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Monte Carlo simulation of absorbing phase transition in the models with a conserved field on diluted lattices

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Abstract

An influence of quenched disorder on absorbing phase transitions of the conserved lattice gas (CLG) model and the conserved threshold transfer process (CTTP) was investigated via Monte Carlo simulations. It was found that when the concentration of disordered site is less than the critical concentration, the critical exponents were similar to those of the pure models for both the CLG and the CTTP models. When the concentration becomes critical, the density of active particles showed nonuniversal power-law behavior for all particle densities for the CLG model, whereas the CTTP model exhibited usual critical behavior but with different critical exponents. The nonuniversal power law was attributed to the dead ends on an infinite percolation network. Eliminating those dead ends, it was found that both the CLG model and the CTTP model exhibited usual critical behavior; the estimated exponents were similar for the two models, and they were also similar to those of the CTTP model on an infinite network.

Keywords: absorbing phase transition, quenched disorder, conserved lattice gas model, conserved threshold transfer process, nonuniversal power-law behavior
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1. Introduction

Nonequilibrium absorbing phase transitions (APTs) have been of great interest during last more than two decades [1, 2]. Various models with various evolution rules were classified into several well-defined universality classes. The most prominent and robust one is the directed percolation (DP) class and a wide variety of models with different evolution rules satisfying the DP hypothesis [3, 4] were known to belong to this class. A typical example of models which belong to the DP class is the contact process (CP) with a diffusion-reaction scheme A $\rightarrow$ 2A and A $\rightarrow$ 0 [5, 6], in which each particle either creates an offspring with a rate λ or annihilates with a rate μ. Another known universality class is the parity-conserving (PC) class [7, 8], observed with additional symmetries, which is most likely represented by the branching-annihilating random walks with even number of offsprings, in which each particle either jumps to a nearest-neighbor site (AO $\rightarrow$ OA, OA $\rightarrow$ AO) with a probability p or produces n offsprings (A $\rightarrow$ $(n+1)$A) on n nearest-neighbor sites with a probability $1-p$. When a particle jumps to already occupied site and when an offspring is created on such a site, both the incoming particle and the occupying particle annihilate each other (AA $\rightarrow$ 0), leaving the site empty. When n is even, the number of particles is conserved modulo 2 and this conservation law is known to lead to a new universality class. Recently, a new universality class was proposed for various models with

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an additional conservation, i.e., with a conserved field [9]. The conserved lattice gas (CLG) model, the conserved threshold transfer process (CTTP), and the stochastic sandpile models were known to belong to this class [10, 11, 12].

Recently, influence of quenched disorder on the critical behavior of the CP model, introduced by applying the Harris criterion [13] established for equilibrium spin systems to nonequilibrium APTs, attracted great attention [14, 15, 16, 17, 18]. The Harris criterion suggests that the pure fixed-point is unstable if the specific-heat exponent α for a pure system is positive [13]. This implies that the amount of disorder is a relevant parameter, i.e., a small amount of disorder (diluted sites) added to the system significantly changes the critical behavior. Adding quenched disorder is, in principle, equivalent to dilution of lattice sites and, therefore, the disordered system is the system on a diluted lattice or on a percolation network. For the CP model in the DP universality class, the specific-heat exponent obtained by the hyperscaling relation $\alpha = 2 - d\nu$ [19], ν being the correlation-length exponent in d dimensions, is known to be positive for $d < 4$ if ν is replaced by the spatial correlation-length exponent $\nu_\perp$. Indeed the critical behavior of the diluted CP model was found to be different from that of the pure CP model.

Motivated by those works, the present author studied influence of the disordered CLG model, and the results presented here were partly published. For the CLG model, the specific-heat exponent calculated in a similar way to the CP model was $\alpha \simeq 0$ in one, two, and four dimensions and was small but positive in three dimensions [20]. The critical behavior of the diluted CLG model in two dimensions is, therefore, expected to be different from that in three dimensions, if the Harris criterion is applicable to the nonequilibrium APTs. It is thus interesting to study the diluted CLG model in two and three dimensions.

In this paper, the critical behavior of the diluted CLG model was investigated. It was found that the critical behavior for the disorder concentration x less than the critical concentration x_c ($x_c = 1 - p_c$, p_c being the percolation threshold) was similar to that of the pure CLG model in both two and three dimensions. However, at the critical concentration, the nonuniversal power-law behavior was observed. In order to investigate the source of such behaviors, the diluted CTTP model was also studied on a percolation network.

