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The nature of ZZ branes

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Abstract

In minimal non-critical string theory we show that the principal (r,s) ZZ brane can be viewed as the basic $(1,1)$ ZZ boundary state tensored with the (r,s) Cardy boundary state. In this sense there exists only one ZZ boundary state, the basic $(1,1)$ boundary state.

1. Introduction

Two-dimensional Euclidean AdS_2 (the pseudosphere) was quantized using Liouville quantum field theory by Zamolodchikov and Zamolodchikov. In [1] they generalized the quantization of Liouville theory on the disk [2] to the non-compact geometry of the pseudosphere. The main difference between the quantization of the disk and the pseudosphere is the assumption with regard to the pseudosphere that the two-point correlation function factorizes when the geodesic distance separating the two operators diverges.

Using conformal bootstrap methods the Zamolodchikovs found a number of conformal invariant boundary conditions, that may be imposed at "infinity" of the pseudosphere, and that are consistent with the above assumption. These boundary conditions were labeled by two positive integers $(\hat{m},\hat{n})$, where the "basic" $(1,1)$ boundary condition played a role quite similar to the $(1,1)$ Cardy boundary state in the minimal conformal field theories.

In the context of (p,q) minimal non-critical string theory the boundary conditions of the Zamolodchikovs were given an interpretation as branes, the so-called ZZ branes [3–6]. In [4] it was shown that any ZZ brane in (p,q) minimal string theory can be viewed as a linear combination of $(p-1)(q-1)/2$ "fundamental" ZZ branes, the so-called principal ZZ branes. While the articles [3–5] provided a lot of new understanding of the target space structure of the ZZ branes and [7,8] of the world sheet aspects of ZZ branes, the question raised by the Zamolodchikovs in their original article remains unanswered. They said: "The most intriguing point is the nature of the "excited" vacua.... A meaning of these quantum excitations of the (physically infinite faraway) absolute remains to be comprehended".

We will show that in (p,q) minimal non-critical string theory the principal ZZ brane labeled by $(\hat{m},\hat{n})$ has the simple interpretation as the basic $(1,1)$ ZZ boundary state tensored with the $(\hat{m},\hat{n})$ Cardy matter boundary state[1]:

$$|1,1\rangle_{\text{cardy}} \otimes |\hat{m},\hat{n}\rangle_{\text{ZZ}} = |\hat{m},\hat{n}\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{ZZ}}. \tag{1}$$

Eq. (1) should be understood in the following way: With regard to expectation values of physical observables it does not matter whether we use the right-hand side or the left-hand side of Eq. (1). Thus, there exists only one ZZ boundary state, the basic $(1,1)$ boundary state, the other principle ZZ branes being matter-dressed $(1,1)$ boundary states. Furthermore, we will provide evidence for the following generalization of (1):

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[1] The possibility of a relation like (1) was first noticed in [5] when they calculated the ZZ-FZZT amplitude.

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$$|k,l\rangle_{\text{cardy}} \otimes |\hat{m},\hat{n}\rangle_{\text{ZZ}} = \left(\sum_{i=|k-\hat{m}|+1}^{\text{top}(k,\hat{m};p)}{}' \sum_{j=|l-\hat{n}|+1}^{\text{top}(l,\hat{n};q)}{}' |i,j\rangle_{\text{cardy}} \right) \otimes |1,1\rangle_{\text{ZZ}}, \tag{2}$$

where

$$\text{top}(k,\hat{m};p) = \min(k+\hat{m}-1, 2p-1-k-\hat{m}), \tag{3}$$

$$\text{top}(l,\hat{n};q) = \min(l+\hat{n}-1, 2q-1-l-\hat{n}). \tag{4}$$

Notice, this summation[2] is precisely the same which appears in the fusion of two primary operators in the (p,q) minimal conformal field theory:

$$O_{k,l} \times O_{\hat{m},\hat{n}} = \sum_{i=|k-\hat{m}|+1}^{\text{top}(k,\hat{m};p)}{}' \sum_{j=|l-\hat{n}|+1}^{\text{top}(l,\hat{n};q)}{}' [O_{i,j}]. \tag{5}$$

Why are Eqs. (1) and (2) true?

