A Brief Report on a Number of Recently Discovered Sets of Notes on Riemann’s Lectures and on the Transmission of the Riemann Nachlass

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The collection of Riemann’s mathematical papers preserved in Göttingen University Library since 1895 includes none of Riemann’s scientific correspondence nor any of his more personal papers. The present report gives an account of the documents (correspondence, lecture notes, etc.) discovered outside Göttingen in the course of a larger research project on Riemann, and briefly describes the history of the Riemann Nachlass. At the same time, readers are kindly requested to inform the author of the whereabouts of any further material relating to Riemann, so that it can be included in the collection of texts and sources currently in preparation.

The mathematical papers of Bernhard Riemann (1826–1866) were first studied immediately after his death by a number of leading 19th century scholars such as A. Clebsch, R. Dedekind, and H. Weber in connection with the publication of Riemann’s Collected Works [Riemann 1876]. The papers are now kept in the University Library in Göttingen (Niedersächsische Staats- und Universitätsbibliothek Göttingen, abbreviated hereafter NSUB), together with numerous sets of
notes on Riemann’s lectures brought together on the initiative of Felix Klein. Since the Riemann collection in Göttingen includes none of Riemann’s personal papers (school reports, diplomas, official documents, etc.), nor any of his scientific or private correspondence, the question may be raised as to the transmission of the Riemann Nachlass as a whole and whether any of these failing items are extant. An attempt to answer these questions was made as part of a research project on Riemann commenced in 1978 [Neuenschwander 1979, 1980, 1981a, 1981b, 1981c], supported from 1982 to 1986 by the Stiftung Volkswagenwerk. In the course of this project, a search was made in almost 100 libraries throughout the German speaking countries and elsewhere for extant manuscripts relating to Riemann. In the present report we propose to describe briefly some of the newly discovered documents, in particular lecture notes and correspondence not included in the Göttingen collection, and furthermore to give an account of the history of the Riemann Nachlass. At the same time, we would be grateful to readers for any information concerning the whereabouts of further material relating to Riemann outside the main collections in Göttingen and Berlin, so that it can be included in our edition of letters and documents currently in preparation.

THE TRANSMISSION OF THE RIEMANN Nachlass

The original extent and condition of the Riemann Nachlass can be reconstructed relatively accurately from the correspondence and writings of Dedekind (NSUB Göttingen, Cod. Ms. Dedekind 14; [Dedekind 1876]; etc.), and from the editorial papers of H. Weber (NSUB, Cod. Ms. Riemann 1 and 2). According to these sources, following her husband’s death, Riemann’s wife first handed the mathematical papers over to R. Dedekind (1831–1916), who proceeded to arrange them systematically as far as he could, and posthumously published Riemann’s treatise on trigonometric series, his paper on the foundations of geometry, and his contribution to electrodynamics (see [Riemann 1876, 227–293]). In 1872, Riemann’s mathematical papers came into the hands of A. Clebsch (1833–1872), Riemann’s successor in Göttingen, and after Clebsch’s death, through the mediation of Dedekind and Wilhelm Weber, were passed on to Heinrich Weber (1842–1913) in 1874. Weber drew upon them in compiling the edition of Riemann’s Collected Works [Riemann 1876] and, after the second edition appeared in 1892, passed them on, with the consent of Riemann’s wife, to the Göttingen Society, now Academy of Sciences [1]. From there, the papers were finally given to the University Library in February 1895, where they have remained since [2].

