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Cauty's space enhanced

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ABSTRACT

The title refers to Cauty's example (Cauty, 1994 [3]) of a metric vector space which is not an absolute retract. It is shown that Cauty's space can be refined to the effect that the completion of the refined space can be isomorphically embedded as a subspace of an F-space which itself is an absolute retract.

1. Introduction

For a compactum X, denote by $E(X)$ the topological vector space spanned by the point mass measures $\{\delta_x\}_{x\in X}$ and equipped with the strongest vector topology. For a certain compactum X, Cauty [3] has constructed a metric d so that the metric vector space $C=(E(X),d)$ is not an absolute retract. The latter is a consequence of the fact that the compactum X has a complex topological structure. The space C is incomplete; its completion is a separable F-space which, as well, is not an absolute retract. Cauty has also shown that the space C can be identified with a closed linear subspace of yet another incomplete metric vector space which itself is an absolute retract. More precisely, for (any) embedding of X into the Hilbert cube $\mathbb{Q}$, $C\subset E(\mathbb{Q})$ and $E(\mathbb{Q})$ is an absolute retract.

In this note, we show that $E(X)$ can be equipped with a metric vector topology whose completion E can be isomorphically embedded in an *F-space* (that is, a complete metric vector space) F which is an absolute retract. This observation, interesting in its own right, implies that certain continuous surjections between F-spaces do not have continuous cross-sections (see [6]). Furthermore, the space F has the FDD-property. As a consequence, (1) F is a topological copy of ℓ_2, and

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(2) the affine structure of locally compact convex subsets of F is the same as those of ℓ_2. So, the topological structure of convex subsets of F begins to be complex beyond the local compactness.

The Cauty space C not only fails to be an absolute retract, but also does not satisfy the so-called compact extension property, which is weaker than the absolute retract property. More precisely, considering X as a subset of $\mathbb{Q}$, the inclusion mapping of X into $C = (E(X), d)$ does not continuously extend to $\mathbb{Q}$. Following the approach of [4] we construct a dense linear subspace E' in the completion of C so that E' continuous to be not an absolute retract, yet has the compact extension property. Hence, every mapping from a closed subset of $\mathbb{Q}$ into E' has a continuous extension (equivalently, E' is Klee-admissible, see [12,13]).

2. The completion preserves polarness

Let (L, τ) be a topological vector space and ρ be a weaker vector topology on L. L is said to be *ρ-polar* if it has a base of ρ-closed neighborhoods of 0 (see [10]).

Lemma 2.1. *If (L, τ) is ρ-polar then the identity operator $(L, \tau) \to (L, \rho)$ extends to a* 1–1 *operator between the respective completions.*

Proof. Write j for the map of the respective completions that extends the identity operator $(L, \tau) \to (L, \rho)$. If $j(\tilde{x}) = 0$ for $\tilde{x}$ in the completion of (L, τ), there is a Cauchy net (x_p) in (L, τ) with $\lim x_p = \tilde{x}$ in the τ-completion of L; hence, the net $(j(x_p))$ converges to 0 in (L, ρ). Then, if V is a τ-neighborhood of 0, there exists p_0 such that, for $p, q \succ p_0$, we have $x_p - x_q \in V$. Write $\tilde{V}$ for the ρ-closure of V in the completion of (L, ρ). Assuming V is ρ-closed, we obviously have $\tilde{V} \cap L = V$. Because the net $(j(x_p - x_q))_q$ converges to x_p in ρ, we see that $x_p \in \tilde{V}$. It follows that $x_p \in \tilde{V} \cap L = V$ for $p \succ p_0$. This shows that the net (x_p) converges to 0 in (L, τ); hence $\tilde{x} = 0$. □

Lemma 2.2. *Let (L, τ) be a topological vector space and ρ be a weaker vector topology on L. Assume L_0 is a dense subspace of (L, τ). If (L_0, τ) is $\rho|_{L_0}$-polar then L is ρ-polar.*

Proof. We will show that every neighborhood U of 0 in (L, τ) contains a neighborhood which is ρ-closed. Pick an open symmetric neighborhood $V_1 \subset U$ so that $V_1 + V_1 \subset U$. Pick a further open symmetric neighborhood $V_2 \subset V_1$ so that the ρ-closure of $V_2 \cap L_0$ relative to L_0 is contained in $V_1 \cap L_0$. Pick an open symmetric neighborhood V_3 so that $V_3 + V_3 \subset V_2$.

