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Interference bands in decays of doubly-charged Higgs bosons to dileptons in the minimal type-II seesaw model at the TeV scale

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ABSTRACT

The dileptonic decays of doubly-charged Higgs bosons $H^{\pm\pm}$ are investigated in the minimal type-II seesaw model with one Higgs triplet Δ and one heavy Majorana neutrino N_1 at the TeV scale. We show that the branching ratios $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$ depend not only on the mass and mixing parameters of three light neutrinos ν_i (for $i = 1, 2, 3$), but also on those of N_1. Assuming the mass of N_1 to lie in the range 200 GeV–1 TeV, we figure out the generous interference bands for the contributions of ν_i and N_1 to $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$: $\sqrt{|\sin\theta_{i4}\sin\theta_{j4}|} \sim 10^{-8}$–$10^{-5}$, where θ_{i4} and θ_{j4} measure the strength of charged-current interactions of N_1. We illustrate some salient features of the interference bands by considering three typical mass patterns of ν_i, and stress that it is very difficult to distinguish the type-II seesaw model from the triplet seesaw model in such a parameter region at the Large Hadron Collider.

1. Introduction

The effort to build neutrino mass models at the TeV scale has recently revived [1], simply because this new energy frontier will soon be explored by the Large Hadron Collider (LHC). A naive but reasonable argument is that possible new physics, if it exists at the TeV scale and is responsible for the electroweak symmetry breaking, might also be responsible for the origin of neutrino masses. The latter is a kind of new physics which has been conceivably established by a number of neutrino oscillation experiments in the past decade [2].

Among many possibilities of generating tiny neutrino masses, a natural one is to extend the standard model by introducing a few heavy right-handed Majorana neutrinos [3] and (or) one Higgs triplet [4]. The gauge-invariant neutrino mass terms can then be written as

$$-\mathcal{L}_{\rm mass} = \overline{l_{\rm L}} Y_\nu \tilde{H} N_{\rm R} + \frac{1}{2}\overline{N^c_{\rm R}} M_{\rm R} N_{\rm R} + \frac{1}{2}\overline{l_{\rm L}} Y_\Delta \Delta i\sigma_2 l^c_{\rm L} + \text{h.c.}, \tag{1}$$

where $M_{\rm R}$ is the mass matrix of right-handed Majorana neutrinos, and

$$\Delta \equiv \begin{pmatrix} H^- & -\sqrt{2}H^0 \\ \sqrt{2}H^{--} & -H^- \end{pmatrix} \tag{2}$$

denotes the Higgs triplet. After the spontaneous gauge symmetry breaking, one obtains the neutrino mass matrices $M_{\rm D} = Y_\nu v/\sqrt{2}$ and $M_{\rm L} = Y_\Delta v_\Delta$, where $\langle H\rangle \equiv v/\sqrt{2}$ and $\langle\Delta\rangle \equiv v_\Delta$ correspond to the vacuum expectation values of the neutral components of H and Δ. To minimize the degrees of freedom associated with $M_{\rm L}$, $M_{\rm D}$ and $M_{\rm R}$, we may assume that there is only a single heavy Majorana neutrino (denoted as N_1) in the model. This assumption implies that $M_{\rm R}$ and $M_{\rm D}$ become 1×1 and 3×1, respectively, but $M_{\rm L}$ remains to be 3×3. Such a simple seesaw scenario is phenomenologically viable and can be referred to as the minimal type-II seesaw model [5]. Its simplicity makes it interesting and instructive to reveal the salient features of the type-II seesaw mechanism. Therefore, we shall concentrate on this model in the present Letter.

Our purpose is to investigate the dileptonic decays of doubly-charged Higgs bosons $H^{\pm\pm}$ in the minimal type-II seesaw model. Such decays can naturally happen because Δ is allowed to couple to the Standard Model Higgs doublet H and thus the lepton number is violated by two units [4]. If the mass scale of Δ is of $\mathcal{O}(1)$ TeV, then $H^{\pm\pm}$ can be produced at the LHC via the Drell–Yan process $q\bar{q} \to \gamma^*, Z^* \to H^{++}H^{--}$ or through the charged-current process $q\bar{q}' \to W^* \to H^{\pm\pm}H^{\mp}$. Note that the masses of $H^{\pm\pm}$ and $H^\pm$ are