As to Riemann’s scientific correspondence, it appears to have first been placed at the disposal of E. Schering (1833–1897), who presented the Riemann Memorial Lecture [Schering 1909 and 1867] to the Göttingen Academy in 1866. In the spring of 1875, after Dedekind had agreed to write a brief biography [Riemann 1876, 539–558] to accompany Riemann’s Collected Works, Frau Riemann asked Schering to return her husband’s letters, which she then sent on to Dedekind on May 1, 1875. Her accompanying letter, which is now in the Dedekind-Collection in Göttingen, yields important information about the more personal papers Riemann left behind,
of which only relatively few have so far been brought to light. According to Frau Riemann’s letter (NSUB, Cod. Ms. Dedekind 14, No. 177), the material sent on to Dedekind included Riemann’s private correspondence with his parents, brother, and sisters, some letters which had passed between Riemann and his wife, a small black book containing records of Riemann’s sojourn in Paris in the spring of 1860, and a large bundle of letters received by Riemann from leading men of his day. From other letters exchanged in connection with the edition of Riemann’s Collected Works, it emerges that, at this time, Riemann’s own lecture notebooks from his student days in Berlin, and the student enrollment lists later consulted by Klein, were also in Frau Riemann’s possession.

Until 1890, Frau Riemann lived in Gottingen, where in 1884 the couple’s only daughter, Ida (1863–1929), married Carl David Schilling (1857–1932), the future director of the Seefahrtschule in Bremen. Schilling had taken his doctor’s degree in 1880 under H. A. Schwarz (1843–1921), who at that time lived next door to Frau Riemann and was on good terms with her [3]. Schilling remained in touch with Schwarz and other Göttingen mathematicians such as Klein even in his later years. Between 1900 and 1909 he published the correspondence between Gauss and Olbers [Schilling 1894–1909, Vol. 2]. Around 1890, Riemann’s wife and his only surviving sister, Ida, moved away from Göttingen to live with Dr. Schilling in Bremen, and we may therefore assume that the entire Riemann Nachlass also found its way to Bremen, at least as regards those items which did not remain in Weber’s or Dedekind’s hands. It seems unlikely that any items were lost or sold up to this point, for on July 22, 1892, Schilling wrote to Weber in connection with the planned donation of the mathematical papers to a public institution (NSUB Göttingen, Cod. Ms. Riemann 1.2, fol. 48–49; translated from German): “At first mother could not come to terms with the idea that Riemann’s papers should no longer remain in private hands; to her, they are something sacred, and she doesn’t like to think of them being made accessible to any student, who would then also be able to read the marginal notes, some of which are purely personal.”

While Riemann’s mathematical papers were passed on by Weber to the Göttingen Academy of Sciences in the spring of 1895, it appears that some of the more personal documents must have remained in the family’s possession. By far the largest known collection of such manuscripts is currently in the Library of the Prussian Cultural Foundation (Staatsbibliothek Preussischer Kulturbesitz) in Berlin, and comprises some 200 letters which had passed between Riemann and various members of his family, as well as a few letters from friends of his. These letters were left by the Bonn mathematician and historian of mathematics Erich Bessel-Hagen (1898–1946) to his brother, Dr. Hermann B. Hagen, who donated them to the Berlin Library in 1966. Unfortunately, a detailed study of Erich Bessel-Hagen’s papers has failed to shed any light on the precise circumstances under which these letters came to be in his possession. Inquiries among Bessel-Hagen’s relatives and friends, however, suggest that he only acquired them during the Second World War. At any rate, Otto Neugebauer, who in 1926 attended Bessel-Hagen’s lectures on Riemann in Göttingen on the occasion of the hun-
dredth anniversary of Riemann’s birth, has no knowledge of any such letters having turned up in Göttingen, and Bessel-Hagen’s sister, now aged 95, thinks that he bought the letters at an auction or from a dealer, most probably during the war. A comparison of Riemann’s family correspondence now in the Berlin Library with those letters referred to by Dedekind in his biography of Riemann [Riemann 1876, 539–558] shows that the Berlin collection is far from complete: of six letters which Dedekind mentions by date, only one original and two excerpts made by Riemann’s sister are now to be found in Berlin. Extensive research has since brought to light some further items of Riemann’s family correspondence, as well as a number of other documents (school reports, university leaving certificates and attendance records, a diary, correspondence from the university curators, etc.), which are distributed among Carl David Schilling’s numerous descendants. Nevertheless, a great part of the family letters and almost all of Riemann’s scientific correspondence has not yet been traced.