Now suppose y is in the ρ-closure of V_3. We want to show y is in U. By density there exists $x \in L_0 \cap (y + V_3)$. If x is in the ρ-closure of $L_0 \cap V_2$ then $x \in V_1$ and $y \in U$. If not, there is an open symmetric ρ-neighborhood W of 0 so that $x + W$ fails to meet $L_0 \cap V_2$. This means that $x + W$ does not meet V_2 at all because they are both τ-open sets; and so $x \notin W + V_2$. But since y is in the ρ-closure of V_3 we can find $z \in (y + W) \cap V_3 \cap L_0$. Then $x, z \in L_0$ and $x - z = (x - y) - (z - y) \in V_3 + W$. Hence $x \in V_3 + V_3 + W \subset V_2 + W$, which is a contradiction. □

Combing the above lemmas we obtain the following:

Corollary 2.3. *Let (L, τ) be a topological vector space and $\tilde{L}_\tau$ be its completion. If (L, τ) is ρ-polar then $\tilde{\rho}$ is a weaker topology on $\tilde{L}_\tau$ and $\tilde{L}_\tau$ is $\tilde{\rho}$-polar; here, $\tilde{\rho}$ is the completion of ρ.*

Remark 1. Clearly, the above facts remain true in the group-topological setting.

The following examples help illustrate Corollary 2.3:

Examples 2.4. Let L be a normed (incomplete) vector space and V be a subspace of the dual space L^* which separates points of L. Let $\rho = \sigma(L, V)$ be the weak topology on L induced by V. Then, if V separates points of $\tilde{L}$, the identity operator $L \to (L, \sigma(L, V))$ extends to a one-to-one operator of the respective completions $\tilde{L}$ and $V^\#$. Specifically, we have:

(1) The $\sigma(L, V)$-closure of the unit ball of L is the unit $\|\cdot\|_1$-ball, where $\|x\|_1 = \sup\{v(x)\colon\ v \in V,\ \|v\| \leqslant 1\}$. Therefore, L is $\sigma(L, V)$-polar if and only if V is norming. In this case, L can be identified with a subspace of V^*, which is a subset of $V^\#$. The required extension is obvious (and nicely illustrates Corollary 2.3).

(2) If V is a non-norming subspace of L^* and separates points of $\tilde{L}$ (see, e.g., the example of $\tilde{L} = c_0$ in [8, p. 94]), then L is not $\sigma(L, V)$-polar but the one-to-one extension still exists.

(3) Letting L be a dense subspace of $\ell_2 = \ell_2(\mathbb{N})$ with $L \cap \ell_2(2\mathbb{N}) = \{0\}$ and $L \cap \ell(2\mathbb{N} - 1) = \{0\}$, and $V = \ell_2(2\mathbb{N})$, we see that V doesn't separate points of $\ell_2 = \tilde{L}$. Consequently, such a one-to-one extension does not exist.

3. Polarness in inductive limits

Following [17], let us recall basic definitions and facts on the vector inductive limits. Let L be a vector space. Suppose $L = \bigcup E_n$, where each E_n is a topological space and a balanced subset of L. Assume that $E_n + E_n \subset E_{n+1}$ for every n; in particular, $E_n \subset E_{n+1}$. Assume the topology of E_n coincides with the subspace topology of E_{n+1}. Suppose that the addition and multiplication maps $E_n \times E_n \to E_{n+1}$ and $[-1, 1] \times E_n \to E_n$ are continuous. Equip L with the strongest **vector** topology that coincides with the topology on each E_n. The resulting topological vector space L is called the vector inductive limit of the sequence (E_n), and is denoted by $\varinjlim E_n$ (see [17]). Actually, it is enough to assume that $E_n + E_n \subset E_{k(n)}$ for some $k(n) \geqslant n + 1$, see [17, 1.1.2]. Let us state a consequence of [17, Theorem 1.1.13]:

Lemma 3.1. *Let F be an F-space and assume that each E_n is closed in F. Then the vector inductive limit $L = \varinjlim E_n$ can be identified with the projective limit of the system $(L, d)_{d \in D(L)}$. Here, the directed set $D(L)$ consists of all metrics d on L such that (L, d) is a metric vector space and the original topology of each E_n coincides with the topology of (E_n, d); $D(L)$ is ordered by the relation $d \prec d'$ iff the identity operator $(L, d') \to (L, d)$ is continuous.*

Hence, the topology of the vector inductive limit $L = \varinjlim E_n$ is the lim sup *of the topologies induced by the metrics from $D(L)$.*

Proposition 3.2. *Let L be the vector inductive limit of the sequence (E_n) that satisfies the assumption of Lemma* 3.1. *Let ρ be a vector topology on F that yields the original topology of each E_n. Then, given a metric vector topology τ on L, there exists a stronger metric vector topology τ' such that (L, τ') is ρ-polar in the completions.*

Proof. The vector inductive limit topology of L is ρ-polar. This is a consequence of the fact that the sets $\overline{U}_0 \cap \bigcap_{n \geqslant 1} \overline{E_n + U_n}$ form a base of ρ-closed neighborhoods of 0, where each U_n is a ρ-neighborhood of 0 and $\overline{U}_0$ and $\overline{E_n + U_n}$ are ρ-closures (see [17]).

Now, given the topology τ, find a sequence (A_n) of balanced neighborhoods of the above form such that $A_n + A_n \subset A_{n-1}$, $\bigcap A_n = \{0\}$, and A_n is contained in the τ-unit ball of radius $1/n$. The metric topology τ' produced by the sequence (A_n) is stronger than τ. Since each A_n is ρ-closed, (L, τ') is ρ-polar. According to Corollary 2.3, (L, τ') is ρ-polar in the completions. □

Corollary 3.3. *If, in Proposition* 3.2*, each E_n is compact and the topology ρ admits a sequence of continuous linear functionals separating points of L (e.g., ρ is locally convex), then the resulting space (L, τ') is weakly polar.*

Proof. Since the original and weak topologies coincide on E_n, an application of Proposition 3.2 (for the weak topology in place of ρ), yields the assertion. □

Let us recall that the **topological** inductive limit of a sequence of topological spaces (E_n), $E_n \subset E_{n+1}$, is the space $L = \bigcup E_n$ with the topology defined by the condition: $U \subset L$ is open if and only if each $U \cap E_n$ is open in E_n. Let us quote the result of [18, p. 48]:

Lemma 3.4. *Let $L = \varinjlim E_n$ be the vector inductive limit of (E_n). If each E_n is compact, then the vector inductive limit topology of L coincides with the topology of the topological inductive limit of (E_n).*

4. Refining the Cauty space

Let X be a metric compactum. Embed X onto a linearly independent set of a metric locally convex space F (e.g., $F = \ell_2$). Define $E_n(X) = \{\sum_{i=1}^n \lambda_i x_i \mid x_1, \ldots, x_n \in X,\ \sum_{i=1}^n |\lambda_i| \leqslant n\} \subset F$. Each $E_n(X)$ is compact, balanced, and $E_n(X) + E_n(X) \subset E_{2n}(X)$. Let $E(X) = \varinjlim E_n(X)$ be the vector inductive limit of the sequence $(E_n(X))$. Then the results 3.1–3.4 hold. Perhaps the easiest way to visualize the above is to embed X into the space $C^*(X)$ via the map $x \to \delta_x$, where δ_x is the point mass at x. Then the topology ρ is simply the weak*-topology and each $E_n(X)$ is compact in this topology and, for F in Lemma 3.1, we let the completion of the space of all measures with finite support. (However, one is free to choose any other metric locally convex space F; recall that every infinite-dimensional Banach space F admits an embedding of X onto a linearly independent set.)