2. Model description

In the CLG model, ρN particles are initially distributed randomly on a lattice of N undiluted sites on an infinite percolation network, and each site may be occupied by at most one particle. Particles with one or more particles on the nearest-neighbor sites are considered to be active, and isolated particles are inactive. The dynamics proceed with hopping of active particles; each active particle attempts to hop to nearest-neighbor empty sites by a repulsive interaction with an increment of the evolution time $1/N_a(t)$, $N_a(t)$ being the number of active sites (particles) at the time of hopping t. In the CTTP model, on the other hand, each undiluted site may be occupied by up to two particles, and doubly occupied sites are considered to be active sites and empty and singly occupied sites are inactive sites. Particles in each active site attempts to hop to nearest-neighbor inactive sites. In both models, there is no particle creation or annihilation, no external fields, and no self-diffusion; therefore, the number of particles is preserved during the dynamic process.

If the particle density ρ is small, the density of active sites n_a will decrease rapidly, and the system goes into an absorbing state. If, on the other hand, ρ is sufficiently large, n_a saturates to a steady-state value of $n_{\rm ss}$, which is considered to be an order parameter. Thus, at the critical density ρ_c, n_a exhibits a power-law decay,

$$n_a(t) \sim t^{-\theta}, \tag{1}$$

θ being the universal critical exponent. Above ρ_c, the order parameter decreases as ρ approaches ρ_c following a power-law behavior

$$n_{\rm ss}(\rho) \sim (\rho - \rho_c)^\beta \tag{2}$$

and becomes 0 at the critical density. The system, therefore, exhibits a critical behavior.

It has been known that the CLG and the CTTP models belong to the same universality class on a regular lattice of the dimensionality $d \geq 2$. However, in one dimension, the two models were known to yield the critical behavior different from each other and the universality splits, being attributed by the different hopping mechanisms [21]. In the CLG model, since each active particle has an occupied site on one side and an empty site on the other, the direction of hopping is selected deterministically, whereas in the CTTP model hopping is mainly stochastic. The different

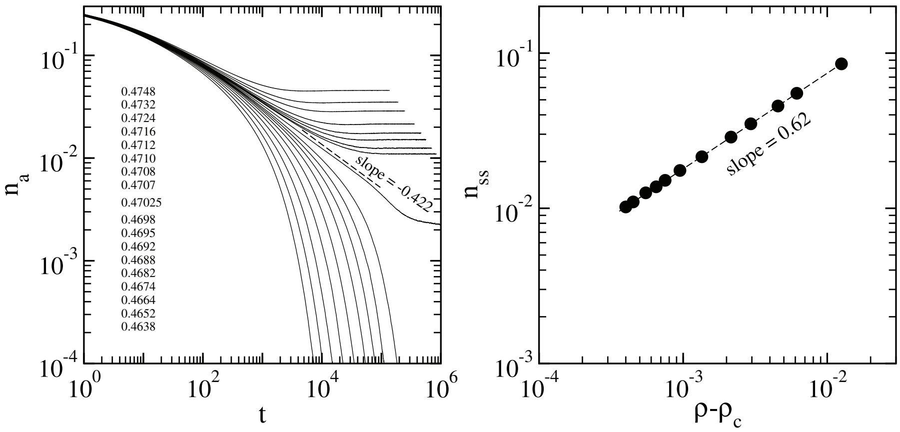

Figure 1: (Left) The densities of active particles for the diluted CLG model with $x = 0.3$ on a square lattice. The numbers in the legend are the particle densities in the same order as the data from top to bottom, and the dashed line is the regression fit over the data for $\rho_c = 0.47025$. (Right) The steady-state densities of active particles plotted as a function of the distance from criticality.

behaviors were also observed on a checkerboard fractal lattice in which hopping of the CLG model is dominantly deterministic while it is stochastic for the CTTP model.

Hopping on a percolation network may also be different for the two models, particularly on the sites at dead ends. In the CLG model, once particles hop to dead ends, they cannot escape forever, while in the CTTP model there are chances to escape. Such trapping events are known to yield the active-particle density slowly decreasing, with a nonuniversal power-law behavior in the CLG model. One can then raise a question regarding whether or not such a nonuniversal power-law behavior can still be observed for the CTTP model. In this study, the answer to this question will be addressed via the Monte Carlo simulations.

3. Results and discussions

Simulations for the CLG and CTTP models were carry out on a percolation network generated on a square lattice of L^d sites, d being the lattice dimensionality, where in most cases $L = 2000$ were used, except the case when the finite size effect is apparent, for which the systems of $L = 4000$ were also simulated.

