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Finite products are biproducts in a compact closed category

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Abstract

If a compact closed category has finite products or finite coproducts then it in fact has finite biproducts, and so is semi-additive.

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1. Introduction

Compact closed categories with biproducts have recently attracted renewed attention from theoretical computer scientists, because of their role in the abstract approach to quantum information initiated by Abramsky and Coecke [1]. Perhaps surprisingly, it seems to have gone unnoticed that finite products or coproducts in a compact closed category necessarily carry a biproduct structure. Here we prove that this is so. In fact we prove a more general result, viz:

Proposition 2. *Let $\mathbb{C}$ be a monoidal category with finite products and coproducts, and suppose that for every object $A \in \mathbb{C}$, the functor $A \otimes -$ preserves products and the functor $- \otimes A$ preserves coproducts. Then $\mathbb{C}$ has finite biproducts.*

A category with finite biproducts is necessarily semi-additive, i.e. enriched over commutative monoids. In other words, each homset has the structure of a commutative monoid, and composition preserves the commutative monoid structure. The converse is also true: a semi-additive category with finite products or coproducts in fact has finite biproducts. Therefore an equivalent statement of our conclusion would be that $\mathbb{C}$ is semi-additive.

The gist of the argument is as follows. Let $\mathbb{C}$ be as in the statement of the proposition. The object $0 \otimes 1$ is initial because $- \otimes 1$ preserves initiality, and terminal because $0 \otimes -$ preserves terminality. So it is a zero object. The binary case is similar, though more intricate. Let $A, B, C, D \in \mathbb{C}$ and consider the object $(A + B) \otimes (C \times D)$. Since the $\otimes$ distributes over both the $+$ and the $\times$ in this expression, it may be multiplied out as either

$$(A \otimes C \times A \otimes D) + (B \otimes C \times B \otimes D) \tag{1}$$

or

$$(A \otimes C + B \otimes C) \times (A \otimes D + B \otimes D), \tag{2}$$

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hence (1) is isomorphic to (2). Letting $C = D = I$ shows that

$$A^2 + B^2 \cong (A + B)^2, \tag{3}$$

and it may be verified (Lemmas 3 and 4) that the canonical natural map

$$A^2 + B^2 \to (A + B)^2,$$

denoted $t_{A,B}$ below, is equal to the left-to-right direction of (3). It follows that $t_{A,B}$ is invertible. From this we derive, via Lemma 5, that the natural map $A + B \to A \times B$ is also invertible, which implies the desired conclusion.

The remainder of this paper contains the detailed proof. The next section recalls the basic facts about finite products and coproducts, and some simple properties of compact closed categories: it will not tax the experienced reader, who may prefer to skip directly to Section 3.

2. Background

This short paper uses only elementary ideas of category theory, which we briefly recall so as to fix our notation.

In a category with finite products, we denote the given terminal object 1, and suppose that for every pair A, B of objects there is a given product cone $(\pi_1 : A \times B \to A, \pi_2 : A \times B \to B)$. For any pair of maps $f : X \to A$, $g : Y \to B$, we denote their pairing as $\langle f, g\rangle : X \to A \times B$, i.e. $\langle f, g\rangle$ is the unique map for which $\pi_1 \circ \langle f, g\rangle = f$ and $\pi_2 \circ \langle f, g\rangle = g$. Given $f : A \to B$ and $g : C \to D$, we write $f \times g$ for the map

$$\langle f \circ \pi_1, g \circ \pi_2\rangle : A \times C \to B \times D.$$

Note that this definition makes $\times$ into a functor, in such a way that π_1 and π_2 constitute natural transformations. For example $\pi_1 \circ (f \times g) = \pi_1 \circ \langle f \circ \pi_1, g \circ \pi_2\rangle = f \circ \pi_1$.

A functor F is said to *preserve products* if the image under F of a product cone is always a product cone (not necessarily the chosen one). We take it to include the nullary case also, i.e. the image of a terminal object must be terminal. If the categories $\mathbb{C}$ and $\mathbb{D}$ have finite products and $F : \mathbb{C} \to \mathbb{D}$ preserves products then the morphism

$$F(A \times B) \xrightarrow{\langle F\pi_1, F\pi_2\rangle} FA \times FB$$

is invertible.