By Lemma 3.1 and Lemma 3.4, the topology of $E(X)$ coincides with the lim sup of the topologies of $(E(X), d)_{d \in D(X)}$, where $D(X)$ abbreviates $D(E(X))$, as well as with the topology of the topological inductive limit of $(E_n(X))$.

Recall [12,13] that a metric vector space L is *Klee-admissible* if every compactum in L admits arbitrarily close to the identity continuous displacements into finite-dimensional sets. This is equivalent to: Every continuous mapping f from a compactum K of a metrizable space M extends to a continuous mapping $\bar{f} : M \to L$ (see [5,4]). Every dense linear subspace L' of such an L is Klee-admissible because every mapping from a finite-dimensional compactum into L can be approximated by mappings into L'.

Theorem 4.1. *(Cauty [3]) There exist a metric compactum X and a metric $d_0 \in D(X)$ such that, for each $d \in D(X)$ with $d_0 \prec d$, the sigma-compact linear space $C_d = (E(X), d)$ is not Klee-admissible. In particular, each space C_d is not an absolute retract. Furthermore, the completion E_d of C_d is a separable F-space which is not an absolute retract.*

Each space C_d will be referred to as a Cauty space. Cauty's construction [3] guarantees that C_d admits a separating sequence of continuous linear functionals. Obviously, in general, the latter property does not carry over to E_d, the completion of C_d. We will show that, for a co-final subset A of $D(X)$, the space $E_{d'}$, $d' \in A$, admits a separating sequence of continuous linear functionals.

Recall that a linear space L has the FDD (finite-dimensional decomposition) property if there is a sequence of finite-dimensional linear subspaces L_n of L and a sequence $Q_n : L \to L_n$ of continuous linear projections such that $Q_n \circ Q_m = 0$ for $n \neq m$ and $x = \sum_{n=1}^{\infty} Q_n x$ for all $x \in L$.

Corollary 4.2. *Every space $(E(X), d)$, $d \in D(X)$, admits a stronger metric d' such that the completion E of $(E(X), d')$ is isomorphic to a subspace of an F-space F with the* FDD *property. In particular, E has a sequence of continuous linear functionals separating points.*

Proof. By our discussion at the beginning of this section, Proposition 3.2 is applicable. Hence, there exists a stronger metric d' such that $E(X)$ has a base of neighborhoods of 0 that is polar in the weak topology (use Corollary 3.3). An application of [10, Theorem 7.4] produces a required F-space F. Since the space F has a sequence of continuous linear functionals that separates points, so does the subspace E. □

Cauty [3] has shown that every space C_d (from Theorem 4.1) can be identified with a closed subspace of another sigma-compact space C', which is Klee-admissible (while C_d is not). As mentioned above, the completion of a metric linear space that is not Klee-admissible remains such; however, the completion of a Klee-admissible space is not necessarily Klee-admissible (see Theorem 5.1). So, our main result below cannot be automatically obtained by just taking the closure of C_d in the completion of the space C'.

Theorem 4.3. *There exists an F-space F which is homeomorphic ℓ_2 (hence, is an absolute retract) and contains a closed subspace E that is not an absolute retract (not even Klee-admissible). Moreover, for a certain co-final subset A of $D(X)$, every space E_d is a subspace of such a space F_d, $d \in A$.*

Proof. Let $d_0 \in D(X)$ be that from Theorem 4.1. Pick $d \in D(X)$ with $d_0 \prec d$. For this d, let F and E be those from Corollary 4.2. By [5, Theorem 2], an F-space which admits a sequence of finite rank operators that converges uniformly on compacta to the identity is an absolute retract. Hence F is an absolute retract. Now, by a result of [7], every separable and infinite-dimensional F-space which is an absolute retract is homeomorphic to ℓ_2. □

Remark 2. Each space E_d from Theorem 4.3 (which itself is not an absolute retract) contains sufficiently many F-spaces that are homeomorphic to ℓ_2. Namely, by a result of [9] (see also [11, Theorem 2.1]), every infinite-dimensional F-space of E_d contains a basic sequence (x_n) (i.e., the sequence of continuous linear functionals that is a Schauder basis for the closure of its span). Applying [5, Theorem 2], this latter space is an absolute retract.