3.1. Diluted conserved lattice gas model

In the first, simulations for the diluted CLG model were carried out for the disorder concentrations $x = 0.3$ and 0.35. Plotted in Fig. 1 are $n_a(t)$ (left) and $n_{ss}(\rho)$ (right). It is clear that the data for $\rho < \rho_c$ decay rapidly and fall into absorbing states implying that the Griffith phase does not exist in the CLG model. The Griffith phase was observed in the CP model in the region $\lambda_c(0) < \lambda < \lambda_c(x)$, $\lambda_c(0)$ being the critical spreading rate of the pure CP model, in which n_a decreases following the nonuniversal power law [14]. The estimated critical exponents were $\theta = 0.42$ and $\beta \simeq 0.62$, which are consistent within 3% error with the values of the pure CLG model. This confirms that the disorder added to the system is irrelevant as long as $x < x_c$. Simulations for $x = 0.35$ were also carried out and the results were found to be qualitatively similar to those for $x = 0.35$ but with narrower power-law regions.

For $x = x_c \simeq 0.407$, the substrate is fractal with the fractal dimension $d_f = \frac{91}{48}$. The main concern in this case is whether the critical behavior follows the activated dynamic scaling as observed for the diluted CP model [17]. Plotted in Fig. 2 are $n_a(t)$ for various particle densities. Comparing the data in the figure with those for $x = 0.3$, it is clear that n_a for $\rho < \rho_c$ decreases more slowly than those for $x = 0.3$. In fact, data for $n_a(t)$ were found to yield power-law decay in the long-time region, $n_a \sim t^{-\phi}$, the power varying according to the particle density, indicating that n_a exhibits nonuniversal power-law behavior. For $\rho = 0.521$ data in the early-time region appear to yield a power-law decay with a power of 0.211. For $\rho = 0.5215$, ρ_a still yields a power law with a power of 0.197; however, data in the long-time region exhibits a precursor of saturation, indicating that the value $\rho = 0.5215$ is in the supercritical region. Considering these two cases, it seems that the critical density lies between the two values and the power is $\theta \simeq 0.20$. The active particle density, however, did not follow the activated dynamic scaling law, $n_a \sim (\ln t)^{-\delta}$.

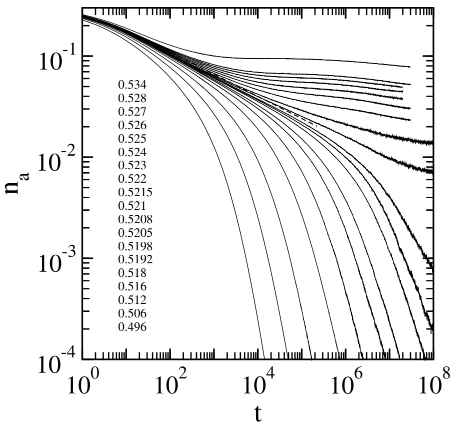

Figure 2: The densities of active particles for the diluted CLG model with $x_c = 0.407$ on a square lattice. The numbers in the legend are the particle densities in the same order as the data from top to bottom, and the dashed line is the regression fit over the data for $\rho = 0.5215$.

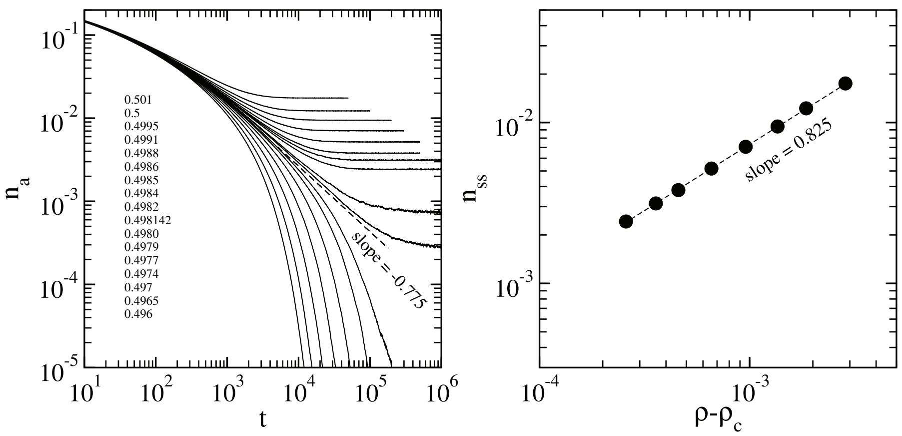

Figure 3: (Left) The densities of active particles for the diluted CLG model with $x = 0.6$ on a simple cubic lattice. The numbers in the legend are the particle densities in the same order as the data from top to bottom, and the regression fit is over the data for $\rho_c = 0.498142$. (Right) The steady-state densities of active particles plotted as a function of the distance from criticality.

