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Wave-induced hydrodynamic responses of an immersed rigid body connected with elastic plates in a two-layer fluid

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Abstract

A new physical model for very large floating structures (VLFSs) connected with an immersed module is investigated for a two-layer fluid, which may provide the basic knowledge to analyze multi-module floating structures in the stratified ocean. Under the hypothesis of small-amplitude wave theory, the case with the coupling effects by the wave motion, elastic deformation of the plate and the rigid body's oscillation is solved by considering a scattering problem and a radiation one. An inner product with orthogonality is used to calculate the undetermined coefficients in eigenfunction expansions. The exciting forces and the coefficients for added mass, damping, and stiffness are obtained.

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Keywords: Eigenfunction expansion; Multi-module VLFS; Orthogonality; Two-layer Fluid

1. Introduction

A floating elastic plate connected with an immersed rigid body may be a representative physical model for multi-module very large floating structures (VLFSs). The body stands for a certain buildings, for example, living facilities, oil storages, airport terminals, on the VLFS. The method of matched eigenfunction expansions is a wildly used technique for the hydroelastic interaction between waves and a marine structure. A prior manner for calculating the numerical coefficients is the error function method presented by Fox and Squire [1,2] for the interaction of waves with a semi-infinite elastic plate. Sahoo et al. [3] adopted an inner product method for the same problem. Xu and Lu [4] employed the vertical eigenfunctions at free surface region to make an inner product with orthogonality, by which the numerical approximation for the series solution can converge quickly after truncating a few terms in the expansions.

Under the small-amplitude wave hypothesis and the linear potential flow theory, we consider the above-mentioned model by a linear superposition of a scattering potential and a radiation one. The difficulty is to match the velocity potentials at vertical interfaces between different regions. We try to employ the vertical eigenfunctions at the free surface region to make an inner product. It is shown that this manner is effective and will yield some orthogonal relations which are helpful for simplifying the matching equations, and thus the computation efficiency is promoted.

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2. Mathematical formulation

We consider a two-dimensional model composed of an immersed rigid body and two pieces of thin elastic plate floating on a two-layer fluid, as shown in Fig. 1. The pitch and heave motions of the rigid body are taken into account while the surge motion is neglected here. The density and the thickness of the upper fluid are denoted by ρ_1 and h_1 while those of the lower fluid by ρ_2 and h_2. With the assumption of linearity and time-harmonic motion, we write the velocity potential as $\Phi(x,z,t) = \Re[\phi(x,z)e^{-i\omega t}]$, where ω refers to the frequency and $\phi(x,z)$ is the spatial potential function which satisfies $\nabla^2\phi(x,z) = 0$.

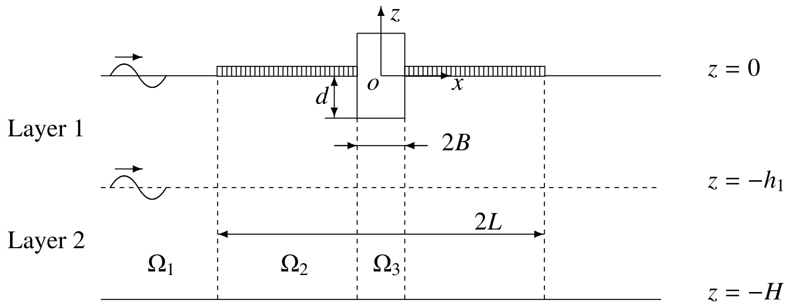


Fig. 1. Schematic diagram of the floating structure

The combined boundary conditions on top surface, the middle interface and the bottom seabed should be

$$-\rho_1\omega^2\phi + \left(D\frac{\partial^4}{\partial x^4} - M\omega^2 + \rho_1 g\right)\frac{\partial\phi}{\partial z} = 0, \qquad (|x| > B, z = 0), \tag{1}$$

$$\gamma\left[K\phi(x,-h_1^+) - \frac{\partial\phi(x,-h_1^+)}{\partial z}\right] = K\phi(x,-h_1^-) - \frac{\partial\phi(x,-h_1^-)}{\partial z}, \qquad (-\infty < x < +\infty), \tag{2}$$