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expected to be nearly degenerate in a class of seesaw models [4,6,7], so only $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ (for $\alpha, \beta = e, \mu, \tau$) and $H^{\pm\pm} \to W^\pm W^\pm$ modes are kinematically open. Note also that the dileptonic channels $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ become dominant when $v_\Delta < 1$ MeV is taken [7]. Therefore, we focus our interest on the same-sign dilepton events of $H^{\pm\pm}$, which signify the lepton number violation and serve for a clean collider signature of new physics beyond the standard model [8]. The rates of $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ decays are given by

$$\Gamma\left(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta\right) = \frac{1}{4\pi(1+\delta_{\alpha\beta})}\left|(Y_\Delta)_{\alpha\beta}\right|^2 M_{H^{\pm\pm}}, \tag{3}$$

from which one obtains the branching ratios [7]

$$\mathcal{B}\left(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta\right) \equiv \frac{\Gamma(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)}{\sum_{\rho,\sigma}\Gamma(H^{\pm\pm} \to l^\pm_\rho l^\pm_\sigma)} = \frac{2}{(1+\delta_{\alpha\beta})}\frac{|(M_{\rm L})_{\alpha\beta}|^2}{\sum_{\rho,\sigma}|(M_{\rm L})_{\rho\sigma}|^2}, \tag{4}$$

where the Greek subscripts run over e, μ and τ. It becomes obvious that the magnitudes of $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$ are only relevant to the matrix elements of $M_{\rm L}$.

We find that the branching ratios $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$ depend not only on the masses (m_1, m_2, m_3), flavor mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$ and CP-violating phases $(\delta_{12}, \delta_{13}, \delta_{23})$ of three light neutrinos ν_1, ν_2 and ν_3, but also on the mass (M_1) and mixing parameters $(\theta_{14}, \theta_{24}, \theta_{34}$ and $\delta_{14}, \delta_{24}, \delta_{34})$ of the heavy Majorana neutrino N_1. When the former contribution is negligibly small, we can reproduce the case discussed in Ref. [6]; but when the contribution of N_1 is negligibly small, our results for $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$ can simply reproduce those obtained in the triplet seesaw model [9,10]. The new and most interesting case, which has not been analyzed before, is the competition or interference between the contributions of light and heavy Majorana neutrinos. Typically assuming $M_1 \sim 200$ GeV–1 TeV and taking three possible mass patterns of ν_i as allowed by current neutrino oscillation data, we figure out the generous interference bands of ν_i and N_1 contributions to $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$: $\sqrt{|\sin\theta_{i4}\sin\theta_{j4}|} \sim 10^{-8}$–$10^{-5}$ (for $i, j = 1, 2, 3$). We stress that both constructive and destructive interference effects are possible in this parameter region, in which it is very difficult to distinguish the type-II seesaw model from the triplet seesaw model at the LHC. We present some detailed numerical calculations of $\mathcal{B}(H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta)$ in the interference bands. Although our numerical results are subject to the minimal type-II seesaw model, they can serve as a good example to illustrate the interplay between light and heavy Majorana neutrinos in a generic type-II seesaw scenario.

2. Interference bands

After the spontaneous electroweak symmetry breaking, we rewrite Eq. (1) as

$$-\mathcal{L}'_{\rm mass} = \frac{1}{2}\overline{(\nu_{\rm L} N^c_{\rm R})}\begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix}\nu^c_{\rm L} \\ N_{\rm R}\end{pmatrix} + \text{h.c.} \tag{5}$$

We assume the existence of only a single heavy Majorana neutrino N_1 in the type-II seesaw scenario. The 4×4 neutrino mass matrix in Eq. (5) is symmetric and can be diagonalized by the following unitary transformation:

$$\begin{pmatrix} V & R \\ S & U\end{pmatrix}^\dagger \begin{pmatrix} M_{\rm L} & M_{\rm D} \\ M^T_{\rm D} & M_{\rm R}\end{pmatrix}\begin{pmatrix} V & R \\ S & U\end{pmatrix}^* = \begin{pmatrix}\hat{M}_\nu & \mathbf{0} \\ \mathbf{0} & M_1\end{pmatrix}, \tag{6}$$

