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Editorial

Special Issue on Hybrid Logics

This special issue is devoted to a family of modal logics called *hybrid logics*. The simplest hybrid logic is an extension of the basic modal logic with the addition of a new set of atomic symbols called *nominals*. The interpretation of nominals is restricted so that they should be interpreted as an element of the domain (or equivalently, a singleton subset of the domain). For example, the nominal **0** can be interpreted as the smallest element in the model $(\mathbb{N}, <)$, the natural numbers ordered by $<$.

The fact that we can treat elements as singleton sets and, vice-versa, singleton sets as elements, let hybrid languages mix 'terms' and 'formulas' as in

$$\mathbf{0} \wedge \mathit{even},$$

where *even* is a propositional symbol. This formula is satisfied by an element e in a model, if **0** is interpreted as e and e satisfies the proposition *even*. Once nominals have been introduced in the language, other operators can be naturally defined. Two, by now standard, hybrid operators are *satisfaction modalities* (usually written as $@_i\varphi$ or $i : \varphi$, for i a nominal), and the *downarrow binder* (usually written as $\downarrow i.\varphi$, for i a nominal). The former are used to *switch* evaluation to the element denoting the nominal; i.e., an arbitrary element satisfies the formula $@_i\varphi$ if the element denoting i satisfies φ. The latter provides on-the-fly naming of the current point of evaluation; i.e., an element e satisfies $\downarrow i.\varphi$ if it satisfies φ when all free instances of i in φ are interpreted as e.

Over the last few years the basic ideas of hybrid logic have become increasingly familiar to logicians, and specially to modal logicians. Nowadays most researchers in the field know how to 'name' elements in a model using nominals; and the 'gains and risks' of using @ and ↓. Most are also aware that the fundamental ideas behind hybrid logics can be traced back to the work of Arthur Prior from the 1960s. Moreover, a wealth of results ranging from detailed complexity analysis, to semantic characterizations, and new model and proof theoretic insights have been established. If we take a look at the *current* state-of-the-art in hybrid logics, it is fair to say that they have left their original, well defined boundaries far behind; and that they are now firmly intertwined into a wide range of related areas like description logics, justification logics, coalgebraic logics, and many others.

In this special issue we provide a (necessarily incomplete) perspective on recent advances in hybrid logics, which we believe are representative of some of the ways in which the field has evolved. After an open call for papers, and the usual round of reviewing we have selected eleven articles to be included in this special issue. We will now discuss them briefly.

Coinductive models and normal forms for modal logics by C. Areces and D. Gorín. The authors present a coinductive definition of models for modal logics and show that it provides a homogeneous framework in which it is possible to include different modal languages including hybrid logics. The framework can be used to uniformly prove general results, like preservation under bisimulations, for a wide family of logics. The framework is then used to investigate normal forms, and obtain a definition of *extracted form* with the aim of obtaining equivalent formulas with the smallest possible modal depth.

Algorithmic correspondence and completeness in modal logic V. Recursive extensions of SQEMA by W. Conradie, V. Goranko, and D. Vakarelov. SQEMA is an algorithm that computes first-order frame equivalents for modal formulae, and also proves their canonicity. In this article the authors extend SQEMA with an additional rule based on a recursive version of Ackermanns lemma, which enables the algorithm to compute local frame equivalents of modal formulae in the extension of first-order logic with monadic least fixed-points FOμ. The computation transforms input formulae into locally frame equivalent formulas in the pure fragment of the hybrid μ-calculus.

Algebraic tableau reasoning for the description logic SHOQ by J. Faddoul and V. Haarslev. Most description logic reasoners employ procedures that are 'arithmetically uninformed' and substitute arithmetic reasoning over cardinality restrictions by "dont know" non-determinism. This lack of information about arithmetic problems dramatically degrades their performance in many cases, especially with ontologies containing nominals (the description logic, 'one-of' operator $\mathcal{O}$) and qualified cardinality restrictions (the operator $\mathcal{Q}$). In this article the authors present a new algebraic tableau algorithm for the logic $\mathcal{SHOQ}$ that combines tableau rules and linear integer programming, to ensure arithmetically better informed reasoning procedures.