According to [1, Corollary 1.4], every locally compact convex subset of an F-space admitting a separating sequence of continuous linear functionals embeds affinely in ℓ_2. This shows yet another "regularity" of our space E_d (it is not clear that the completion of C_d, the original Cauty's space, has this property).

Remark 3. Every locally compact convex subset of the space E from Theorem 4.3 is affinely homeomorphic to a subset of ℓ_2 (and hence, is an absolute retract).

Below, we use the spaces F and E from Theorem 4.3 to answer the question of non-existence of a continuous cross-section for a continuous linear operator $T : Z \to L$ of the metric vector space Z onto L. It is known that such a cross-section exists when the kernel $T^{-1}(0)$ is locally convex. As observed below, in general, such a continuous cross-section does not exist (see also [6]). Moreover, in our example, the space $Z = F$ is homeomorphic to ℓ_2.

Corollary 4.4. *The quotient mapping $F \to F/E$ has no continuous cross-section.*

Proof. If the quotient map $\kappa : F \to F/E$ had a continuous cross-section, then F would be homeomorphic to the product $E \times F/E$ (see [2]). Since F is an absolute retract, E would be an absolute retract, a contradiction with 4.3. □

Another example of a quotient map with no continuous cross-section can be obtained using a co-universal F-space Z with Schauder basis (see [10, Theorem 5.4]). (By the co-universality property, the F-space E which is not an absolute retract, admits a continuous linear operator T of Z onto E. If a continuous cross-section of T existed, then Z would be homeomorphic to $T^{-1}(0) \times E$, a contradiction because E is not an absolute retract while Z (being a space with a Schauder basis) not only is an absolute retract but also is homeomorphic to ℓ_2.)

It would be interesting to find a continuous linear surjection $T : Z \to L$ between metric vector spaces Z and L with no continuous cross-section and such that $T^{-1}(0)$ is an absolute retract (or, $T^{-1}(0)$ has a Schauder basis for the case of F-spaces). According to [11, Theorem 2.1], one can choose a basic sequence (x_n) as referred to in Remark 2 in the space $C = C_d = (E(X), d)$ (actually, in the symmetric convex hull of X). The intersection C' of the closed span (in E_d) of (x_n) with the space C provides an absolute retract, which is sigma-compact and closed in C. An affirmative answer to the following question would provide a suitable example for non-existence of the continuous cross-section.

Question 4.5. Does the map $C \to C/C'$ have a continuous cross-section?

We also ask

Question 4.6. Is there a vector subspace F_0 of the space F (from Theorem 4.3) such that $E \cap F_0 = \{0\}$, $E + F_0$ is dense in F, and F_0 is closed in $E + F_0$? Can one find such an F_0 that additionally is an absolute retract (has a Schauder basis)?

For such an F_0, the quotient map $E + F_0 \to (E + F_0)/F_0$ does not admit a near cross-section because, otherwise, the quotient $(E + F_0)/F_0 = E$ would also be an absolute retract (for $E + F_0$ is an absolute retract as a dense subspace of the absolute retract F).

5. The complete Cauty space contains a Klee-admissible dense subspace which is not an absolute retract

In [4], assuming the existence of an F-space which is not an absolute retract, van der Bijl and van Mill provided an example of a certain F-space, which itself is not an absolute retract, but contains a dense vector subspace with only countable-dimensional compacta. The latter property yields that the subspace is Klee-admissible. Below, we will mimic their construction [4, Theorem 3.8] and obtain, in the completion of any Cauty space, a dense Klee-admissible vector subspace which is not an absolute retract. Such a dense space, however, has a rather complex Borel structure. (It is unknown whether every Klee-admissible F-space must be an absolute retract.)