where $\hat{M}_\nu = {\rm Diag}\{m_1, m_2, m_3\}$ with m_i being the masses of three light neutrinos ν_i, and M_1 denotes the mass of N_1. After this diagonalization, the flavor states of three light neutrinos ν_α (for $\alpha = e, \mu, \tau$) can be expressed in terms of the masses states of both three light Majorana neutrinos ν_i (for $i = 1, 2, 3$) and the heavy Majorana neutrino N_1; namely, $\nu_\alpha = V_{\alpha i}\nu_i + R_{\alpha 1}N_1$. Then it is straightforward to write out the standard charged-current interactions between ν_α and α in the basis of mass states:

$$-\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}}\left[\overline{(e\,\mu\,\tau)_{\rm L}}V\gamma^\mu\begin{pmatrix}\nu_1 \\ \nu_2 \\ \nu_3\end{pmatrix}_{\rm L} W^-_\mu + \overline{(e\,\mu\,\tau)_{\rm L}}R\gamma^\mu N_{1\rm L}W^-_\mu\right] + \text{h.c.} \tag{7}$$

We see that V describes the flavor mixing of three light neutrinos and three charged leptons, while R determines how strong the heavy Majorana neutrino interacts with three charged leptons. In other words, V and R are responsible for neutrino oscillations of ν_i and collider signatures of N_1, respectively. Note that V itself is not unitary, because $VV^\dagger + RR^\dagger = \mathbf{1}$ holds as a consequence of unitarity of the 4×4 transformation matrix in Eq. (6). The correlation between V and R can be parametrized as [11]

$$\begin{aligned} V &= \begin{pmatrix} c_{14} & 0 & 0 \\ -\hat{s}_{14}\hat{s}^*_{24} & c_{24} & 0 \\ -\hat{s}_{14}c_{24}\hat{s}^*_{34} & -\hat{s}_{24}\hat{s}^*_{34} & c_{34}\end{pmatrix}\begin{pmatrix} c_{12}c_{13} & \hat{s}^*_{12}c_{13} & \hat{s}^*_{13} \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{12}c_{23} - \hat{s}^*_{12}\hat{s}_{13}\hat{s}^*_{23} & c_{13}\hat{s}^*_{23} \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}^*_{12}\hat{s}_{13}c_{23} & c_{13}c_{23}\end{pmatrix}, \\ R &= \begin{pmatrix}\hat{s}^*_{14} \\ c_{14}\hat{s}^*_{24} \\ c_{14}c_{24}\hat{s}^*_{34}\end{pmatrix}, \end{aligned} \tag{8}$$

where $c_{ij} \equiv \theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}}s_{ij}$ with θ_{ij} and δ_{ij} (for $1 \leqslant i < j \leqslant 4$) being the rotation angles and phase angles, respectively. If the heavy Majorana neutrino N_1 is decoupled (i.e., $\theta_{14} = \theta_{24} = \theta_{34} = 0$), V will become a unitary matrix and take the standard form as advocated in Refs. [2,12]. Hence non-vanishing R measures the non-unitarity of V.

Now we make use of Eqs. (6) and (8) to reconstruct $M_{\rm L}$, which determines the branching ratios of $H^{\pm\pm} \to l^\pm_\alpha l^\pm_\beta$ decay modes. We obtain

$$M_{\rm L} = V\hat{M}_\nu V^T + M_1 RR^T. \tag{9}$$

Then the explicit expressions of $(M_{\rm L})_{\alpha\beta}$ can be given in terms of the relevant neutrino masses, mixing angles and CP-violating phases. In view of current experimental constraints $s_{13} < 0.16$ [13] and $s_{i4} \lesssim 0.1$ (for $i = 1, 2, 3$) [14], we may simplify the exact results of $(M_{\rm L})_{\alpha\beta}$ by taking $c_{13} \approx c_{i4} \approx 1$. This good approximation allows us to arrive at