For $\rho > \rho_c$, data of n_a deviated and veered up from the power law, but they did not saturate; they veered down again and eventually yielded slow power-law behavior in the late time. This made impossible to calculate the order parameters. The source of nonuniversal behavior will be discussed later.

Simulations in three dimensions were also carried out in order to observe the dimensional differences, if any. For $x = 0.6$, i.e., for $x < x_c$, $n_a(t)$ and $n_{ss}(\rho)$ are plotted in Fig. 3. Data for $\rho < \rho_c$ decay rapidly with downward curvature and those for $\rho > \rho_c$ yield steady-state values, indicating the universal critical behavior. The critical exponents estimated for $\rho_c = 0.498142$ are $\theta = 0.775(3)$ and $\beta = 0.825(3)$, which are consistent, within 3% error, with the values for the pure CLG model in three dimensions, $\theta = 0.755$ and $\beta = 0.848$ [22]. This confirms that the disorder is irrelevant for $x < x_c$, and the results in three dimensions are qualitatively similar to those in two dimensions.

For $x = x_c = 0.688$, the results were qualitatively similar to those in two dimensions, and dimensional difference predicted by Harris criterion assuming that it is applicable to nonequilibrium systems was not observed. The nonuniversal power-law behavior was also observed in the subcritical region, and the data in the supercritical region also exhibited a slow power-law decrease in the long-time region.

In order to investigate the cause of the nonuniversal power-law behavior, the morphologies of the active particles and inactive particles were carefully examined. Initially active particles were distributed uniformly over the network and, as time progressed, most inactive particles were located on the dead ends. Once particles hop to dead ends, they will be trapped and cannot escape forever because those particles cannot hop even though they have particles on the nearest-neighbor sites due to the restriction of the CLG model that two particles cannot occupy the same

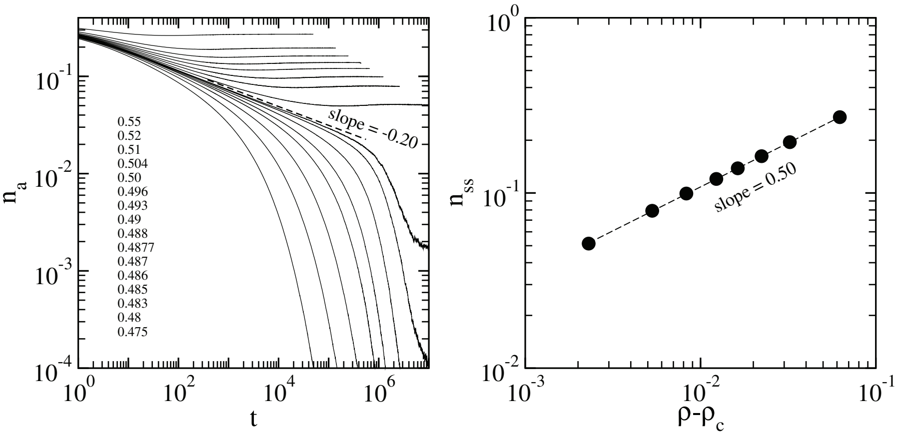

Figure 4: (Left) The densities of active particles for the CLG model on a percolation backbone generated with respect to $x_c = 0.4073$, on a square lattice. The numbers in the legend are the particle densities in the same order as the data from top to bottom, and the data with a regression fit is for $\rho_c = 0.4877$. (Right) The steady-state densities of active particles plotted as a function of the distance from criticality.

site. Therefore, all active particles were located on the blobs and the connected links, and they migrated via singly connected paths to dead ends to become inactive, and such movement appeared to be slow and yielded a power-law decay. Thus, the observed nonuniversal power-law decay of active particles was attributed to the dead ends of an infinite network.

This observation may be confirmed by carrying out simulations for the CLG model on a backbone network, obtained by eliminating all dead ends and dangling blobs. Fig. 4 shows $n_a(t)$ and $n_{ss}(\rho)$ on a backbone network generated for $x_c = 0.4073$ on a square lattice. Data clearly exhibit the universal critical behavior, with the estimates of the exponents $\theta \simeq 0.20$ and $\beta \simeq 0.50$, which are distinctly different from those for the pure CLG model. It should be noted that $\theta \simeq 0.41$ and $\beta \simeq 0.64$ for the pure CLG model in two dimensions.