Theorem 5.1. *The completion E of any Cauty space C contains a dense vector subspace E' that is Klee-admissible but is not an absolute retract.*

Lemma 5.2. *For the spaces E and C, the algebraic dimension of E/C is continuum.*

Proof. Recall that $C = \operatorname{span}(X)$, where X is a compactum. Also $C = \bigcup_{n=1}^{\infty} E_n$, where each $E_n = E_n(X)$ is a compactum. We say that a set A is linearly independent over E_n if, for every $a \in A$, $a \notin \operatorname{span}(A \setminus \{a\}) + E_n$. For every (n, m) define

$$R_{n,m} = \bigl\{(a_1, \dots, a_n) \in C^n \bigm| \{a_1, \dots, a_n\} \text{ is linearly dependent over } E_m\bigr\}.$$

Use induction on n and compactness of E_m to see that each $R_{n,m}$ is closed and nowhere dense in E^n. By a result of Mycielski [15, Theorem 1], there exists a copy of the Cantor set K, $K \subset E$, which is linearly independent with respect to the set of relations $(R_{n,m})$. (This means that, for every choice of $k_1, \dots, k_r \in K$, and every relation $R = R_{n,m}$, if $R(k_1, \dots, k_r)$ then $R(f(k_1), \dots, f(k_r))$ for every mapping $f : \{k_1, \dots, k_r\} \to E$, see [15, p. 140].)

We claim that, for every $k \in K$, $k \notin \operatorname{span}((K \setminus \{k\}) + C)$. Otherwise, for some k, $k - \sum_{i=1}^{n} \lambda_i k_i = \sum \mu_j x_j \in E_m$ for some m and $k_i \in K$, where $k \neq k_i$ for all i. It follows that $R_{n+1,m}(k, x_1, \dots, x_n)$. Letting $f(x_i) = 0$ for every i, and $f(k) = v$ for some $v \notin E_m$, we see that $R_{n+1,m}(f(k), f(x_1), \dots, f(x_n))$ is false; this is a contradiction. Hence, the set K is linearly independent over C. □

Proof of Theorem 5.1. Define $\mathcal{K} = \{K \subset E \mid K \text{ is compact with algebraic dimension } \mathfrak{c}\}$. Using a known approach (see e.g., [14]), we construct two linear subspaces E' and E'' of E such that

(1) $K \cap E' \neq \emptyset$ and $K \cap E'' \neq \emptyset$ for every $K \in \mathcal{K}$;
(2) E' is dense in E;
(3) $E' \cap E'' = \{0\}$.

We will show that the space E' is Klee-admissible and is not an absolute retract. Let A be a compactum in E'. If the algebraic dimension of A is $\mathfrak{c}$, then A contains a linearly independent Cantor set K (see e.g., [14]). But K can be partitioned into two disjoint Cantor sets. By (1), E' and E'' intersect both of these sets, a contradiction. Hence, every compact subset of E' is contained in a subspace spanned by countably many vectors. This easily implies that A can be continuously mapped into a finite-dimensional subspace by a small displacement. This shows that E' is Klee-admissible.

Now, assume that E' is an absolute retract. Applying [16, Proposition 4.2], there exists an absolute retract $\tilde{M}$, which is a G_δ-subset of E and contains E'. Hence, $E \setminus \tilde{M} = \bigcup_{n=1}^{\infty} K_n$, where each K_n is closed in E. Since, for every n, $K_n \cap E' = \emptyset$, each K_n has the algebraic dimension less than $\mathfrak{c}$. It follows that the algebraic dimension of $E \setminus \tilde{M}$ is less than $\mathfrak{c}$. Applying Lemma 5.2, it is easy to find $x \in E$ such that $C + x \subset \tilde{M}$. Being a dense convex subset of an absolute retract $\tilde{M}$, $C + x$ is an absolute retract as well. (In other words, $C + x$ is locally homotopy negligible in $\tilde{E}$ and, therefore, by [16, Theorem 3.1], $C + x$ is an absolute retract.) Hence, C would be an absolute retract, a contradiction. □

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