RIEMANN’S SCIENTIFIC CORRESPONDENCE

At the present time, the clearest picture of Riemann’s scientific correspondence can be formed from drafts scattered among his mathematical papers. Some difficulty arises, however, from the fact that Riemann frequently drafted and redrafted his letters on quite different pieces of paper, which are now interspersed among the 4000 or so sheets comprising the Göttingen collection. Outside Göttingen and Berlin, originals of Riemann’s letters have so far been located only in London (addressed to Kronecker), Paris (to Élie de Beaumont), Pisa (to Betti), and Zürich (to the ETH). A systematic search through the Göttingen collection, together with the few originals of Riemann’s letters discovered to date, yields documentary evidence of some 70 letters written by Riemann. An important complement to this list was provided by our recent discovery of a list of letters written to Riemann, possibly compiled by Riemann himself, which is now in the possession of his descendants. The latter list mentions a total of 39 letters to Riemann written by 19 different correspondents between June 1854 and February 1866. If we collate the two lists, the following persons emerge as Riemann’s academic correspondents [4]:


According to both lists, Riemann’s most important correspondent by far was Borchardt, with whom Riemann exchanged letters about the publication of his articles in Crelle’s journal, followed by Betti, Kohlrausch, Kronecker, and Wilhelm Weber.
NOTES BASED ON RIEMANN’S LECTURES

The decisive impetus to collect notes based on Riemann’s lectures came from Felix Klein (1849–1925). In the summer of 1890, Klein placed a written proposal before the Göttingen Academy of Sciences that surviving sets of notes on Riemann’s lecture courses be gathered together, and that the Riemann collection in general be systematically reexamined, with a view to the subsequent publication of the most important items. Unfortunately, Klein’s proposal met with opposition at the meeting held on July 5, 1890, and so he withdrew it; nevertheless, it appears likely that Klein pursued his efforts to collect Riemann-related items over the next several years [5]. Thus, as emerges from the correspondence between H. Weber and Schilling (NSUB Göttingen, Cod. Ms. Riemann 1.2), it is partly due to Klein that Riemann’s mathematical papers could be handed over to the Göttingen University Library in 1895. In 1896, Klein contacted W. von Bezold (1837–1907), who agreed to give his lecture notes to the Göttingen University Library. At the same time, the Academy purchased a number of Riemann-related items from the estate of K. Hattendorff (1834–1882). In the meetings held on May 29 and July 31, 1897, Klein reported on these acquisitions to the Academy and published his report in the Academy journal [Klein 1897]. A short time afterward, further Riemann-related items were acquired from the Schering estate; Klein mentions this in a footnote at the end of a report on the current state of progress on the publication of the works of Gauss, and expresses his desire to see sets of lecture notes from all of Riemann’s courses brought together in the Göttingen library [Klein 1898, 18]. In the end, Klein’s efforts and the appeal he made to his contemporaries effected the results he had hoped for: between 1896 and 1902, M. Noether (1844–1921) and W. Wirtinger (1865–1945) combed through the newly collected material and the Riemann collection, and in 1902 published the “Nachträge” (Supplement) [Riemann 1902] to Riemann’s Collected Works. Thanks to the efforts of Klein, Noether, and Wirtinger, the Göttingen University Library collection has subsequently expanded and now includes a total of no fewer than 20 different sets of lecture notes from Riemann’s courses, taking into account the material in the Dedekind Nachlass, which was later deposited there.

As the sets of lecture notes kept in Göttingen are relatively well known and have in many cases already been described or partly published elsewhere (see [Goedecker 1872, 1879; Hattendorff 1869, 1876; Klein 1897, 1898; Neuenschwander 1979, 1980, 1981b, 1981c, 1987; Noether 1900, 1909; Riemann 1876, 1902; Stahl 1896, 1899; Wirtinger 1905]), we shall confine ourselves here to describing those sets of notes to be found outside the Göttingen University Library collection which have come to light as a result of our systematic research conducted between 1978 and 1986. Most of these sets of lecture notes were not previously known to exist and provide a significant addition to our understanding of Riemann’s ideas.