3.2. Diluted conserved threshold transfer process

In the CLG model, since each lattice site is occupied by a single particle, particles on dead ends cannot escape forever, and this behavior yielded the nonuniversal power-law behavior. For the CTTP model, on the other hand, particles on dead ends may escape since each site can be doubly occupied. This suggests that the critical behavior of the CTTP model on an infinite percolation network might be different from that of the CLG model.

Simulations for the diluted CTTP model were carried out for $x = 0$ (pure lattice), 0.2, 0.3, 0.35, and $x_c = 0.407$. The critical density was estimated from the power-law plot of $n_{ss} \sim (\rho - \rho_c)^\beta$ assuming ρ_c as a fitting parameter, and results were refined from the power-law behavior of $n_a(t)$. In Fig. 5, the left plot is the $n_a(t)$, and the right one is $n_{ss}(\rho)$, with the inset x versus estimated ρ_c. From both plots, it is clear that the critical exponent changes only at the critical concentration. The estimates are $\theta \simeq 0.42$ and $\beta \simeq 0.64$ for $x < x_c$, and $\theta \simeq 0.20$ and $\beta \simeq 0.48$ at x_c The estimates for $x < x_c$ are close to those of the CLG model, indicating that the critical behavior of the CTTP model is similar to that of the CLG model. The results at x_c are also similar to those of the CLG model on a backbone network generated with respect to x_c.

The latter observation is rather surprising because the CLG and CTTP models were known to belong to the same universality class, and the fractal dimension of an infinite network is quite different from that of a backbone network. One might, then, raise a question of whether or not the critical behavior of the CTTP model on a backbone network is similar to that on an infinite network. In order to address the answer to this question, simulations on a backbone network were also carried out, and the active particle density and the order parameter yielded the critical behavior similar to those of the CTTP model on an infinite network (not shown). The critical exponents were obtained as $\theta \simeq 0.20$ and $\beta \simeq 0.48$, which are close to the values obtained on an infinite network. This implies that dead ends and dangling blobs yield null effect on the critical behavior of the CTTP model.

Simulations for the CTTP model on an infinite network in three dimensions were also carried out and the preliminary results appeared to show the critical behavior similar to those in two dimensions. The source of such rather unusual behavior requires further works.

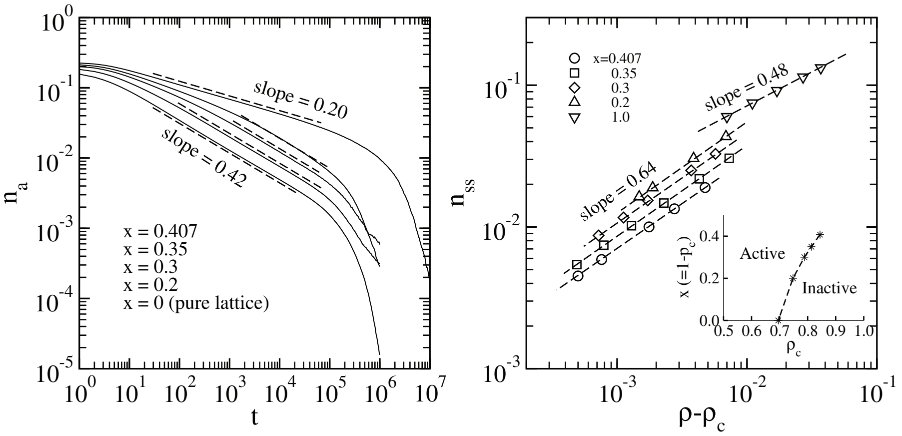

Figure 5: (Left) The densities of active particles at the critical density, for the CLG model on a percolation backbone generated with respect to $x_c = 0.4073$, on a square lattice. The numbers in the legend are the disorder concentration in the same order as the data from top to bottom, and the dashed line on each set of data is the power-law fit of $n_a \sim t^{-\theta}$. (Right) The steady-state density of active particles plotted as a function of the distance from criticality for various disorder concentrations.

4. Summary

The critical behaviors of the CLG model and the CTTP model were investigated via Monte Carlo simulations in two and three dimensions. It was found that the disorder was irrelevant as long as the concentration of disorder is less than the critical concentration both in two and three dimensions. When the concentration becomes critical, a nonuniversal power-law behavior was observed for the CLG model, whereas universal critical behavior was observed for the CTTP model. The nonuniversal power law for the CLG model appeared to be attributed to the dead ends on an infinite network.

On a backbone network which is free of dead ends, the critical behavior of both the CLG model and the CTTP model was universal and the critical exponents for the two models were found to be similar, indicating a strong universality between the two models. However, on an infinite network, the two models exhibited distinct critical behaviors.

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