The EHTA for the Modified Nizhnik-Novikov-Vesselov Equation

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Abstract

In this paper, by using the extended homoclinic test approach, a new type of two-wave solutions are constructed for the Modified Nizhnik-Novikov-Vesselov equation. Their dynamic properties of some exact solutions are discussed and their profiles of these solutions are given by Maple.

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1. Introduction

The Modified Nizhnik-Novikov-Vesselov equation

\[
\begin{align*}
    u_t + u_{xxx} + u_{yyy} + 3u_x v_{xx} + 3u_y v_{yy} - u_x^3 - u_y^3 &= 0 \\
    v_{xy} &= u_x u_y
\end{align*}
\]

arises in a large number of areas of physics, which can be used to simulate many phenomena in physics. Therefore solving nonlinear problems plays an important role in nonlinear sciences. There is a wide variety of approaches to nonlinear problems to seek for exact solutions, such as the tanh function method [1], the homogeneous balance method[2], the auxiliary function method [3], the Hirota method [4], and the Exp-function method [5] and so on.

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A new technique called "extended homoclinic test approach" is proposed very recently [6], which is a fairly effective method to seek periodic solitary wave solutions of integrable wave equations [7-9]. By the method, Dai gets some new exact soliton solutions for nonlinear evolution equations [10-12].

In this paper, we apply the novel approach, extended homoclinic test approach, to Modified Nizhnik-Novikov-Vesselov equation (1). We first review the bilinear form of the Modified Nizhnik-Novikov-Vesselov equation (1).

\[
(D_y + D_x^3 + D_y^3) f(x, y, t) \cdot g(x, y, t) = 0
\]

which is obtained by substituting the following dependent variable transformation into Eq. (1)

\[
\begin{align*}
\left\{ 
\begin{array}{l}
u = -2 \arctan \frac{f}{g} \\
v = \ln(f^2 + g^2)
\end{array}
\right.
\]

where the bilinear operator [13] is defined as:

\[
D_x^m D_y^n D_z^k f(x, y, t) \cdot g(x, y, t) = \left( \partial_x - \partial_x^m \right) \left( \partial_y - \partial_y^n \right) \left( \partial_z - \partial_z^k \right) f(x, y, t) g(x, y, t) \big|_{x' = x, y' = y, t' = t}.
\]

In this case, we choose extended homoclinic test function

\[
\begin{align*}
f(x, y, t) &= e^{\eta_1} + p_1 \cos(\eta_2) + p_2 e^{-\eta_1}, \\
g(x, y, t) &= e^{\eta_1} + p_2 \cos(\eta_2) + p_4 e^{-\eta_1}.
\end{align*}
\]

where \( \eta_i = a_i x + b_i t + c_i y, (i = 1, 2) \) and \( a_i, b_i, c_i (i = 1, 2) \) are parameters to be determined later. Using Eq. (4), we will investigate exact solutions of Eq. (1). Some new exact solutions will be obtained and some interesting phenomena will be found.

2. The exact solutions of Eq. (1)

In Eq. (2), let the test function be Eq. (4). Substituting (4) into (2) yields

\[
\begin{align*}
L_1 + \{ L_2 \cos \alpha + L_3 \sin \alpha \} e^\beta + \{ L_4 \cos \alpha + L_5 \sin \alpha \} e^{-\beta} &= 0, \\
L_6 + \{ L_7 \cos \alpha + L_8 \sin \alpha \} e^\beta + \{ L_9 \cos \alpha + L_{10} \sin \alpha \} e^{-\beta} &= 0,
\end{align*}
\]