Recall the definition of the Cardy matter boundary states in the (p,q) minimal conformal field theory:

$$|k,l\rangle_{\text{cardy}} \equiv \sum_{i,j} \frac{S(k,l;i,j)}{\sqrt{S(1,1;i,j)}} |i,j\rangle\rangle, \tag{6}$$

where the summation runs over all the different Ishibashi states $|i,j\rangle\rangle$ in the (p,q) minimal model,

$$S(k,l;i,j) = 2\sqrt{\frac{2}{pq}}(-1)^{1+kj+li} \sin(\pi b^2 lj) \sin(\pi ki/b^2) \tag{7}$$

is the modular S-matrix in the (p,q) minimal model and $b^2 = p/q$. The Cardy matter boundary states are labeled by two integers (k,l), which satisfy that $1 \leqslant k \leqslant p-1$, $1 \leqslant l \leqslant q-1$ and $kq - lp > 0$.

On the other hand the principal ZZ boundary states are defined as

$$|\hat{m},\hat{n}\rangle_{\text{ZZ}} = \int_0^\infty \mathrm{d}P \frac{\sinh(2\pi\hat{m}P/b)\sinh(2\pi\hat{n}Pb)}{\sinh(2\pi P/b)\sinh(2\pi Pb)} \Psi_{1,1}(P)|P\rangle\rangle, \tag{8}$$

where $b = \sqrt{p/q}$. $\Psi_{1,1}(P)$ is the basic $(1,1)$ ZZ wave function [1]:

$$\Psi_{1,1}(P) = \beta \frac{iP\mu^{-iP/b}}{\Gamma(1-2iPb)\Gamma(1-2iP/b)}, \tag{9}$$

where the constant β is independent of the cosmological constant μ and P. Finally, $|P\rangle\rangle$ denotes the Ishibashi state corresponding to the non-local primary operator $\exp(2(Q/2+iP)\phi)$ in Liouville theory, where $Q = b + 1/b$. The principal ZZ branes are labeled by two integers $(\hat{m},\hat{n})$, where $1 \leqslant \hat{m} \leqslant p-1$, $1 \leqslant \hat{n} \leqslant q-1$, $\hat{m}q - \hat{n}p > 0$.

Notice, the ranges of the indices k, l labeling the different Cardy matter boundary states and the indices $\hat{m}$, $\hat{n}$ labeling the principal ZZ branes are the same. As noted already by the Zamolodchikovs in [1], the modular bootstrap equations for the ZZ boundary states are surprisingly similar to the bootstrap equations for the Cardy matter boundary states in the minimal models. The key point is now that the physical operators in minimal string theory, to be discussed below, carry both a matter "momentum" and a Liouville "momentum" and these are not independent, but related by the requirement that the operators scale in a specific way. In particular, the Liouville momenta P of the physical observables are imaginary and the imaginary i explains the shift from sin to sinh going from (6) to (8). The coupling between the matter and Liouville momenta implies, that physical expectation values will be the same irrespectively of whether we use the left or the right side of Eq. (1).

Below we will verify (1) and (2) for physical bulk operators evaluated on AdS_2, as well as (2) with regard to the FZZT–ZZ cylinder amplitude and the ZZ–ZZ cylinder amplitude.

2. The disk amplitude

According to [4] the physical operators in minimal noncritical string theory are the tachyon operators, the ground ring operators and the copies of the ground ring operators at negative ghost number.