We shall first present a list of the titles, locations, and other bibliographical data concerning these manuscripts and then go on to describe each of them in greater detail.


Manuscripts Nos. 1–5 are to be found in the Schwarz Nachlass kept by the GDR Academy of Sciences (Akademie der Wissenschaften der DDR, abbreviated hereafter to AdW der DDR) in Berlin, which was sorted and cataloged by the archive staff in 1986. This collection also includes a list, compiled by Schwarz, of Riemann’s papers (AdW der DDR, Nachl. Schwarz, Nr. 682), and an unspecified excerpt from notes on Riemann’s lectures, entitled “Untersuchung der Abel’schen Funktionen für den Fall $p = 3$” (AdW der DDR, Nachl. Schwarz, Nr. 680). The lecture notes in the Schwarz Nachlass were, for the most part, copied later by third persons, substantial sections probably being in Schwarz’s own hand, and in several cases there are also explanatory marginal notes made by Schwarz himself. In October 1986, when we first examined these lecture notes, the records of borrowers revealed manuscripts Nos. 3 and 4 to be the only ones previously consulted, a situation which had not changed when we returned in April 1987 to reexamine them. Current restrictions on copying some of the manuscripts kept in the GDR have unfortunately made it difficult to carry out exhaustive comparisons with the sets of notes located elsewhere, so that we are presently unable to make a final evaluation of some of these lecture notes.

Manuscript No. 1 was written by the future Berlin astronomer F. Tietjen (1834–1895), who studied in Göttingen during the summer semester of 1858 and the winter semester of 1858/59. It is to be found alongside a number of other manuscripts dating from Tietjen’s student days (notes on a lecture course given by Stern and copies of papers by Abel, Gauss, and Malmsten) in a bound volume bearing the title “Mathematik. Wintersemester 1858/59” on the spine. Tietjen’s notes are of particular interest, as the only previously known record of the Riemann lecture course in question was a set of relatively brief shorthand notes by Bezold (NSUB Göttingen, Cod. Ms. Riemann 29) entitled “Höhere Mechanik” (Higher Mechanics), and as compared with the lecture courses published by Hattendorff [Hattendorff 1869, 1876], Riemann changed both his choice of material and the order of presentation that semester. According to the record left by Tietjen, for the semester in question Riemann dealt not only with “Höhere Mechanik,” as announced in the lecture-list, but also with the theory of electricity, mechanical theory of heat, and the theory of elasticity. It is possible that Riemann decided upon these changes as a result of Dirichlet’s falling ill, and it appears that he also delivered Dirichlet’s lectures on partial differential equations in his indisposed colleague’s place [6].

Manuscript No. 2 is based upon the same lecture course by Riemann as the book [Hattendorff 1876], and covers broadly the same ground. It is a copy of a set of notes by Eduard Schultzze, who studied in Göttingen during the winter semester of 1860/61 and the summer semester of 1861, when he also attended Riemann’s lectures on function theory (cf. MS No. 3 of our list, as well as Schultzze’s leaving certificate from the University of Göttingen). From 1866 to 1867, Schultzze taught at the Friedrich-Wilhelm-Gymnasium in Berlin, as Schwarz notes on the title page of MS No. 2. Further copies of Schultzze’s notes are extant in Göttingen (our Manuscript No. 8) and Berlin (our Manuscript No. 9) and are, as far as we were
able to determine, verbatim copies of No. 2. Indeed, the correspondence between Nos. 8 and 2 is so close that identical passages of the text are consistently found on the same pages, with a discrepancy of a few lines at most. In addition, MS No. 8 includes all the marginal notes contained in No. 2, some of them being Schwarz's own, while at the same time containing additional marginal notes on the concordance with the edition by Hattendorff [Hattendorff 1876]. It is reasonable to suppose, therefore, that No. 8 is a later copy of No. 2. Manuscript No. 9, on the other hand, is marked "Aus dem Nachlaß des GR. Bertram" (from privy councillor Bertram’s estate), and bears the stamp of the library of the Mathematical Society of the University of Berlin.