$$\begin{aligned}
(M_{\rm L})_{ee} &= m_1 c_{12}^2 + m_2 \hat{s}_{12}^{*2} + m_3 \hat{s}_{13}^{*2} + M_1 \hat{s}_{14}^{*2},\\
(M_{\rm L})_{\mu\mu} &= m_1 \hat{s}_{12}^2 c_{23}^2 + m_2 c_{12}^2 c_{23}^2 + m_3 \hat{s}_{23}^{*2} + M_1 \hat{s}_{24}^{*2},\\
(M_{\rm L})_{\tau\tau} &= m_1 \hat{s}_{12}^2 \hat{s}_{23}^2 + m_2 c_{12}^2 \hat{s}_{23}^2 + m_3 c_{23}^2 + M_1 \hat{s}_{34}^{*2};\\
(M_{\rm L})_{e\mu} &= -m_1 c_{12} \hat{s}_{12} c_{23} + m_2 c_{12} \hat{s}_{12}^* c_{23} + m_3 \hat{s}_{13}^* \hat{s}_{23}^* + M_1 \hat{s}_{14}^* \hat{s}_{24}^*,\\
(M_{\rm L})_{e\tau} &= m_1 c_{12} \hat{s}_{12} \hat{s}_{23} - m_2 c_{12} \hat{s}_{12}^* \hat{s}_{23} + m_3 \hat{s}_{13}^* c_{23} + M_1 \hat{s}_{14}^* \hat{s}_{34}^*,\\
(M_{\rm L})_{\mu\tau} &= -m_1 \hat{s}_{12}^2 c_{23} \hat{s}_{23} - m_2 c_{12}^2 c_{23} \hat{s}_{23} + m_3 c_{23} \hat{s}_{23}^* + M_1 \hat{s}_{24}^* \hat{s}_{34}^*.
\end{aligned} \tag{10}$$

As a consequence,

$$\begin{aligned}
\sum_{\rho,\sigma} \left|(M_{\rm L})_{\rho\sigma}\right|^2 = {}& \left(m_1^2 + m_2^2 + m_3^2\right) + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 + 2 m_1 M_1 \,{\rm Re}\!\left[(c_{12}\hat{s}_{14} - \hat{s}_{12} c_{23} \hat{s}_{24} + \hat{s}_{12}\hat{s}_{23}\hat{s}_{34})^2\right]\\
& + 2 m_2 M_1 \,{\rm Re}\!\left[(\hat{s}_{12}^* \hat{s}_{14} + c_{12} c_{23} \hat{s}_{24} - c_{12} \hat{s}_{23} \hat{s}_{34})^2\right] + 2 m_3 M_1 \,{\rm Re}\!\left[(\hat{s}_{13}^* \hat{s}_{14} + \hat{s}_{23}^* \hat{s}_{24} + c_{23} \hat{s}_{34})^2\right].
\end{aligned} \tag{11}$$

By combining Eqs. (10) and (11) with Eq. (4), we are then able to calculate the branching ratios $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$. There are two extreme cases.

(1) If the heavy Majorana neutrino N_1 is essentially decoupled (i.e., $\theta_{i4} \approx 0$ for $i = 1, 2, 3$), the unitarity of V will be restored. In this case, the results of $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ are the same as those obtained in the triplet seesaw model [9,10].

(2) If the contribution of N_1 to $(M_{\rm L})_{\alpha\beta}$ is dominant, one may simplify Eqs. (10) and (11) by neglecting the terms proportional to m_i (for $i = 1, 2, 3$). In this case,

$$\begin{aligned}
\mathcal{B}\big(H^{\pm\pm} \to e^\pm e^\pm\big) &\approx \frac{s_{14}^4}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2},\\
\mathcal{B}\big(H^{\pm\pm} \to \mu^\pm \mu^\pm\big) &\approx \frac{s_{24}^4}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2},\\
\mathcal{B}\big(H^{\pm\pm} \to \tau^\pm \tau^\pm\big) &\approx \frac{s_{34}^4}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2};\\
\mathcal{B}\big(H^{\pm\pm} \to e^\pm \mu^\pm\big) &\approx \frac{2 s_{14}^2 s_{24}^2}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2},\\
\mathcal{B}\big(H^{\pm\pm} \to e^\pm \tau^\pm\big) &\approx \frac{2 s_{14}^2 s_{34}^2}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2},\\
\mathcal{B}\big(H^{\pm\pm} \to \mu^\pm \tau^\pm\big) &\approx \frac{2 s_{24}^2 s_{34}^2}{(s_{14}^2 + s_{24}^2 + s_{34}^2)^2},
\end{aligned} \tag{12}$$

which only rely on the mixing angles θ_{i4} (for $i = 1, 2, 3$). Given $s_{14} \approx 0$, possible signatures of $H^{\pm\pm} \to \mu^\pm\mu^\pm$, $\mu^\pm\tau^\pm$ and $\tau^\pm\tau^\pm$ modes at the LHC have been analyzed in Ref. [6].