where \( \alpha = a_2 x + b_2 t + c_2 y \),

\( \beta = a_1 x + b_1 t + c_1 y \),

\[
\begin{align*}
L_1 &= -8 p_4 c_1^2 + 8 c_1^3 p_2 + 8 a_1^2 p_2 - 2 p_2 b_1 - 8 p_4 a_2^3 + 2 b_2 p_2, \\
L_2 &= - p_3 b_1 + b_1 p_1 + 3 p_2 a_2^2 a_1 - p_3 c_1^3 + a_1^3 p_1 + 3 p_3 c_2^2 c_1 - 3 a_1 p_3 a_2^3 + c_1^3 p_1 - p_3 a_1^3 - 3 c_1 p_3 c_2^2, \\
L_3 &= 3 a_2^3 + p_1 a_2^3 + p_3 b_2 + 3 c_1^2 p_2 c_2 - 3 p_3 a_2^3 + p_3 a_2^3 - p_3 c_2^3 - 3 p_3 c_2 c_2^2, \\
L_4 &= - p_4 b_1 + p_4 a_2^2 a_1 - 3 p_3 c_2^3 p_2 c_1 - 3 p_2 a_2^3 + p_3 a_2^2 + p_3 c_2^3 c_1 + 3 p_3 c_2 c_2^2 - 3 p_3 a_2^3 + p_4 a_3 c_2^2, \\
L_5 &= - p_4 b_2 + p_4 b_2 - 3 p_3 a_2^2 a_1 - p_4 a_2^2 + p_3 a_2^2 + p_3 c_2^3 p_2 c_1 + 3 p_3 c_2 c_2^2 - 3 p_3 a_2^3 + p_4 a_3 c_2^2, \\
L_6 &= 4 p_2 a_1 c_1 + 4 p_4 a_1 c_1 - p_1 c_2 a_2 + 3 p_4 c_2 a_2, \\
L_7 &= - p_1 c_2 a_2 + a_1 c_1 p_1 + a_1 c_2 p_3 - p_1 c_2 a_2, \\
L_8 &= p_1 a_2 c_1 + p_3 a_2 c_1 + a_1 p_3 c_2 + a_1 p_3 c_2, \\
L_9 &= p_1 a_2 c_1 + p_3 a_2 c_1 + a_1 p_3 c_2 + a_1 p_3 c_2.
\end{align*}
\]
Equating all the coefficients of the above equations, we obtain
\[ L_1 = 0, L_2 = 0, L_3 = 0, L_4 = 0, L_5 = 0, L_6 = 0, L_7 = 0, L_8 = 0, L_9 = 0, L_{10} = 0. \] (6)

Solving the above the set of algebraic equation with the aid of Maple, we obtain

Case (1). \( a_1 = (-b_1 / 4)^{1/3}, b_2 = -4(-(-b_1 / 4)^{2/3})^{1/2}(-(-b_1 / 4)^{2/3}, c_1 = 0, P_2 = P_2, \)
\[ P_3 = P_3, P_4 = P_4, P_4 = P_1, a_2 = (-(-b_1 / 4)^{2/3})^{1/2}, c_2 = 0. \]

Case (2). \( c_1 = c_1, p_1 = (4p_2 + 4p_4 - p_3^2)^{1/2}, c_2 = c_1I, a_2 = -I(-c_1^3 - b_1 / 4)^{1/3}, \)
\[ b_2 = -I(8c_1^3 + b_1), a_1 = (-c_1^3 - b_1 / 4)^{1/3}, P_2 = P_2, P_3 = P_3, P_4 = P_4. \]

Under the condition of case (1), we have a two-soliton solution of Eq.(1)
\[ u_1 = -2\text{arctan} \frac{e^\alpha + p_1\cos\beta + p_2e^{-\alpha}}{e^\alpha + p_3\cos\beta + p_4e^{-\alpha}}, \]
\[ v_1 = \ln\{(e^\theta + p_3\cos\gamma + p_4e^{-\theta})^2 + (e^\theta + p_1\cos\gamma + p_4e^{-\theta})^2\}, \] (7)
where \( b_1, p_1, p_2, p_3, p_4 \) are free constants and
\[ \alpha = \frac{2^{1/3}(-b_1)^{1/3}x}{2} + bt, \]
\[ \beta = \frac{2^{1/3}(-(-b_1)^{2/3})^{1/2}x}{2} - ((-b_1)^{2/3})^{1/2}(-b_1)^{2/3}t, \]
\[ \theta = (-b_1 / 4)^{1/3}x + bt, \]
\[ \gamma = -(((-b_1 / 4)^{2/3})^{1/2}x + 4(-(-b_1 / 4)^{2/3})^{1/2}(-b_1 / 4)^{2/3})t. \]

Under the condition of case (2), We obtain a homoclinic breather-wave solution of Eq.(1) as follows:
\[ u_2 = -2\text{arctan} \frac{e^\alpha + (4p_2 + 4p_4 - p_3^2)^{1/2}\cos\beta + p_2e^{-\alpha}}{e^\alpha + p_3\cos\beta + p_4e^{-\alpha}}, \]
\[ v_2 = \ln\{(e^\theta + p_3\cos\gamma + p_4e^{-\theta})^2 + (e^\theta + (4p_2 + 4p_4 - p_3^2)^{1/2}\cos\beta + p_4e^{-\theta})^2\}, \] (8)
where \( \alpha = \frac{(-8c_1^3 - 2b_1)^{1/3}x}{2} + bt + c_1y, \)
\[ \beta = a_2x - 8Itc_1^3 - Itb_1 + c_1yI, \]
\[ \theta = (-c_1^3 - b_1 / 4)^{1/3}x + bt + c_1y. \]

The Fig.1 shows that the solution \( u_2 \) has a periodic wave. This is a interesting physical phenomena.
The result shows complexity and variety of dynamical behavior for Nizhnik-Novikov-Vesselov system.
The Fig.1 shows the profiles of solution \( u_2 \) under the fixed parameters \( b_1 = 1, c_1 = 1, a_2 = 1, p_2 = 1, p_3 = 1, p_4 = 1. \)
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References