Manuscript No. 3 appears, on the basis of the sections we compared, to accord almost completely with MS No. 10, which can therefore most likely also be ascribed to the above-mentioned Schultze. In marginal notes and on separate sheets pasted into MS No. 3, Schwarz has noted deviations both from Prym’s lithographed notes (our No. 11) and from a copy of Hattendorff’s notes at that time in Weber’s possession. From page 116 on, the text of MS No. 3 appears to have been written out by another person. The University Library in Göttingen possesses several other sets of notes on Riemann’s summer semester 1861 lectures on function theory, compiled by Abbe (NSUB, Cod. Ms. Riemann 32c), Hankel (NSUB, Cod. Ms. Riemann 32b), and Hattendorff (NSUB, Cod. Ms. Riemann 31 and 32); their interdependence was studied in [Neuenschwander 1981b, 240]. For a provisional edition of the first part of Hankel’s lecture notes, we refer the reader to [Neuenschwander 1979], and for a more authoritative edition of Riemann’s introductory lectures on the general foundations of function theory, based upon all available sets of lecture notes together with Riemann’s own preparation notes, to [Neuenschwander 1987].

Manuscript No. 4 is in the same handwriting as the first part of No. 3, and is most likely an excerpt copied by Schwarz out of a set of lecture notes by Gustav Roch (1839–1866). A penciled note in the same handwriting at the top of folio 2 reads: “G. Roch’s Ausarbeitung nach Riemann’s Vorlesung” (G. Roch’s notes based on Riemann’s lectures). It is possible that the unspecified excerpt mentioned above (AdW der DDR, Nachl. Schwarz, Nr. 680) is also based on Roch’s lecture notes, from which the two fragments XXX and XXXI, published by Weber in Riemann’s Collected Works [Riemann 1876, 483–504], are taken; these fragments correspond closely to the two excerpts mentioned here. Regarding Roch’s original notes, and a copy of certain parts of them for F. Casorati, see [Neuenschwander 1978, 40, 63].

Manuscript No. 5 is a revised transcription of Bezold’s shorthand notes (Cod. Ms. Riemann 29), which are now in the Göttingen University Library. It was compiled at Schwarz’s suggestion by H. Franzen between August 1892 and June 1894, when Bezold’s original notes were still in Berlin. According to the foreword provided by Franzen, pages 1–160 are a revised version based on Bezold’s shorthand manuscript, and pages 161–248 a verbatim transcription of it. Franzen states that it proved necessary to revise the text due to numerous shortcomings in
Bezold’s shorthand script, but he reproduced the original text of the passages he altered in a separate booklet, entitled ‘‘Nachträgliche Bemerkungen’’ (Supplement), which he included with his revised version.

Manuscript No. 6 is, as far as we were able to ascertain by comparing individual pages, for the most part a verbatim typewritten copy of No. 5. It bears the comment: ‘‘15.12.26. Geschenkt von Prof. Bieberbach’’ (15.12.26. Presented by Prof. Bieberbach), and the stamp of the Mathematical Seminar of the University of Berlin. According to Mr. Hadan, long-standing chief librarian of the Zweigstelle Mathematik of the Library of the Humboldt-University, Berlin (East), this manuscript has not been consulted by any borrowers from outside the University for more than 15 years.