The tachyon operators are given by

$$\mathcal{T}_{r,s} = c\bar{c}\mathcal{O}_{r,s}e^{2\beta_{r,s}\phi}, \tag{10}$$

where c is the ghost field, $\mathcal{O}_{r,s}$ the primary matter operators,

$$\beta_{r,s} = \frac{1}{2}(Q - r/b + sb), \qquad rq - ps > 0 \tag{11}$$

and

$$Q = b + 1/b, \quad b = \sqrt{p/q}. \tag{12}$$

In order to provide evidence for (1) we calculate the tachyon one-point function on the pseudosphere with the states $|k,l\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{ZZ}}$ and $|1,1\rangle_{\text{cardy}} \otimes |k,l\rangle_{\text{ZZ}}$ imposed at infinity.

Since the Liouville "momentum" corresponding to $\mathcal{T}_{r,s}$ is $P_{r,s} = i(Q/2 - \beta_{r,s})$ one obtains from (8):

$$\langle \mathcal{T}_{r,s} | \big(|k,l\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{ZZ}}\big) = \alpha \frac{S(k,l;r,s)}{\sqrt{S(1,1;r,s)}} \Psi_{1,1}(P_{r,s}), \tag{13}$$

and

$$\langle \mathcal{T}_{r,s} | \big(|1,1\rangle_{\text{cardy}} \otimes |k,l\rangle_{\text{ZZ}}\big) = \alpha\sqrt{S(1,1;r,s)} \times \frac{\sinh(2\pi kP_{r,s}/b)\sinh(2\pi lP_{r,s}b)}{\sinh(2\pi P_{r,s}/b)\sinh(2\pi P_{r,s}b)} \Psi_{1,1}(P_{r,s}), \tag{14}$$

where α is a constant independent of r, s, k and l.

Using the expression (7) for the modular S-matrix it is now simple algebra to verify that the right-hand sides of Eqs. (13) and (14) are equal.

One can also arrive at this conclusion starting from a FZZT boundary state tensored with a Cardy matter state, $|k,l\rangle_{\text{cardy}} \otimes |\sigma\rangle_{\text{fzz}}$, where σ is related to the boundary cosmological constant μ_b and the bulk cosmological constant μ by

$$\frac{\mu_b}{\sqrt{\mu}} = \cosh(\pi b\sigma). \tag{15}$$

[2] The prime in the summation symbol $\sum'$ means that the summation runs in steps of two.

According to [4] (by a calculation similar to the one leading to (13) and (14)) one has:

$$\langle \mathcal{T}_{r,s}|\big(|k,l\rangle_{\text{cardy}} \otimes |\sigma\rangle_{\text{fzz}}\big) = A_{r,s}(-1)^{ks+lr}\cosh\left(\frac{\pi\sigma(r-sb^2)}{b}\right) \times \sin\left(\frac{\pi kr}{b^2}\right)\sin\big(\pi lsb^2\big). \tag{16}$$

Using [3]

$$|k,l\rangle_{\text{zz}} = \big|i(k/b+lb)\big\rangle_{\text{fzz}} - \big|i(k/b-lb)\big\rangle_{\text{fzz}}, \tag{17}$$

which is valid for principal ZZ boundary states, one can now calculate both $\langle \mathcal{T}_{r,s}|(|k,l\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{zz}})$ and $\langle \mathcal{T}_{r,s}|(|1,1\rangle_{\text{cardy}} \otimes |k,l\rangle_{\text{zz}})$ and verify that they agree. For future reference we present one of the calculations:

$$\begin{aligned}\langle \mathcal{T}_{r,s}|\big(|k,l\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{zz}}\big) &= \langle \mathcal{T}_{r,s}|\Big(|k,l\rangle_{\text{cardy}} \otimes \big(\big|i(1/b+b)\big\rangle_{\text{fzz}} - \big|i(1/b-b)\big\rangle_{\text{fzz}}\big)\Big) &(18)\\ &= -2A_{r,s}(-1)^{ks+lr}\sin\left(\frac{\pi kr}{b^2}\right)\sin\big(\pi lsb^2\big) \times \sin\left(\frac{\pi r}{b^2}-\pi s\right)\sin\big(\pi r-\pi sb^2\big) &(19)\\ &= 2A_{r,s}(-1)^{ks+lr-s-r}\sin\left(\frac{\pi kr}{b^2}\right)\sin\big(\pi lsb^2\big) \times \sin\left(\frac{\pi r}{b^2}\right)\sin\big(\pi sb^2\big). &(20)\end{aligned}$$

A similar calculation of $\langle \mathcal{T}_{r,s}|(|1,1\rangle_{\text{cardy}} \otimes |k,l\rangle_{\text{zz}})$ gives the same result.