Here let us explore the third interesting case, in which the contributions of ν_i and N_1 to $(M_{\rm L})_{\alpha\beta}$ are comparable in magnitude and may give rise to significant interference effects on the branching ratios of $H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm$ decays. To be explicit, we take $\Delta m_{21}^2 \sim 8.0 \times 10^{-5}$ eV^2 and $|\Delta m_{32}^2| \sim 2.5 \times 10^{-3}$ eV^2 [13] as the typical inputs and assume M_1 to lie in the range 200 GeV—1 TeV. There are three possible patterns of the light neutrino mass spectrum: (1) the normal hierarchy: $m_3 \sim 5.1 \times 10^{-2}$ eV, $m_2 \sim 8.9 \times 10^{-3}$ eV, and m_1 is much smaller than m_2; (2) the inverted hierarchy: $m_2 \sim 5.0 \times 10^{-2}$ eV, $m_1 \sim 4.9 \times 10^{-2}$ eV, and m_3 is much smaller than m_1; (3) the near degeneracy: $m_1 \sim m_2 \sim m_3 \sim 0.1$ eV to 0.2 eV, which is consistent with the cosmological upper bound $m_1 + m_2 + m_3 < 0.61$ eV [13]. In each case, the contributions of ν_i and N_1 to $(M_{\rm L})_{\alpha\beta}$ in Eq. (10) will be of the comparable magnitude if the mixing angles θ_{i4} satisfy the following condition[1]:

$$s_{i4} s_{j4} \sim \frac{\max\{m_1, m_2, m_3\}}{M_1} \sim 10^{-14} \cdots 10^{-12}, \tag{13}$$

where $i, j = 1, 2, 3$. In view of this rough estimate, which is essentially compatible with a more careful numerical analysis, we can generously set $\sqrt{s_{i4} s_{j4}} \sim 10^{-8}$–$10^{-5}$ as the interference bands of $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ for $M_1 \sim 200$ GeV–1 TeV. Because the CP-violating phases δ_{i4} are completely unrestricted, they may cause either constructive or destructive effects in the interference bands. We shall numerically calculate $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ in the subsequent section to illustrate the interference effects for different patterns of the light neutrino mass hierarchy.

If $M_1 \lesssim \mathcal{O}(1)$ TeV and the values of s_{i4} lie in the interference bands obtained above, it will be impossible to produce and observe N_1 at the LHC. The reason is simply that the interaction of N_1 with three charged leptons is too weak to be detected in this parameter space. Given the integrated luminosity to be 100 fb^{-1}, for example, the resonant signature of N_1 in the channel $p\bar{p} \to \mu^\pm N_1$ with $N_1 \to \mu^\pm W^\mp$

[1] Here we have taken account of $\theta_{12} \sim 34^\circ$, $\theta_{13} < 10^\circ$ and $\theta_{23} \sim 45^\circ$ given by a global analysis of current neutrino oscillation data in the unitary limit of V [13].

at the LHC has been analyzed and the sensitivity of the cross section $\sigma(p\bar{p} \to \mu^\pm\mu^\pm W^\mp) \approx \sigma(p\bar{p} \to \mu^\pm N_1)\mathcal{B}(N_1 \to \mu^\pm W^\mp)$ to the effective mixing parameter $S_{\mu\mu} \approx s_{24}^4/(s_{14}^2 + s_{24}^2 + s_{34}^2)$ has been examined in Ref. [15]. It is found that $S_{\mu\mu} \geqslant 7.2 \times 10^{-4}$ (or equivalently, $s_{24}^2 \geqslant 2.1 \times 10^{-3}$ for $s_{14} \sim s_{24} \sim s_{34}$) is required in order to get a signature at the 2σ level for $M_1 \geqslant 200$ GeV. This result illustrates that there will be no chance to probe the existence of N_1 in the interference bands at the LHC.