Bezold’s original shorthand notes (NSUB Göttingen, Cod. Ms. Riemann 29) already aroused great interest in the 19th century, and were consulted and transcribed by both H. A. Schwarz and L. Fuchs [7]. As can be gathered from letters written by Max Noether and Frau Riemann, this soon led to a dispute over publication rights for the original notes, and as a result Bezold was happy to hand over both the manuscript itself and the publication rights to Göttingen in 1897. The notes were then studied by Wirtinger, who published excerpts from them in the Supplement to Riemann’s Collected Works [Riemann 1902, 69–94] and subsequently devoted a lecture to them [Wirtinger 1905].

Manuscript No. 7 is a set of notes compiled by the Norwegian mathematician C. A. Bjerknes (1825–1903), who studied in Göttingen and Paris from 1855 to 1857 and attended Riemann’s lectures on function theory in the winter semester of 1855/56 and in the summer semester of 1856 [8]. Bjerknes’ notes are written in German, with marginal annotations in Norwegian summarizing the main points; they present a welcome addition to the notes on the same lectures compiled by Schering (NSUB Göttingen, Cod. Ms. Riemann 37), and to Riemann’s own treatise on the theory of Abelian functions [Riemann 1876, 88–144], which was based on these courses. Apart from Riemann’s lectures, Bjerknes also attended lectures given by Dirichlet, Stern, and Dedekind during his time in Göttingen. Bjerknes’ Nachlass also includes notes on some of these lectures, as we were able to ascertain from photocopies kindly provided by the Royal University Library in Oslo.

Manuscript No. 10 bears the names B. Kerékjártó (1898–1946) and D. E. Smith (1860–1944) on a flyleaf. On the same sheet, the latter has written: ‘‘Student’s copy of Riemann’s lectures at Göttingen, 1861. David Eugene Smith—The student was Bela von Kerékjártó. His son (?) was at one time Privatdozent at Szeged, then Ujpest, and later at Göttingen. He wrote numerous monographs.’’ In fact, this information given by Smith regarding the authorship of the notes seems extremely questionable to us, since there is no student by the name of Kerékjártó to be found in the Göttingen matriculation register [Ebel 1974]. Moreover, a comparison with the other sets of notes on Riemann’s lectures on function theory reveals that those passages of MS No. 10 we have examined are, for the most part, identical with the corresponding passages of MS No. 3. We therefore regard
it as more likely that the name of KerékJártó on the flyleaf is to be interpreted as an indication of ownership, and that the notes themselves were compiled by the above-mentioned Schultze. In fact, an examination of the handwriting and a consideration of the numerous crossings-out in MS No. 10, sometimes even involving the deletion of whole sections, make it probable that No. 10 might actually be Schultze’s original notes on the lecture course.

Manuscript No. 11 comprises a set of lithographed notes compiled by F. Prym (1841–1915), a small number of copies of which he distributed among his personal acquaintances [Riemann 1902, 116; Stahl 1896, foreword]. It contains two separately numbered parts, the first roughly covering the lectures Riemann gave in the summer semester of 1861, from which the sets of notes mentioned in our commentary on MS No. 3 also derive; the second part is devoted to the continuation of the lectures in the winter semester of 1861/62. The first part of Prym’s lithographed notes bears on page 69 the comment “finis: 5. Mai 1862. Nachm. 5 Uhr.” (completed: May 5, 1862, 5 P.M.), and ends with the note: “finis 18./5. 62.” In compiling this part, Prym relied on the lecture notes taken down by Hattendorff (NSUB Göttingen, Cod. Ms. Riemann 31 and 32), since he was still studying in Heidelberg in the summer semester of 1861 (cf. Prym’s statements contained in letters dating from the year 1898 in NSUB, Cod. Ms. Klein 11, Nos. 388 and 1097). The second part of Prym’s lithographed compilation is considerably less extensive than the notes he took himself during Riemann’s lectures on function theory in the winter semester of 1861/62, or those taken by Minnigerode (NSUB, Cod. Ms. Riemann 38), which Noether and Wirtinger drew upon for their edition of extracts from this lecture course [Riemann 1902, 1–66].