The case of coproducts is dual to the above. In a category that has finite coproducts, we assume that there is an initial object 0 and that for every pair of objects A, B, there is a given coproduct cocone $(i_1 : A \to A+B, i_2 : B \to A+B)$. Given maps $f : A \to Y$ and $g : B \to Y$, we write their co-pairing as

$$[f, g] : A + B \to Y;$$

if $\mathbb{C}$ and $\mathbb{D}$ have finite coproducts and $F : \mathbb{C} \to \mathbb{D}$ preserves coproducts then the map

$$FA + FB \xrightarrow{[Fi_1, Fi_2]} F(A + B)$$

is invertible.

Now suppose we are in a category that has both finite products and finite coproducts. A morphism

$$f : A + B \to C \times D$$

is determined by the four maps

$$\begin{aligned} f_{11} &:= \pi_1 \circ f \circ i_1 : A \to C, \qquad & f_{12} &:= \pi_1 \circ f \circ i_2 : B \to C \\ f_{21} &:= \pi_2 \circ f \circ i_1 : A \to D, \qquad & f_{22} &:= \pi_2 \circ f \circ i_2 : B \to D, \end{aligned}$$

since $f = [\langle f_{11}, f_{21}\rangle, \langle f_{12}, f_{22}\rangle] = \langle [f_{11}, f_{12}], [f_{21}, f_{22}]\rangle$. We refer to this as the *matrix representation* of f, and write it as

$$f = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}.$$

A technique that is used several times below is to check that two maps are equal by calculating and comparing their matrix representations.

There are several equivalent ways of defining what it means for a category to have finite biproducts. The one most convenient for our purposes is as follows (see Exercise VIII.2.4 of Mac Lane [4]).

Definition. A category $\mathbb{C}$ has *finite biproducts* if it has finite products and finite coproducts, such that:

- the unique morphism $0 \to 1$ is invertible, thus there is a (unique) zero map $0_{A,B} : A \to 1 \cong 0 \to B$ between any objects A and B, and
- the morphism

  $$\begin{bmatrix} 1_A & 0_{B,A} \\ 0_{A,B} & 1_B \end{bmatrix} : A + B \to A \times B$$

  is invertible for all A and B in $\mathbb{C}$.

Compact closed categories were first defined (almost in passing) by Kelly [2], and later studied in depth by Kelly and Laplaza [3]. The reader may consult either of those references for the precise definition. For the purposes of this paper, it suffices to know that a compact closed category is a monoidal category $(\mathbb{C}, \otimes, I)$ that has – among other things – the following two properties:

- $\mathbb{C}$ is self-dual, i.e. $\mathbb{C}$ is equivalent to $\mathbb{C}^{\mathrm{op}}$,
- for every object $A \in \mathbb{C}$, the functors $A \otimes -$ and $- \otimes A$ have both a left and a right adjoint.

Examples include the category Rel of sets and relations, with the tensor as cartesian product, and the category FinVect of finite-dimensional vector spaces, with the usual tensor product of vector spaces.

3. Main result

Our main result is as follows.

Theorem 1. *Let $\mathbb{C}$ be a compact closed category. If $\mathbb{C}$ has finite products (or coproducts) then it has finite biproducts.*

We shall deduce the theorem from a somewhat more general proposition:

Proposition 2. *Let $\mathbb{C}$ be a monoidal category with finite products and coproducts, and suppose that for every object $A \in \mathbb{C}$, the functor $A \otimes -$ preserves products and the functor $- \otimes A$ preserves coproducts. Then $\mathbb{C}$ has finite biproducts.*

The nullary case may be dispensed with immediately:

Proof (*That the unique morphism* $0 \to 1$ *is invertible*). The functor $0 \otimes -$ preserves products, thus $0 \otimes 1$ is terminal. But also the functor $- \otimes 1$ preserves coproducts, so $0 \otimes 1$ is also initial. Therefore 0 is isomorphic to 1, and the claim follows. □

From now on, we assume that we have a category that satisfies the conditions of Proposition 2, and which therefore has a zero object. We shall omit the subscripts when referring to a zero map, since the type is always obvious from the context. We have no further occasion to refer explicitly to an initial object, so the symbol ‘0’ below always denotes a zero map. Also we shall follow the common practice of abbreviating the identity morphism 1_A to A.