$$\frac{\partial\phi(x,-h_1^+)}{\partial z} = \frac{\partial\phi(x,-h_1^-)}{\partial z}, \qquad (-\infty < x < +\infty), \tag{3}$$

$$\frac{\partial\phi}{\partial z} = 0, \qquad (-\infty < x < +\infty, z = -H), \tag{4}$$

where g is the gravitational acceleration; $\gamma = \rho_1/\rho_2$ and $K = \omega^2/g$; D and M are piecewise parameters with $D = Ed_e^3/[12(1-\nu^2)]$ and $M = \rho_e d_e$ for the plate-covered region ($B < |x| < L$) while $D = 0$ and $M = 0$ for the free-surface region ($|x| > L$); E, d_e, ρ_e, and ν are Young's Modulus, thickness, density, and Poisson's ratio of the plate respectively. The conditions for the submerged surface of the rigid body and the edge of the plates are given below in the procedure for solving the problem.

3. Method of solution

The spatial potential is linear superposition of two parts, namely $\phi = \phi^r + \phi^s$, in which ϕ^r refers to the radiation potential due to the rigid body's motion and ϕ^s accounts for the wave scattering by the rigid body without motion. Given corresponding boundary conditions, ϕ^r and ϕ^s can be solved respectively.

3.1. Radiation potential

The radiation potential can be written in the form of $\phi^r = -i\omega\sum_{r=1}^{2} A_r^r\phi_r^r$, where A_1^r and A_2^r are the amplitudes for the heave and pitch motions, respectively. ϕ_1^r is the heave component per unit response speed and ϕ_2^r for the pitch one. On the submerged surface of the rigid body, the potential ϕ_r^r with $r = 1, 2$ satisfies

$$\frac{\partial\phi_r^r}{\partial x} = (z - z_c)\delta_{2r}, \qquad (|x| = B, -d < z < 0), \tag{5}$$

$$\frac{\partial\phi_r^r}{\partial z} = \delta_{1r} - x\delta_{2r}, \qquad (|x| < B, z = -d), \tag{6}$$

where $(0, z_c)$ is the barycenter coordinate of the rigid body, δ_{1r} and δ_{2r} is the Kronecker delta with $r = 1, 2$. The radiation potential ϕ^{r}_r with $r = 1, 2$ should satisfy the body boundary and free edge conditions as follow

$$\frac{\partial \phi^{\mathrm{r}}_r(\pm B^{\pm}, 0)}{\partial z} = \delta_{1r} \mp B\delta_{2r}, \quad \frac{\partial^2 \phi^{\mathrm{r}}_r(\pm B^{\pm}, 0)}{\partial x \partial z} = -\delta_{2r}, \quad \frac{\partial^3 \phi^{\mathrm{r}}_r(\pm L^{\mp}, 0)}{\partial x^2 \partial z} = 0, \quad \frac{\partial^4 \phi^{\mathrm{r}}_r(\pm L^{\mp}, 0)}{\partial x^3 \partial z} = 0. \tag{7}$$

The dispersion relations in every region are derived as

$$(\gamma t_1 t_2 + 1)\omega^4 - (t_1 + t_2)\omega_0^2\omega^2 + \varepsilon t_1 t_2 \omega_0^4 = 0, \qquad \text{(for Region } \Omega_1), \tag{8}$$

$$[\gamma t_1 t_2 + 1 + (t_1 + \gamma t_2)G_3]\omega^4 - [G_1 t_1 + G_2 t_2 + \varepsilon t_1 t_2 G_3]\omega_0^2\omega^2 + \varepsilon G_1 t_1 t_2 \omega_0^4 = 0, \qquad \text{(for Region } \Omega_2), \tag{9}$$

$$(t_0 + \gamma t_2)\omega^2 - \varepsilon t_0 t_2 \omega_0^2 = 0, \qquad \text{(for Region } \Omega_3), \tag{10}$$