Finally, we should like to refer to two little-known excerpts from Riemann’s lectures, which were published by E. Goedecker [1872, 1879] and which are not mentioned in Riemann’s Collected Works [Riemann 1876, 1902]. These are concerned with problems that Riemann dealt with, according to Goedecker, toward the end of his series of lectures in the winter semester of 1860/61, namely on the distribution of heat within a sphere and of the movement of a circular ring in an infinite and incompressible fluid.

Furthermore, it would appear that the editors of the Supplement to Riemann’s Collected Works [Riemann 1902] had no access to the lecture notes taken by Dedekind, now available in the Göttingen University Library (NSUB, Cod. Ms. Dedekind I, 13–16). These refer to Riemann’s lectures on partial differential equations (WS 1854/55), definite integrals (SS 1855), hypergeometric series (WS 1856/57), and elliptic and Abelian functions (WS 1857/58). At the same time, we take the opportunity in note [9] of citing a number of what we believe to be hitherto unknown sets of notes on lectures given by Riemann’s great rival, Karl Weierstrass (1815–1897), which were discovered in the course of our present research.

NOTES

1. Akademie der Wissenschaften in Göttingen, which bore the name Gesellschaft der Wissenschaften zu Göttingen until October 1940.

3. From approx. 1877–1892, Schwarz lived at Weender Chaussee 17A (the street name was changed during the course of these years to Bertheaustr., and the house number subsequently from 17A to 3), while Frau Riemann lived with her daughter and Riemann’s only surviving sister, Ida, at Weender Chaussee 17 (see the official Göttingen directory of addresses, published every 2–3 years). On September 5, 1890, Frau Riemann moved to Bremen, apparently together with Riemann’s sister, Ida (see the official residents’ register of the city of Göttingen, and the Göttingen and Bremen directories of addresses). In view of the fact that Schilling lived in Professor Schwarz’s house during 1879 and 1880, it is quite possible that he already got to know Riemann’s daughter during his student days. Regarding Schwarz’s cordial relationship with both Frau Riemann and Schilling, see [Neuenschwander 1981b, note 48]. For further information concerning Schilling, see [Mücke 1969], as well as the numerous letters written by Schilling to be found in Klein’s and Schwarz’s papers.

4. The Christian names of the correspondents are missing in the second list. In those cases where these cannot be unmistakably ascertained on the basis of the letters preserved or other documents, they have been indicated in square brackets in the following list.

5. Regarding Klein’s proposal and his subsequent endeavors in this matter, see Archiv d. Akad. d. Wiss. in Göttingen, Scient. 116 and Chron. 4. 6, as well as Klein’s own records in NSUB Göttingen, Cod. Ms. Klein 22 L. 3. fol. 9. Further information concerning the acquisition and editing of these Riemann-related items may be found in the articles mentioned in the bibliography, as well as in the “Chronik der Georg-Augusts-Universität,” the “Nachrichten von der Königl. Gesellschaft der Wissenschaften zu Göttingen. Geschäftliche Mittheilungen,” and Klein’s own correspondence.

6. There is various evidence to support this conclusion, including a set of lecture notes by Nägelsbach in Göttingen entitled “Partielle Differentialgleichungen nach Lejeune Dirichlet vorgetragen von Professor Riemann in Göttingen. H. Nägelsbach, Wintersemester 1858/59” (Partial Differential Equations as developed by Lejeune Dirichlet, delivered by Professor Riemann in Göttingen. H. Nägelsbach, winter semester 1858/59) (NSUB, Cod. Ms. Riemann 41). Furthermore, a comparison of leaving certificates reveals extraordinarily wide variations in the wording of the titles of the lecture courses for the semester in question, which reflect the above-mentioned changes. From the sets of notes still in existence, we can see that, contrary to the assumption made by Noether [1909, 25], Riemann gave two different lecture courses on physics that semester. According to a handwritten notice by Riemann in the Nachlass (NSUB, Cod. Ms. Riemann 2, fol. 41'), the exact titles of the three lecture courses Riemann finally offered that semester were as follows:


2) Theoretische Physik, insbesondere Theorie der Elasticität und mechanische Wärmetheorie an denselben Wochentagen 9-10 Uhr. Anfang d. 1. Nov.