Remark. Since $A \otimes -$ preserves products, we know that for all objects A,B,C, the distribution map

$$\langle A \otimes \pi_1, A \otimes \pi_2 \rangle : A \otimes (B \times C) \to (A \otimes B) \times (A \otimes C)$$

is invertible, and since $- \otimes C$ preserves coproducts, we know that for all objects A,B,C, the distribution map

$$[i_1 \otimes C, i_2 \otimes C] : (A \otimes C) + (B \otimes C) \to (A + B) \otimes C$$

is invertible.

$$\begin{tikzcd}[column sep=small]
\begin{matrix}((A_1 \otimes B_1) \times (A_1 \otimes B_2)) \\ + ((A_2 \otimes B_1) \times (A_2 \otimes B_2))\end{matrix} \arrow[rr, dotted] & & (A_1 + A_2) \otimes (B_1 \times B_2) \arrow[rr, dotted] \arrow[dr, "\begin{matrix}\langle (A_1+A_2)\otimes\pi_1, \\ (A_1+A_2)\otimes\pi_2\rangle\end{matrix}", "\sim"'] \arrow[ddr, bend left, "(A_1+A_2)\otimes\pi_k"] & & \begin{matrix}((A_1 \otimes B_1) + (A_2 \otimes B_1)) \\ \times ((A_1 \otimes B_2) + (A_2 \otimes B_2))\end{matrix} \arrow[dl, "\begin{matrix}[i_1\otimes B_1, i_2\otimes B_1] \\ \times [i_1\otimes B_2, i_2\otimes B_2]\end{matrix}"', "\sim"] \arrow[d, "\pi_k"] \\
\begin{matrix}(A_j \otimes B_1) \\ \times (A_j \otimes B_2)\end{matrix} \arrow[u, "i_j"] \arrow[ddd, "\pi_k"'] & \begin{matrix}(A_1 \otimes (B_1 \times B_2)) \\ + (A_2 \otimes (B_1 \times B_2))\end{matrix} \arrow[ul, "\begin{matrix}\langle A_1\otimes\pi_1, A_1\otimes\pi_2\rangle \\ + \langle A_2\otimes\pi_1, A_2\otimes\pi_2\rangle\end{matrix}", "\sim"'] \arrow[ur, "\begin{matrix}[i_1\otimes(B_1\times B_2), \\ i_2\otimes(B_1\times B_2)]\end{matrix}"', "\sim"] & & \begin{matrix}((A_1 + A_2) \otimes B_1) \\ \times ((A_1 + A_2) \otimes B_2)\end{matrix} \arrow[d, "\pi_k"'] & \begin{matrix}(A_1 \otimes B_k) \\ + (A_2 \otimes B_k)\end{matrix} \arrow[dl, "\begin{matrix}[i_1\otimes B_k, i_2\otimes B_k]\end{matrix}"', "\sim"] \\
& A_j \otimes (B_1 \times B_2) \arrow[ul, "\langle A_j\otimes\pi_1, A_j\otimes\pi_2\rangle", "\sim"'] \arrow[u, "i_j"] \arrow[uur, bend left, "i_j\otimes(B_1\times B_2)"] \arrow[rr, "i_j\otimes\pi_k"'] \arrow[dl, "A_j\otimes\pi_k"] & & (A_1 + A_2) \otimes B_k & \\
A_j \otimes B_k \arrow[rrrr, "A_j\otimes B_k"'] & & & & A_j \otimes B_k \arrow[ul, "i_j\otimes B_k"'] \arrow[uu, "i_j"']
\end{tikzcd}$$

Fig. 1. Diagram used in the proof of Lemma 3. The arrows marked '$\sim$' are invertible, by the remark preceding Lemma 3. A dotted arrow represents the unique (iso)morphism for which the triangle below it commutes, so that the composite along the top edge is equal, by definition, to y.