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Justification logics and hybrid logics by M. Fitting. Hybrid logics internalize their own semantics via the use of nominals and satisfaction operators. On the other hand, so called *justification logics* internalize their own proof methodology. In this article, the author pursues the goal to combine these two ideas into a single system, and present a hybrid justification version of the modal logic T, providing a semantics, a proof theory, a completeness theorem, and a realization theorem.

A family of Gödel hybrid logics by D. Galmiche and Y. Salhi. In this paper, the authors define a family of fuzzy hybrid logics based on Gödel logic. It is composed of two infinite-valued versions and a sequence of finitary valued versions. Sequent based decision procedures for these logics are then introduced and investigated.

Axiomatizing hybrid logic using modal logic by I. Hodkinson and L. Paternault. The authors use a method originally introduced by ten Cate, Marx, and Viana to simulate hybrid logic using classical modalities and 'nice' frames. They show that the hybrid logic of a class of frames is the modal logic of the class of its corresponding nice frames. Then, they introduce the notion of 'fairly nice frames,' to capture their closure under disjoint union. Using these results, they show how to axiomatize the hybrid logic of any elementary class of frames. They also study quasimodal logics (hybrid logics axiomatized by modal axioms together with basic hybrid axioms common to any hybrid logic, using only orthodox inference rules). They show that the hybrid logic of any elementary modally definable class of frames, or of any elementary class of frames closed under disjoint unions, bounded morphic images, ultraproducts and generated subframes, is quasimodal.

Lightweight hybrid tableaux by G. Hoffmann. The author presents a decision procedure for hybrid logic with nominals, the satisfaction operator and existential, difference, converse, reflexive, symmetric and transitive modalities. This decision procedure is a prefixed tableau method based on ideas introduced by Bolander and Blackburn. The calculus is then extended to handle the difference modality, following ideas inspired by the work of Kaminski and Smolka.

The complexity of satisfiability for fragments of hybrid logic – Part I by A. Meier, M. Mundhenk, T. Schneider, M. Thomas, V. Weber, and F. Weiss. As the satisfiability problem of hybrid logics with the $\downarrow$ binder is undecidable it is interesting to look for decidable and tractable fragments. In this paper, the authors investigate the effect on complexity of restricting the propositional part of the language over various frames classes, tracing the border of decidability, and providing the exact complexity of most fragments.

Complexity of hybrid logics over transitive frames by M. Mundhenk, T. Schneider, T. Schwentick, and V. Weber. This article examines the complexity of hybrid logics over transitive frames, transitive trees, and linear frames. That satisfiability over transitive frames of the hybrid logic with $\downarrow$ is shown to be NEXPTIME-complete, but adding the @ operator or the past modality leads to undecidability. In contrast, on transitive trees and linear frames, these languages are shown to be nonelementarily decidable. They also establish 2EXPTIME and EXPTIME upper bounds for satisfiability over transitive frames and transitive trees, respectively, for the hybrid Until/Since language.

Hybrid logic with the difference modality for generalisations of graphs by R. Myers and D. Pattinson. The authors investigate ways of generalising the basic hybrid logic with the difference modality to different notions of transition, from subrelational transitions such as monotone neighbourhood frames or selection function models to models with more structure such as Markov chains and alternating temporal frames. They provide a generic canonical cut-free sequent system, with a proof-search strategy that terminates for the fragment with the global (but without the difference) modality.

Axiomatizing hybrid products by K. Sano. The author proposes a two-dimensional hybrid logic to formalize the combination of spatial and temporal reasoning. Intuitively, the idea is to name both horizontal and vertical lines in a two-dimensional plane by two different kinds of nominals. The author provides an axiomatization of the proposed logic and show a general completeness result for frames axiomatized by pure formulas. To capture a particular kind of frames introduced by Thomason, the author define dependent product frames to represent the dependence of space over time, an provides a complete axiomatization.

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