The ground ring operators as well as the physical operators at negative ghost number have FZZT one-point functions which are similar to the tachyon one-point functions (16), except that the range of s is different from the one for the tachyon operators [4]. Using calculations similar to (18)–(20) one can thus verify that the expectation values of these operators are independent of which of the two branes in Eq. (1) we impose at infinity.

We finally turn to the verification of (2). Let $\mathcal{B}_{r,s}$ denote a physical operator, i.e., a tachyon operator, a ground ring operator or one of the copies of a ground ring operator at negative ghost number. In this case $s<q$, $s \neq 0 \bmod q$ and $qr-ps>0$. These operators satisfy (16) with $\mathcal{B}$ substituted for $\mathcal{T}$. Our task is to calculate the matrix element

$$\langle \mathcal{B}_{r,s}|\big(|k,l\rangle_{\text{cardy}} \otimes |\hat{m},\hat{n}\rangle_{\text{zz}}\big). \tag{21}$$

Using (17) for $|\hat{m},\hat{n}\rangle_{\text{zz}}$ and then (16) and the following identity

$$\sinh(nx)\sinh(n'x) = \sum_{l=0}^{\min(n,n')-1}\sinh(x)\sinh\big((n+n'-2l-1)x\big) \tag{22}$$

one obtains after some algebra:

$$\langle \mathcal{B}_{r,s}|\big(|k,l\rangle_{\text{cardy}} \otimes |\hat{m},\hat{n}\rangle_{\text{zz}}\big) = \sum_{i=|k-\hat{m}|+1}^{k+\hat{m}-1}{}' \sum_{j=|l-\hat{n}|+1}^{l+\hat{n}-1}{}' \langle \mathcal{B}_{r,s}|\big(|i,j\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{zz}}\big). \tag{23}$$

This is still not in agreement with (2) since the range of summation does not always agree with (2). However, assume that $\hat{m}+k>p$. We now split the sum over i in two at $i=2p-1-\hat{m}-k$ in accordance to (2). With regard to the second part of the sum we then obtain using (19):

$$\begin{aligned}&\sum_{i=2p+1-\hat{m}-k}^{k+\hat{m}-1}{}' \sum_{j=|l-\hat{n}|+1}^{l+\hat{n}-1}{}' \langle \mathcal{B}_{r,s}|\big(|i,j\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{zz}}\big)\\ &\quad\propto \sum_{i=2p+1-\hat{m}-k}^{k+\hat{m}-1}{}' (-1)^{is}\sin\big(\pi ir/b^2\big) = 0.\end{aligned} \tag{24}$$

Similar arguments for $\hat{n}+l>q$ leads to

$$\begin{aligned}&\sum_{i=|\hat{m}-k|+1}^{k+\hat{m}-1}{}' \sum_{j=2q+1-\hat{n}-l}^{l+\hat{n}-1}{}' \langle \mathcal{B}_{r,s}|\big(|i,j\rangle_{\text{cardy}} \otimes |1,1\rangle_{\text{zz}}\big)\\ &\quad\propto \sum_{j=2q+1-\hat{n}-l}^{l+\hat{n}-1}{}' (-1)^{jr}\sin\big(\pi jsb^2\big) = 0.\end{aligned} \tag{25}$$

Hence, the range of summation in (23) may be expressed as in (2).