Lemma 3. *For all objects A_1, A_2, B_1, B_2, the canonical map*

$$\begin{bmatrix} i_1 \circ \pi_1 & i_2 \circ \pi_1 \\ i_1 \circ \pi_2 & i_2 \circ \pi_2 \end{bmatrix} \tag{$*$}$$

($= [i_1 \times i_1, i_2 \times i_2] = \langle \pi_1 + \pi_1, \pi_2 + \pi_2 \rangle$) of type

$$\begin{aligned} &((A_1 \otimes B_1) \times (A_1 \otimes B_2)) + ((A_2 \otimes B_1) \times (A_2 \otimes B_2)) \\ &\quad \to ((A_1 \otimes B_1) + (A_2 \otimes B_1)) \times ((A_1 \otimes B_2) + (A_2 \otimes B_2)) \end{aligned}$$

is invertible.

Proof. We'll show that $(*)$ is equal to the map y defined as the composite

$$\begin{aligned} &((A_1 \otimes B_1) \times (A_1 \otimes B_2)) + ((A_2 \otimes B_1) \times (A_2 \otimes B_2)) \\ &\quad \to (A_1 \otimes (B_1 \times B_2)) + (A_2 \otimes (B_1 \times B_2)) \\ &\quad \to (A_1 + A_2) \otimes (B_1 \times B_2) \\ &\quad \to ((A_1 + A_2) \otimes B_1) \times ((A_1 + A_2) \otimes B_2) \\ &\quad \to ((A_1 \otimes B_1) + (A_2 \otimes B_1)) \times ((A_1 \otimes B_2) + (A_2 \otimes B_2)) \end{aligned}$$

of distribution maps and their inverses. Clearly y is invertible, since it is composed of isomorphisms.

Take $j, k \in \{1, 2\}$ and consider the diagram in Fig. 1. All the regions commute for obvious reasons, so the outside commutes and $\pi_k \circ y \circ i_j = i_j \circ \pi_k$. Since this is true for all j and k, it follows that $y = (*)$, as required. □

Definition. Given objects A and B, let $t_{A,B}$ denote the map

$$\begin{bmatrix} i_1 \circ \pi_1 & i_2 \circ \pi_1 \\ i_1 \circ \pi_2 & i_2 \circ \pi_2 \end{bmatrix} : (A \times A) + (B \times B) \to (A + B) \times (A + B).$$

Lemma 4. *For all objects A, B, the map $t_{A,B}$ is invertible.*

Proof. Use Lemma 3 with $A_1 = A$, $A_2 = B$ and $B_1 = B_2 = I$, and apply the right-unit isomorphism. □

Definition. Given objects A and B, let $e_{A,B}$ denote the composite

$$(A \times A) + (B \times B) \xrightarrow{\pi_1 + \pi_2} A + B \xrightarrow{\langle A,0\rangle + \langle 0,B\rangle} (A \times A) + (B \times B)$$

which is clearly an idempotent that splits on $A + B$, and let $e'_{A,B}$ denote the composite

$$(A + B) \times (A + B) \xrightarrow{[A,0]\times[0,B]} A \times B \xrightarrow{i_1 \times i_2} (A + B) \times (A + B)$$

which is an idempotent that splits on $A \times B$.

Lemma 5. *$t_{A,B}$ is a map of idempotents from $e_{A,B}$ to $e'_{A,B}$, i.e. the diagram*

$$\begin{tikzcd}
(A \times A) + (B \times B) \arrow[r, "t_{A,B}"] \arrow[d, "e_{A,B}"'] & (A + B) \times (A + B) \arrow[d, "e'_{A,B}"] \\
(A \times A) + (B \times B) \arrow[r, "t_{A,B}"'] & (A + B) \times (A + B)
\end{tikzcd}$$

commutes.