Nevertheless, it is possible to produce $H^{\pm\pm}$ at the LHC provided $M_{H^{\pm\pm}} \lesssim \mathcal{O}(1)$ TeV, and it is also possible to observe the signatures of $H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm$ decays [6,7,9,10]. In this case, however, the measurements of $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ themselves are very difficult to tell whether the existence of $H^{\pm\pm}$ is due to a pure triplet seesaw model or due to a (minimal) type-II seesaw model.

3. Numerical examples

For the sake of simplicity, here we take $\theta_{12} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$, $\theta_{13} = 0^\circ$ and $\theta_{23} = 45^\circ$, implying that V takes the well-known tri-bimaximal mixing pattern [16] in its unitary limit (i.e., $\theta_{i4} = 0$). In addition, we switch off the CP-violating phases δ_{12}, δ_{13} and δ_{23} so as to clearly examine the role of new CP-violating phases δ_{i4} in $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$. We fix $\Delta m_{21}^2 = 8.0 \times 10^{-5}$ eV^2, $|\Delta m_{32}^2| = 2.5 \times 10^{-3}$ eV^2 and $M_1 = 500$ GeV in our numerical calculations. To further reduce the number of free parameters, we shall consider two special cases for the mixing angles θ_{i4}: (a) $\theta_{14} = \theta_{24} = \theta_{34}$ and (b) $\theta_{14} = 0$ and $\theta_{24} = \theta_{34}$; and two special cases for the CP-violating phases δ_{i4}: (a) $\delta_{14} = \delta_{24} = \delta_{34} = 0$ and (b) $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. Our discussions can be classified into three parts according to three possible patterns of the light neutrino mass hierarchy.

3.1. Normal hierarchy

We simply take $m_1 = 0$, such that $m_2 \approx 8.9 \times 10^{-3}$ eV and $m_3 \approx 5.1 \times 10^{-2}$ eV can be extracted from the given values of Δm_{21}^2 and $|\Delta m_{32}^2|$. For chosen values of θ_{12}, θ_{13}, θ_{23} and δ_{12}, δ_{13}, δ_{23}, Eqs. (10) and (11) can now be simplified to

$$
\begin{aligned}
(M_{\rm L})_{ee} &= \frac{1}{3}m_2 + M_1\hat{s}_{14}^{*2},\\
(M_{\rm L})_{\mu\mu} &= \frac{1}{3}m_2 + \frac{1}{2}m_3 + M_1\hat{s}_{24}^{*2},\\
(M_{\rm L})_{\tau\tau} &= \frac{1}{3}m_2 + \frac{1}{2}m_3 + M_1\hat{s}_{34}^{*2};\\
(M_{\rm L})_{e\mu} &= \frac{1}{3}m_2 + M_1\hat{s}_{14}^{*}\hat{s}_{24}^{*},\\
(M_{\rm L})_{e\tau} &= -\frac{1}{3}m_2 + M_1\hat{s}_{14}^{*}\hat{s}_{34}^{*},\\
(M_{\rm L})_{\mu\tau} &= -\frac{1}{3}m_2 + \frac{1}{2}m_3 + M_1\hat{s}_{24}^{*}\hat{s}_{34}^{*},
\end{aligned}
\tag{14}
$$

and

$$
\sum_{\rho,\sigma}\big|(M_{\rm L})_{\rho\sigma}\big|^2 = \big(m_2^2 + m_3^2\big) + M_1^2\big(s_{14}^2 + s_{24}^2 + s_{34}^2\big)^2 + \frac{2}{3}m_2M_1\,{\rm Re}\big[(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34})^2\big] + m_3M_1\,{\rm Re}\big[(\hat{s}_{24} + \hat{s}_{34})^2\big].
\tag{15}
$$

Our numerical results for the branching ratios $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ are shown in Fig. 1. Some comments and discussions are in order.

Fig. 1(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. We see that $\mathcal{B}(H^{\pm\pm} \to e^\pm\mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm\tau^\pm)$ are approximately equal beyond the interference band ($3 \times 10^{-7} \lesssim \theta \lesssim 2 \times 10^{-6}$), but their near degeneracy is lifted in the interference band. In contrast, $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\mu^\pm) = \mathcal{B}(H^{\pm\pm} \to \tau^\pm\tau^\pm)$ holds in the whole parameter space.

