt{p^2-4qr}}{2}\xi)}{\sqrt{p^2-4qr}\sinh(\sqrt{p^2-4qr}\xi) - p\cosh(\sqrt{p^2-4qr}\xi) \pm i\sqrt{p^2-4qr}},$$

$$u_{11} = a_0 - \frac{4kqr\sinh(\frac{\sqrt{p^2-4qr}}{2}\xi)}{-p\sinh(\sqrt{p^2-4qr}\xi) + \sqrt{p^2-4qr}\cosh(\sqrt{p^2-4qr}\xi) \pm \sqrt{p^2-4qr}},$$

$$u_{12} = a_0 - \frac{8kqr\sinh(\frac{\sqrt{p^2-4qr}}{4}\xi)\cosh(\frac{\sqrt{p^2-4qr}}{4}\xi)}{-2p\sinh(\frac{\sqrt{p^2-4qr}}{4}\xi)\cosh(\frac{\sqrt{p^2-4qr}}{4}\xi) + 2\sqrt{p^2-4qr}\cosh^2(\frac{\sqrt{p^2-4qr}}{4}\xi) - \sqrt{p^2-4qr}},$$

where $\xi = kx + ly + mz + \frac{k(4k^2qrl+3m-k^2p^2l)}{2l}t$, and A, B are two non-zero real constants with $B^2 - A^2 > 0$.

Case II. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $qr \neq 0$), we have the periodic solutions as:

$$u_{13} = a_0 - k\left[-p + \sqrt{4qr-p^2}\tan\left(\frac{\sqrt{4qr-p^2}}{2}\xi\right)\right],$$

$$u_{14} = a_0 + k\left[p + \sqrt{4qr-p^2}\cot\left(\frac{\sqrt{4qr-p^2}}{2}\xi\right)\right],$$

$$u_{15} = a_0 - k\left[-p + \sqrt{4qr-p^2}\left(\tan(\sqrt{4qr-p^2}\xi) \pm \sec(\sqrt{4qr-p^2}\xi)\right)\right],$$

$$u_{16} = a_0 + k\left[p + \sqrt{4qr-p^2}\left(\cot(\sqrt{4qr-p^2}\xi) \pm \csc(\sqrt{4qr-p^2}\xi)\right)\right],$$

$$u_{17} = a_0 - \frac{k}{2}\left[-2p + \sqrt{4qr-p^2}\left(\tan\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right) - \coth\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right)\right)\right],$$

$$u_{18} = a_0 - k\left[-p + \frac{\pm\sqrt{(A^2-B^2)(4qr-p^2)} - A\sqrt{4qr-p^2}\cos(\sqrt{4qr-p^2}\xi)}{A\sinh(\sqrt{4qr-p^2}\xi)+B}\right],$$

$$u_{19} = a_0 - k\left[-p - \frac{\pm\sqrt{(A^2-B^2)(4qr-p^2)} - A\sqrt{4qr-p^2}\cos(\sqrt{4qr-p^2}\xi)}{A\sin(\sqrt{4qr-p^2}\xi)+B}\right],$$

$$u_{20} = a_0 + \frac{4kqr\cos(\frac{\sqrt{4qr-p^2}}{2}\xi)}{\sqrt{4qr-p^2}\sin(\frac{\sqrt{4qr-p^2}}{2}\xi) + p\cos(\frac{\sqrt{4qr-p^2}}{2}\xi)},$$

$$u_{21} = a_0 - \frac{4kqr\sin(\frac{\sqrt{4qr-p^2}}{2}\xi)}{-p\sin(\frac{\sqrt{4qr-p^2}}{2}\xi) + \sqrt{4qr-p^2}\cos(\frac{\sqrt{4qr-p^2}}{2}\xi)},$$

$$u_{22} = a_0 + \frac{4kqr\cos(\frac{\sqrt{4qr-p^2}}{2}\xi)}{\sqrt{4qr-p^2}\sin(\sqrt{4qr-p^2}\xi) + p\cos(\sqrt{4qr-p^2}\xi) \pm \sqrt{4qr-p^2}},$$

$$u_{23} = a_0 - \frac{4kqr\sin(\frac{\sqrt{4qr-p^2}}{2}\xi)}{-p\sin(\sqrt{4qr-p^2}\xi) + \sqrt{4qr-p^2}\cos(\sqrt{4qr-p^2}\xi) \pm \sqrt{4qr-p^2}},$$

$$u_{24} = a_0 - \frac{8kqr\sin\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right)}{-2p\sin\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right)\cos\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right) + 2\sqrt{4qr-p^2}\cos^2\left(\frac{\sqrt{4qr-p^2}}{4}\xi\right) - \sqrt{4qr-p^2}},$$

where $\xi = kx + ly + mz + \frac{k}{2l}(4k^2qrl + 3m - k^2p^2l)t$, A and B are two non-zero real constants and satisfies $A^2 - B^2 > 0$.

Case III. When $r = 0$ and $pq \neq 0$, we obtain the soliton-like solutions

$$u_{25} = a_0 + \frac{2kpd}{d + \cosh(p\xi) - \sinh(p\xi)},$$

$$u_{26} = a_0 + \frac{2kp[\cosh(p\xi) + \sinh(p\xi)]}{d + \cosh(p\xi) + \sinh(p\xi)},$$

where $\xi = kx + ly + mz + (-\frac{1}{2}k^3p^2 + \frac{3km}{2l})t$ and d is an arbitrary constant.

Case IV. When $q \neq 0$ and $r = p = 0$, we obtain the rational solutions

$$u_{27} = a_0 + \frac{2kq}{q\xi + c_1},$$

where $\xi = kx + ly + mz + \frac{3km}{2l}t$ and c_1 is an arbitrary constant.

3. Conclusion

In this paper, based on the generalized Riccati equation mapping method, we obtain abundant solutions to the $(3+1)$-dimensional Jimbo–Miwa equation. To our knowledge, these solutions have not been reported in previous literatures.

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