where $\omega_0^2 = gk$, $t_0 = \tanh kh_0$, $t_1 = \tanh kh_1$, $t_2 = \tanh kh_2$, $h_0 = h_1 - d$, $G_1 = \Gamma k^4 + 1$, $G_2 = \varepsilon + \gamma G_1$, $G_3 = \sigma k$, $\Gamma = D/\rho_1 g$, and $\sigma = M/\rho_1$. For a given ω, we can find roots from Eqs. (8)–(10) with respect to k. Equation (8) has four real roots $\pm k_{0_1}$, $\pm k_{0_2}$ and infinite numbers of pure imaginary roots $\pm \mathrm{i}k_i$ ($i = 1, 2, 3 \ldots$). Equation (9) has four real roots $\pm\kappa_{0_1}$, $\pm\kappa_{0_2}$, infinite numbers of pure imaginary roots $\pm\mathrm{i}\kappa_j$ ($j = 1, 2, 3 \ldots$), and two couples of complex conjugates $\pm\mathrm{i}\kappa_{\mathrm{I}}$ and $\pm\mathrm{i}\kappa_{\mathrm{II}}$. Equation (10) has one zero root λ_{0_1}, two non-zero real roots $\pm\lambda_{0_2}$ and infinite numbers of pure imaginary roots $\pm\mathrm{i}\lambda_l$ ($l = 1, 2, 3 \ldots$).

In terms of the symmetry and far-field conditions, we have the radiation potentials expanded in series as follow

$$\phi^{\mathrm{r}}_{r1}(x,z) = \mathcal{A}_{r0_1}\mathrm{e}^{-\mathrm{i}k_{0_1}(x+L)}Z_{0_1} + \mathcal{A}_{r0_2}\mathrm{e}^{-\mathrm{i}k_{0_2}(x+L)}Z_{0_2} + \sum_{i=1}^{\infty}\mathcal{A}_{ri}\mathrm{e}^{k_i(x+L)}Z_i, \qquad (\Omega_1), \tag{11}$$

$$\begin{aligned}\phi^{\mathrm{r}}_{r2}(x,z) = {} & (\mathcal{B}_{r0_1}\mathrm{e}^{\mathrm{i}\kappa_{0_1}(x+B)} + C_{r0_1}\mathrm{e}^{-\mathrm{i}\kappa_{0_1}(x+B)})\widetilde{Z}_{0_1} + (\mathcal{B}_{r0_2}\mathrm{e}^{\mathrm{i}\kappa_{0_2}(x+B)} + C_{r0_2}\mathrm{e}^{-\mathrm{i}\kappa_{0_2}(x+B)})\widetilde{Z}_{0_2} \\ & + \sum_{j=1}^{\infty}(\mathcal{B}_{rj}\mathrm{e}^{-\kappa_j(x+B)} + C_{rj}\mathrm{e}^{\kappa_j(x+B)})\widetilde{Z}_j + \sum_{j=\mathrm{I,II}}(\mathcal{B}_{rj}\mathrm{e}^{-\kappa_j(x+B)} + C_{rj}\mathrm{e}^{\kappa_j(x+B)})\widetilde{Z}_j, \qquad (\Omega_2),\end{aligned} \tag{12}$$

$$\phi^{\mathrm{r}}_{r3}(x,z) = \mathcal{W}_r + \mathcal{D}_{r0_1}\Big(\frac{-x}{B}\Big)^{\delta_{2r}}\overline{Z}_{0_1} + \mathcal{D}_{r0_2}\frac{\cos(\lambda_{0_2}x - \frac{\pi}{2}\delta_{2r})}{\cos(\lambda_{0_2}B + \frac{\pi}{2}\delta_{2r})}\overline{Z}_{0_2} + \sum_{l=1}^{\infty}\mathcal{D}_{rl}\frac{\cosh(\lambda_l x - \mathrm{i}\frac{\pi}{2}\delta_{2r})}{\cosh(\lambda_l B + \mathrm{i}\frac{\pi}{2}\delta_{2r})}\overline{Z}_l, \qquad (\Omega_3), \tag{13}$$

where $\{Z_{0_1}(z), Z_{0_2}(z), Z_i(z)\} = \{V(k_{0_1}, z), V(k_{0_2}, z), V(\mathrm{i}k_i, z)\}$, $\{\widetilde{Z}_{0_1}(z), \widetilde{Z}_{0_2}(z), \widetilde{Z}_j(z)\} = \{V(\kappa_{0_1}, z), V(\kappa_{0_2}, z), V(\mathrm{i}\kappa_j, z)\}$, $\{\overline{Z}_{0_1}(z), \overline{Z}_{0_2}(z), \overline{Z}_l(z)\} = \{V(\lambda_{0_1}, z), V(\lambda_{0_2}, z), V(\mathrm{i}\lambda_l, z)\}$, and