Fig. 1(b) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. One can see more obvious interference effects for θ changing from 10^{-7} to 10^{-6}. In particular, $\mathcal{B}(H^{\pm\pm} \to e^\pm\tau^\pm)$ is strongly enhanced, while $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\mu^\pm)$, $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\tau^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to \tau^\pm\tau^\pm)$ are strongly suppressed at $\theta \sim 2 \times 10^{-7}$.

Fig. 1(c) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$. In this case, there is little interference between the contributions of ν_i and N_1 to $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$. It is straightforward to observe that $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$, $\mathcal{B}(H^{\pm\pm} \to e^\pm\mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm\tau^\pm)$ are considerably suppressed due to the vanishing of θ_{14}.

Fig. 1(d) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$. In this case, all the decay modes involve significant interference effects around $\theta \sim 2 \times 10^{-7}$. Note that $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\tau^\pm)$ undergoes both a minimum and a maximum, which result from the minimums of its numerator and denominator, respectively. So do $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to \tau^\pm\tau^\pm)$. In comparison, the branching ratio of $H^{\pm\pm} \to e^\pm e^\pm$, $e^\pm\mu^\pm$ or $e^\pm\tau^\pm$ only undergoes a maximum, because its numerator does not have an appreciable minimum in the interference band.

3.2. Inverted hierarchy

We simply take $m_3 = 0$, such that $m_1 \approx 4.9 \times 10^{-2}$ eV and $m_2 \approx 5.0 \times 10^{-2}$ eV can be extracted from the given values of Δm_{21}^2 and $|\Delta m_{32}^2|$. For chosen values of θ_{12}, θ_{13}, θ_{23} and δ_{12}, δ_{13}, δ_{23}, Eqs. (10) and (11) can now be simplified to

$$
\begin{aligned}
(M_{\rm L})_{ee} &= \frac{2}{3}m_1 + \frac{1}{3}m_2 + M_1\hat{s}_{14}^{*2},\\
(M_{\rm L})_{\mu\mu} &= \frac{1}{6}m_1 + \frac{1}{3}m_2 + M_1\hat{s}_{24}^{*2},
\end{aligned}
$$