3. The cylinder amplitude

To simplify the discussion we restrict ourselves to $(2,2m-1)$ minimal string theory. However, we expect the result to be valid in any (p,q) minimal string theory. In $(2,2m-1)$ non-critical string theory the Cardy matter boundary states are labeled by only one integer r, which satisfies $1 \leqslant r \leqslant m-1$. Our basic object is the FZZT–FZZT cylinder amplitude $Z(r,\sigma';s,\sigma)$ obtained in [8]. The principal $(\hat{m},\hat{n})$ ZZ boundary states in the $(2,2m-1)$ minimal string theories have $\hat{m}=1$ and $1 \leqslant \hat{n} \leqslant m-1$. We may calculate the FZZT–ZZ cylinder amplitude from (17):

$$Z\big(r,(1,\hat{n})_{\text{zz}};s,\sigma\big) = Z\big(r,\sigma'=i(1/b+\hat{n}b);s,\sigma\big) - Z\big(r,\sigma'=i(1/b-\hat{n}b);s,\sigma\big). \tag{26}$$

Let us now define the following quantity:

$$\begin{aligned}K(r,\sigma';s,\sigma) &\equiv \sqrt{2}\pi^2\int_{\gamma_u}\mathrm{d}P\,\frac{P}{P^2+\varepsilon^2} \times \frac{e^{2\pi i\sigma P}\cos(2\pi\sigma' P)\sinh(2\pi rbP)\sinh(2\pi sbP)}{\sinh(4\pi P/b)\sinh^2(2\pi bP)}\\ &\quad+\sqrt{2}\pi^2\int_{\gamma_l}\mathrm{d}P\,\frac{P}{P^2+\varepsilon^2} \times \frac{e^{-2\pi i\sigma P}\cos(2\pi\sigma' P)\sinh(2\pi rbP)\sinh(2\pi sbP)}{\sinh(4\pi P/b)\sinh^2(2\pi bP)},\end{aligned} \tag{27}$$

where γ_u is a path enclosing the singularities in the upper half plane anti-clockwise, while γ_l is a path enclosing the singularities in the lower half plane clockwise. We have followed [5] and used the regularization

$$\frac{1}{P} \to \frac{P}{P^2+\varepsilon^2}$$

in (27). The FZZT–FZZT cylinder amplitude is actually given by (27) for $r+s \leqslant m$. For $0<\sigma'<\sigma$ this may be seen by deforming the integration contour appearing in the integral expression for the FZZT–FZZT cylinder amplitude in [8]. However, the integral expression in [8] is not convergent for generic values of σ and σ' and we therefore define the cylinder amplitude for generic values of σ and σ' by the analytic continuation of (27).[3] Hence, one has

$$K(r,\sigma';s,\sigma) = Z(r,\sigma';s,\sigma) \quad \text{for } r+s \leqslant m. \tag{28}$$

As argued in [8], for $r+s>m$ the expression for $Z(r,\sigma';s,\sigma)$ is more complicated as a simple consequence of the fusion rules (5) in the matter sector. This complication does not affect the FZZT–ZZ cylinder amplitude in the sense that the additional terms present in $Z(r,\sigma';s,\sigma)$ compared to $K(r,\sigma';s,\sigma)$ for $r+s>m$ cancel in the difference (26). We thus have

$$\begin{aligned} Z\big(r,(1,\hat{n})_{\text{ZZ}};s,\sigma\big) &= K\big(r,\sigma'=i(1/b+\hat{n}b);s,\sigma\big) \\ &\quad - K\big(r,\sigma'=i(1/b-\hat{n}b);s,\sigma\big), \end{aligned} \tag{29}$$

for all values of the Cardy matter labels r and s.

We can now use (29) and (27) to perform a calculation similar to the ones already performed in (18)–(20) and (21)–(23). The algebraic operations are performed on the K-integrands and using (22) it is easily seen that

$$\begin{aligned} Z\big(r,(1,\hat{n})_{\text{ZZ}};s,\sigma\big) &= \sum_{k=|r-\hat{n}|+1}^{r+\hat{n}-1}{}' K\big(k,\sigma'=i(1/b+b);s,\sigma\big) \\ &\quad - K\big(k,\sigma'=i(1/b-b);s,\sigma\big) \\ &= \sum_{k=|r-\hat{n}|+1}^{r+\hat{n}-1}{}' Z\big(k,(1,1)_{\text{ZZ}};s,\sigma\big), \end{aligned} \tag{30}$$

which is consistent with (2).