$$V(k,z) = \begin{cases} \dfrac{K\cosh kh_2 - \varepsilon k\sinh kh_2}{\gamma K\cosh kH}\cosh k(z+h_1) + \dfrac{\sinh kh_2}{\cosh kH}\sinh k(z+h_1), & (-h_1 < z < 0), \\ \dfrac{\cosh k(z+H)}{\cosh kH}, & (-H < z < -h_1), \end{cases} \tag{14}$$

$$\mathcal{W}_r = \begin{cases} \dfrac{(-x)^{\delta_{2r}}}{h_0 + \gamma h_2}\Big[\dfrac{z^2}{2} - \dfrac{x^2}{2\cdot 3^{\delta_{2r}}} + (h_1 + \gamma h_2)z - \varepsilon h_2\Big(h_1 + \dfrac{1}{K}\Big)\Big], & (-h_1 < z < -d), \\ \dfrac{\gamma(-x)^{\delta_{2r}}}{h_0 + \gamma h_2}\Big(\dfrac{z^2}{2} - \dfrac{x^2}{2\cdot 3^{\delta_{2r}}} + Hz\Big), & (-H < z < -h_1). \end{cases} \tag{15}$$

The first subscript for $\phi^{\mathrm{r}}(x,z)$ denotes the state of motion while the second for the fluid region.

Along the matching boundaries, the continuity of pressure and the conservation of mass flux lead to

$$\phi^{\mathrm{r}}_{r1} = \phi^{\mathrm{r}}_{r2}, \quad \frac{\partial\phi^{\mathrm{r}}_{r1}}{\partial x} = \frac{\partial\phi^{\mathrm{r}}_{r2}}{\partial x}, \qquad (x = -L, -H < z < 0), \tag{16}$$

$$\phi^{\mathrm{r}}_{r2} = \phi^{\mathrm{r}}_{r3}, \qquad (x = -B, -H < z < -d), \tag{17}$$

$$\frac{\partial\phi^{\mathrm{r}}_{r2}}{\partial x} = \begin{cases} (z - z_c)\delta_{2r}, & (x = -B, -d < z < 0), \\ \dfrac{\partial\phi^{\mathrm{r}}_{r3}}{\partial x}, & (x = -B, -H < z < -d). \end{cases} \tag{18}$$

For this case, it is difficult to make the velocities beside $x = -B$ be well matched for Eq. (18). We try to employ the vertical eigenfunctions $Z_m(z)$ ($m = 0_1, 0_2, 1, 2, 3, \dots$) at the free surface region to make inner product for Eqs. (16) and (18), and $\overline{Z}_n(z)$ ($n = 0_1, 0_2, 1, 2, 3, \dots$) for Eq. (17), where the inner product is in a general form of

$$\langle U, V; a\rangle = \int_{-H}^{-h_1} U \cdot V \,\mathrm{d}z + \gamma \int_{-h_1}^{a} U \cdot V \,\mathrm{d}z, \qquad (a = 0, -d), \tag{19}$$

where U and V denote the vertical eigenfunctions. It is easy to validate the orthogonality for the inner products

$$\langle Z_m, Z_n; 0\rangle = 0, \quad \langle \overline{Z}_m, \overline{Z}_n; -d\rangle = 0, \qquad (m, n = 0_1, 0_2, 1, 2, 3, \dots), \tag{20}$$

$$\langle \widetilde{Z}_m, Z_n; 0\rangle - D_{mn} = 0, \qquad (m = 0_1, 0_2, \mathrm{I}, \mathrm{II}, 1, 2, 3, \dots; n = 0_1, 0_2, 1, 2, 3, \dots), \tag{21}$$

where

$$D_{mn} = \frac{\gamma(D\kappa_m^4 - M\omega^2)}{\rho_1\omega^2(k_n^4 - \kappa_m^4)}\left[\frac{\partial \widetilde{Z}_m}{\partial z}\frac{\partial^3 Z_n}{\partial z^3} + \frac{\partial^3 \widetilde{Z}_m}{\partial z^3}\frac{\partial Z_n}{\partial z}\right]\Bigg|_{z=0} = 0. \tag{22}$$

Although Eq. (21) is not a completely orthogonal relation without the explicit differential term D_{mn}, it is remarkably helpful in improving the efficiency of numerical computation. After truncating the expansions at $i, j = N$, $l = N'$ for the inner products and adding the edge conditions of Eq. (7), we can obtain a set of $3N + N' + 12$ equations for $3N + N' + 12$ coefficients, from which the radiation potentials can be solved numerically.