Proof. We claim that both paths have the matrix representation

$$\begin{bmatrix} i_1 \circ \pi_1 & 0 \\ 0 & i_2 \circ \pi_2 \end{bmatrix}.$$

Consider the diagram

$$\begin{tikzcd}
A^2 + B^2 \arrow[r, "\pi_1 + \pi_2"] & A + B \arrow[r, "\langle A,0\rangle + \langle 0,B\rangle"] & A^2 + B^2 \arrow[r, "t_{A,B}"] & (A+B)^2 \arrow[dd, "\pi_1"] \\
 & & A^2 \arrow[u, "i_1"'] \arrow[d, "\pi_1"] & \\
A^2 \arrow[uu, "i_1"] \arrow[r, "\pi_1"'] \arrow[dd, "i_1"'] & A \arrow[uu, "i_1"'] \arrow[ur, "\langle A,0\rangle"] \arrow[r, "A"] \arrow[d, "i_1"] & A \arrow[r, "i_1"'] & A + B \\
 & A + B \arrow[ur, "{[A,0]}"'] & & \\
A^2 + B^2 \arrow[r, "t_{A,B}"'] & (A+B)^2 \arrow[u, "\pi_1"] \arrow[r, "{[A,0] \times [0,B]}"'] & A \times B \arrow[uu, "\pi_1"'] \arrow[r, "i_1 \times i_2"'] & (A+B)^2 \arrow[uu, "\pi_1"']
\end{tikzcd}$$

where the composite along the top edge is equal to $t_{A,B} \circ e_{A,B}$, and the bottom edge is equal to $e'_{A,B} \circ t_{A,B}$. Since all the cells commute, it follows that

$$\pi_1 \circ (t_{A,B} \circ e_{A,B}) \circ i_1 = i_1 \circ \pi_1 = \pi_1 \circ (e'_{A,B} \circ t_{A,B}) \circ i_1,$$

and a similar argument shows that $\pi_2 \circ (t_{A,B} \circ e_{A,B}) \circ i_2 = i_2 \circ \pi_2 = \pi_2 \circ (e'_{A,B} \circ t_{A,B}) \circ i_2$. Similar diagrams also show that $\pi_1 \circ (t_{A,B} \circ e_{A,B}) \circ i_2 = 0 = \pi_1 \circ (e'_{A,B} \circ t_{A,B}) \circ i_2$ and $\pi_2 \circ (t_{A,B} \circ e_{A,B}) \circ i_1 = 0 = \pi_2 \circ (e'_{A,B} \circ t_{A,B}) \circ i_1$. For example, for the former we have

$$\begin{tikzcd}
A^2 + B^2 \arrow[r, "\pi_1 + \pi_2"] & A + B \arrow[r, "\langle A,0\rangle + \langle 0,B\rangle"] & A^2 + B^2 \arrow[r, "t_{A,B}"] & (A+B)^2 \arrow[ddd, "\pi_1"] \\
 & & B^2 \arrow[u, "i_2"'] \arrow[d, "\pi_1"] & \\
 & B \arrow[uu, "i_2"] \arrow[ur, "\langle 0,B\rangle"] \arrow[r, "0"'] & A \arrow[dr, "i_2"] & \\
B^2 \arrow[uuu, "i_2"] \arrow[ur, "\pi_2"] \arrow[dr, "\pi_1"'] \arrow[ddd, "i_2"'] & & & A + B \\
 & B \arrow[r, "0"] \arrow[d, "i_2"] & A \arrow[ur, "i_1"'] & \\
 & A + B \arrow[ur, "{[A,0]}"'] & & \\
A^2 + B^2 \arrow[r, "t_{A,B}"'] & (A+B)^2 \arrow[u, "\pi_1"] \arrow[r, "{[A,0] \times [0,B]}"'] & A \times B \arrow[uu, "\pi_1"'] \arrow[r, "i_1 \times i_2"'] & (A+B)^2. \arrow[uuu, "\pi_1"']
\end{tikzcd}$$