3) Über Functionen einer veränderlichen complexen Größe, insbesondere über hypergeometrische Reihen und verwandte Transzendenten, an denselben Tagen 4-5 Uhr. Anfang d. 26. October.

7. Regarding the transcription produced for Schwarz, see our commentary on MS No. 5; regarding that produced for Fuchs, see [Wirtinger 1905, 124]. For the dispute between Fuchs and Schwarz, and for an account of the donation of the original manuscript to Göttingen, see NSUB Göttingen, Cod. Ms. Klein 11, Nos. 127 and 528, and Klein 8, Nos. 87–90, respectively. Wirtinger’s claim [1905, 124] that Fuchs had the manuscript transcribed as early as 1894 would appear to conflict with statements made by Bezold in letters to Klein that Fuchs had the manuscript at his disposal from approximately May 1896 to July 1897, in order to have it transcribed before it was passed on to Göttingen.

8. We are indebted to Jan-Erik Roos for bringing the existence of these lecture notes to our attention. In July 1987 the Library staff discovered a second manuscript bearing the title “9Funktioner
Riemann” on the spine, which seems to be based on the continuation of Riemann’s lectures in the summer semester 1856.

9. More than 10 such sets of lecture notes, as well as seminar papers and notes on lectures delivered by other Berlin mathematicians are to be found in the Nachlass of the mathematician V. Dantscher (1847–1921) in the Graz University Library; according to information received from the library, no publications have appeared to date regarding this material. Dantscher was a professor at the University of Graz, and in 1908 published a book on Weierstrass’ theory of irrational numbers, entitled “Vorlesungen über die Weierstrasssche Theorie der irrationalen Zahlen.” Further substantial collections of notes on Weierstrass’ lectures (comprising 5 or more sets of lecture notes in each case) which are not listed at all (or only in part) in the relevant literature are to be found in the library of the Mathematical Institute of the University of Göttingen; in the Aussenstelle Sektion Mathematik of the library of the Karl-Marx-University, Leipzig; in the various departments of the Humboldt-University, Berlin (East); in the Deutsche Staatsbibliothek, Berlin (East); in the library of the Mathematical Institute of the University of Giessen; in the library of the Sektion Mathematik of the University of Halle-Wittenberg; and in the manuscript department of the ETH Library, Zürich. Smaller collections or individual sets of notes which, as far as we were able to ascertain, had hitherto remained undiscovered, are to be found in the library of the Fachbereich Mathematik of the Technische Universität in Berlin (West); in the Mathematical Institute of the University and in the University Archives, Bonn; in the University of Chicago Library; in the Hessische Landes- und Hochschulbibliothek, Darmstadt; in the library of the Mathematical Institute of the University of Erlangen-Nuremberg; in the University Library of the Technische Universität, Hanover; in the Mathematical Institute of the University and in the University Library, Vienna; in the University Library, Wroclaw (PL); as well as in the possession of the author, who will be pleased to provide more detailed information regarding the individual manuscripts. For information concerning those notes on Weierstrass’ lectures already known, see: P. Dugac, Eléments d’analyse de Karl Weierstrass. Archiv für Geschichte der Wissenschaften 10 (1973), 41–176; [Neuenschwander 1981b]; G. Richenhagen, Carl Runge (1856–1927): Von der reinen Mathematik zur Numerik, Göttingen: Vandenhoeck & Ruprecht 1985; C. Weierstrass, Einführung in die Theorie der analytischen Funktionen nach einer Vorlesungsnachricht von Wilhelm Killing aus dem Jahr 1868. Herausgegeben von W. Scharlau. Schriftenreihe des Mathematischen Instituts der Universität Münster, 2. Serie, Heft 38, 1986; etc.

REFERENCES