3.2. Scattering potential

We decompose the scattering potential into a symmetric part ϕ_1^{s} and an antisymmetric one ϕ_2^{s} as

$$\phi^{\mathrm{s}}(x,z) = \frac{1}{2}[\phi_1^{\mathrm{s}}(x,z) + \phi_2^{\mathrm{s}}(x,z)], \quad \phi_1^{\mathrm{s}}(x,z) = \phi^{\mathrm{s}}(x,z) + \phi^{\mathrm{s}}(-x,z), \quad \phi_2^{\mathrm{s}}(x,z) = \phi^{\mathrm{s}}(x,z) - \phi^{\mathrm{s}}(-x,z). \tag{23}$$

For the scattering potential, the velocity on the immersed surface and all of edge conditions should be equal to zero. The matching conditions about the continuities of mass flux and velocity at $x = -L$ and of mass flux at $x = -B$ are the same as those in Eqs. (16)–(17), and the velocity at $x = -B$ is changed to

$$\frac{\partial \phi_{r2}^{\mathrm{s}}}{\partial x} = \begin{cases} 0, & (x = -B, \ -d < z < 0), \\ \dfrac{\partial \phi_{r3}^{\mathrm{s}}}{\partial x}, & (x = -B, \ -H < z < -d). \end{cases} \tag{24}$$

Subsequently by taking the same method in Sec. 3.1, we can finally obtain the scattering potentials.

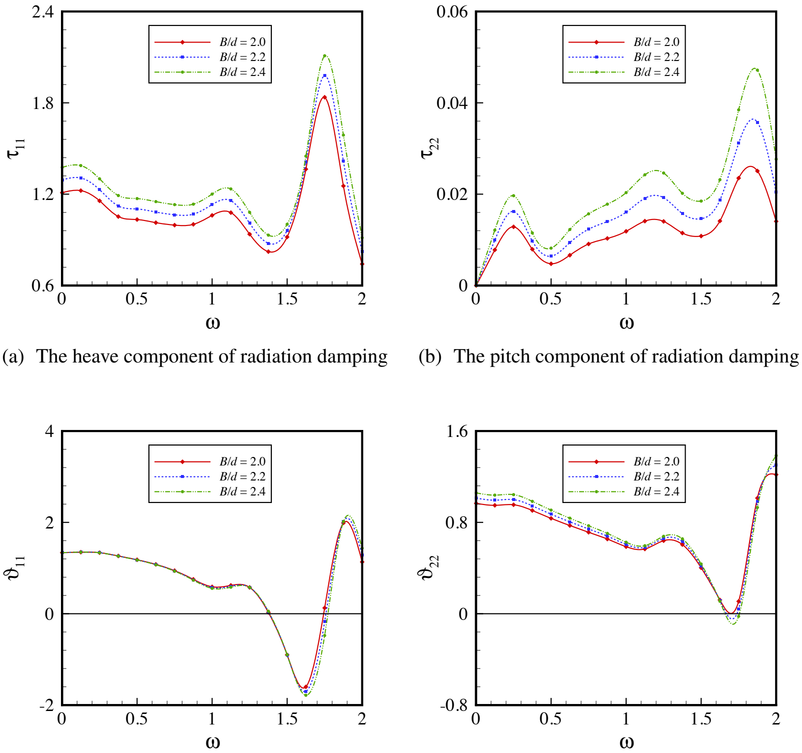
3.3. Hydrodynamic coefficients

The components for exciting forces (f_{w1}, f_{w2}, f_{e1}, f_{e2}), hydrodynamic coefficients of added mass (μ_{11}, μ_{12}, μ_{21}, μ_{22}), damping (τ_{11}, τ_{12}, τ_{21}, τ_{22}, ϖ_{11}, ϖ_{12}, ϖ_{21}, ϖ_{22}), and stiffness (ϑ_{11}, ϑ_{12}, ϑ_{21}, ϑ_{22}) are given by