We finally consider the ZZ–ZZ cylinder amplitude. Using (17) and (30) one finds:

$$\begin{aligned} &Z\big(r,(1,\hat{n})_{\text{ZZ}};s,(1,\hat{m})_{\text{ZZ}}\big) \\ &\quad = \sum_{k=|r-\hat{n}|+1}^{r+\hat{n}-1}{}' \sum_{l=|s-\hat{m}|+1}^{s+\hat{m}-1}{}' Z\big(k,(1,1)_{\text{ZZ}};l,(1,1)_{\text{ZZ}}\big), \end{aligned} \tag{31}$$

again in accordance with (2).

[3] If one performs the summation over residues, one realizes that (27) is actually symmetric in σ and σ'.

4. Discussion

We have provided some evidence that (1) and (2) are valid for physical expectation values in minimal non-critical string theory. It answers partly the question raised by the Zamolodchikovs and quoted in the introduction. In the cases where one actually has a concrete model like the (p,q) minimal conformal field theory as the matter content in non-critical string theory, one can view the principal ZZ branes as matter-dressed basic $(1,1)$ ZZ boundary states. Because of the special form (7) of the fusion matrix, imposing a Cardy matter condition at infinity different from the basic $(1,1)$ Cardy matter condition is equivalent to (at least for the observables we have considered) multiplying the basic ZZ wave function $\Psi_{1,1}(P)$ with a factor:

$$\begin{aligned} \Psi_{1,1}(P) \to \frac{\sinh(2\pi\hat{m}P/b)\sinh(2\pi\hat{n}Pb)}{\sinh(2\pi P/b)\sinh(2\pi Pb)}\Psi_{1,1}(P) \\ \equiv \Psi_{\hat{m},\hat{n}}(P), \end{aligned} \tag{32}$$

$\Psi_{\hat{m},\hat{n}}(P)$ being the wave function of the principal $(\hat{m},\hat{n})$ ZZ boundary state. The Zamolodchikovs derived the set of wave functions $\Psi_{\hat{m},\hat{n}}(P)$ by demanding that the two-point function factorize into a product of one-point functions when the geodesic distance diverges, but they only considered Liouville theory, i.e., the matter part had been integrated out. What we have shown in this article is that the effect of imposing a $(\hat{m},\hat{n})$ Cardy matter boundary condition at infinity is precisely to produce a $\Psi_{\hat{m},\hat{n}}(P)$ wave function in Liouville theory. The integral expression for the FZZT–ZZ cylinder amplitude in (29) and (27) is indeed obtained after integrating over the matter degrees of freedom.

Within the context of Liouville theory it was observed in [1] that the two-point functions diverge as a function of the geodesic distance on the pseudosphere if one consider a $(\hat{m},\hat{n})$ ZZ brane with $(\hat{m},\hat{n}) \neq (1,1)$. It is interesting if this can be understood in terms of Cardy matter boundary states. In principle we have the machinery to address such questions not only in terms of the "background geometry" of the pseudosphere, but in terms of the full quantum geometry using the so-called loop-loop propagator $G_\mu(l_1,l_2;d)$ [9–12]. As shown in [8] it allows us to describe the transition from compact to non-compact geometry as the geodesic distance $d\to\infty$, and in some cases it also allows us to relate very explicitly to the matter states at the boundaries [10,13,14]. In the so-called CDT-model of 2d quantum gravity [15], which is closely related to ordinary Euclidean 2d quantum gravity [16], we have already observed a drastic change in the behavior of the correlation functions due to the geometry of the pseudosphere [17] and the same thing might happen in the full Euclidean theory.

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