We can now complete the proof of Proposition 2, and hence of Theorem 1. □

Proof (*That* $\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$ *is invertible*). By Lemma 5, we know that the map $c_{A,B} :=$

$$A + B \xrightarrow{\langle A,0\rangle + \langle 0,B\rangle} (A \times A) + (B \times B) \xrightarrow{t_{A,B}} (A + B) \times (A + B) \xrightarrow{[A,0]\times[0,B]} A \times B$$

is invertible with inverse

$$A \times B \underset{i_1 \times i_2}{\longrightarrow} (A + B) \times (A + B) \underset{t_{A,B}^{-1}}{\longrightarrow} (A \times A) + (B \times B) \underset{\pi_1 + \pi_2}{\longrightarrow} A + B,$$

so it suffices to check that $c_{A,B} = [\langle A, 0\rangle, \langle 0, B\rangle]$. But that's easy to check: for example, the diagram

$$\begin{tikzcd}
 & & A \arrow[dr, "i_1"] \arrow[drr, bend left, "A"] & & \\
A \arrow[urr, bend left, "A"] \arrow[r, "\langle A,0\rangle"'] \arrow[d, "i_1"'] & A^2 \arrow[ur, "\pi_1"'] \arrow[d, "i_1"] & & A+B \arrow[r, "{[A,0]}"'] & A \\
A+B \arrow[r, "\langle A,0\rangle + \langle 0,B\rangle"'] & A^2+B^2 \arrow[rr, "t_{A,B}"'] & & (A+B)^2 \arrow[u, "\pi_1"] \arrow[r, "{[A,0]\times[0,B]}"'] & A\times B \arrow[u, "\pi_1"']
\end{tikzcd}$$

shows that $\pi_1 \circ c_{A,B} \circ i_1$ is the identity on A, and the diagram

$$\begin{tikzcd}
 & & A \arrow[dr, "i_1"] \arrow[drr, bend left, "0"] & & \\
A \arrow[urr, bend left, "0"] \arrow[r, "\langle A,0\rangle"'] \arrow[d, "i_1"'] & A^2 \arrow[ur, "\pi_2"'] \arrow[d, "i_1"] & & A+B \arrow[r, "{[0,B]}"'] & B \\
A+B \arrow[r, "\langle A,0\rangle + \langle 0,B\rangle"'] & A^2+B^2 \arrow[rr, "t_{A,B}"'] & & (A+B)^2 \arrow[u, "\pi_2"] \arrow[r, "{[A,0]\times[0,B]}"'] & A\times B \arrow[u, "\pi_2"']
\end{tikzcd}$$

shows that $\pi_2 \circ c_{A,B} \circ i_1 = 0$. The other two cases are similar. □

Proof of Theorem 1. A compact closed category is equivalent to its opposite, therefore has finite coproducts iff it has finite products. For every object A, the functors $A \otimes -$ and $- \otimes A$ have both a left and a right adjoint, hence preserve limits and colimits. So Proposition 2 applies, in particular, to a compact closed category that has finite products (or coproducts). □

4. Final remarks

It is significant that the zero object plays a crucial role in our argument. A compact closed category may very well have finite *non-empty* products and coproducts that are not biproducts. A simple example, due to Masahito Hasegawa, is the ordered group of integers under addition. Indeed *any* linearly ordered abelian group constitutes an example, for the following reason. A partially ordered abelian group may be regarded as a compact closed category: the underlying partial order is regarded as a category in the usual way, the group operation provides a symmetric tensor product, and the adjoint of an object is its group inverse. If in fact the group is *linearly* ordered then every non-empty finite set of elements has a minimum (which is their product) and a maximum (coproduct).

This degenerate example may also be used to construct non-degenerate examples, by taking its product with Rel, say.

One last observation: Proposition 2's requirement that $\mathbb{C}$ be a monoidal category is stronger than necessary. We didn't actually need the associativity of tensor, nor the left unit isomorphism. So instead of the full monoidal structure it suffices merely to have a functor $\otimes : \mathbb{C} \times \mathbb{C} \to \mathbb{C}$ with a right unit.

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