$$\begin{aligned}
&f_{\mathrm{w1}} = \mathrm{i}\omega\rho_1 \int_{-B}^{0} \phi_{13}^{\mathrm{s}}(x,-d)\,\mathrm{d}x, \quad f_{\mathrm{w2}} = \mathrm{i}\omega\rho_1\left(\int_{-d}^{0}(z - z_{\mathrm{c}})\phi_{22}^{\mathrm{s}}(-B,z)\,\mathrm{d}z - \int_{-B}^{0} x\phi_{23}^{\mathrm{s}}(x,-d)\,\mathrm{d}x\right),\\
&f_{\mathrm{e1}} = \frac{\mathrm{i}D}{\omega}\frac{\partial^4\phi_{12}^{\mathrm{s}}(-B,0)}{\partial x^3\partial z}, \quad f_{\mathrm{e2}} = \frac{\mathrm{i}D}{\omega}\left(B\frac{\partial^4\phi_{22}^{\mathrm{s}}(-B,0)}{\partial x^3\partial z} + \frac{\partial^3\phi_{22}^{\mathrm{s}}(-B,0)}{\partial x^2\partial z}\right),\\
&\mu_{11} + \frac{\mathrm{i}}{\omega}\tau_{11} = 2\rho_1\int_{-B}^{0}\phi_{13}^{\mathrm{r}}(x,-d)\,\mathrm{d}x, \quad \mu_{22} + \frac{\mathrm{i}}{\omega}\tau_{22} = 2\rho_1\left(\int_{-d}^{0}(z - z_{\mathrm{c}})\phi_{22}^{\mathrm{r}}(-B,z)\,\mathrm{d}z - \int_{-B}^{0} x\phi_{23}^{\mathrm{r}}(x,-d)\,\mathrm{d}x\right),\\
&\mu_{12} + \frac{\mathrm{i}}{\omega}\tau_{12} = \mu_{21} + \frac{\mathrm{i}}{\omega}\tau_{21} = \vartheta_{12} - \mathrm{i}\omega\varpi_{12} = \vartheta_{21} - \mathrm{i}\omega\varpi_{21} = 0,\\
&\vartheta_{11} - \mathrm{i}\omega\varpi_{11} = -2D\frac{\partial^4\phi_{12}^{\mathrm{r}}(-B,0)}{\partial x^3\partial z}, \quad \vartheta_{22} - \mathrm{i}\omega\varpi_{22} = -2D\left(B\frac{\partial^4\phi_{22}^{\mathrm{r}}(-B,0)}{\partial x^3\partial z} + \frac{\partial^3\phi_{22}^{\mathrm{r}}(-B,0)}{\partial x^2\partial z}\right).
\end{aligned}$$

4. Result and conclusion

We formulate a type of VLFS which is a combination of an immersed rigid body and two elastic plates by rigid joints. An effective manner of making inner product with orthogonality is presented to process the matching relations. After applying this approach, the difficulty in handling the velocity continuity along the lateral surface of the rigid body is reduced. The components induced directly by the velocity potentials, namely (f_{w1}, f_{w2}), (μ_{11}, μ_{22}) and (τ_{11}, τ_{22}) will become larger as the ratio B/d increases. The components (f_{e1}, f_{e2}), $(\vartheta_{11}, \vartheta_{22})$ and $(\varpi_{11}, \varpi_{22})$ due to the elastic forces at the joints are almost unaffected by B/d. The variations of radiation damping (τ_{11}, τ_{22}) and added stiffness $(\vartheta_{11}, \vartheta_{22})$ versus the frequency are shown in Fig. 2, where $Bd = 0.1$ is constant.



(a) The heave component of radiation damping (b) The pitch component of radiation damping

(c) The heave component of added stiffness (d) The pitch component of added stiffness

Fig. 2. The radiation damping (τ_{11}, τ_{22}) and the added stiffness $(\vartheta_{11}, \vartheta_{22})$ against the frequency with different aspect ratio B/d

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