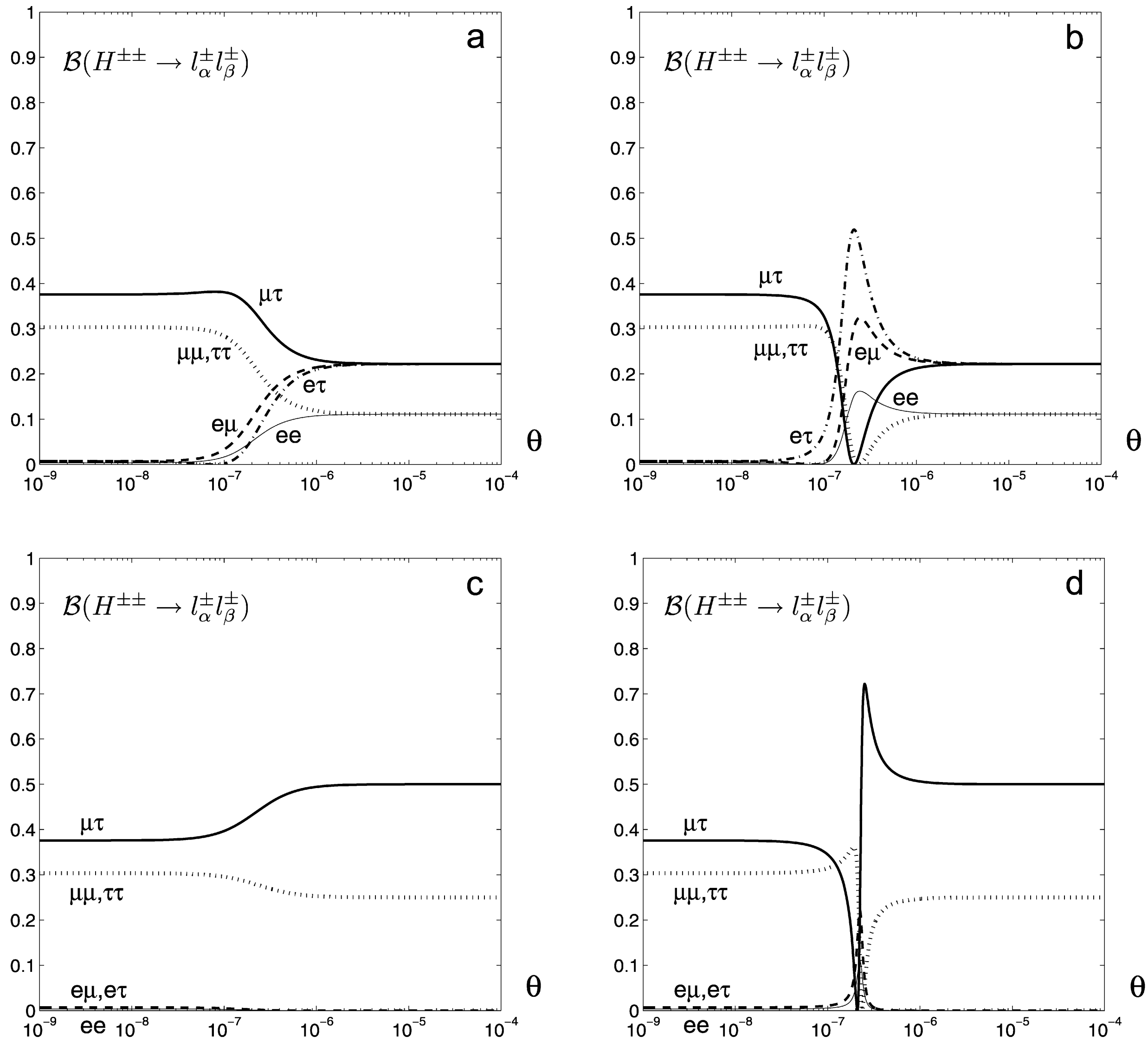

Fig. 1. Branching ratios of $H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm$ decays for the normal hierarchy of m_i with $m_1 = 0$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$; (d) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$.

$$
\begin{aligned}
(M_{\rm L})_{\tau\tau} &= \frac{1}{6}m_1 + \frac{1}{3}m_2 + M_1 \hat{s}_{34}^{*2};\\
(M_{\rm L})_{e\mu} &= \frac{1}{3}(m_2 - m_1) + M_1 \hat{s}_{14}^{*} \hat{s}_{24}^{*},\\
(M_{\rm L})_{e\tau} &= \frac{1}{3}(m_1 - m_2) + M_1 \hat{s}_{14}^{*} \hat{s}_{34}^{*},\\
(M_{\rm L})_{\mu\tau} &= -\frac{1}{6}m_1 - \frac{1}{3}m_2 + M_1 \hat{s}_{24}^{*} \hat{s}_{34}^{*},
\end{aligned}
\tag{16}
$$

and

$$
\sum_{\rho,\sigma} \left|(M_{\rm L})_{\rho\sigma}\right|^2 = \left(m_1^2 + m_2^2\right) + M_1^2\left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 + \frac{1}{3}m_1 M_1 \,{\rm Re}\left[(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34})^2\right] + \frac{2}{3}m_2 M_1 \,{\rm Re}\left[(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34})^2\right]. \tag{17}
$$

As a consequence of $m_1 \approx m_2$, the contributions of ν_1 and ν_2 are approximately canceled in $(M_{\rm L})_{e\mu}$ and $(M_{\rm L})_{e\tau}$. Our numerical results for the branching ratios $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ are shown in Fig. 2. Some comments and discussions are in order.

Fig. 2(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. We see that $\mathcal{B}(H^{\pm\pm} \to e^\pm\mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm\tau^\pm)$ are essentially degenerate in the whole parameter space, so are $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to \tau^\pm\tau^\pm)$. Different from other branching ratios, $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\tau^\pm)$ undergoes a minimum just because of the minimum of $|(M_{\rm L})_{\mu\tau}|$ at $\theta \sim 2 \times 10^{-7}$.

Fig. 2(b) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. In this case, the contribution of N_1 to $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ flips the sign such that $\mathcal{B}(H^{\pm\pm} \to \mu^\pm\tau^\pm)$ undergoes a maximum because of the minimum in its denominator. Due to the appearance of a minimum in its numerator, the branching ratio of $H^{\pm\pm} \to e^\pm e^\pm$, $\mu^\pm\mu^\pm$ or $\tau^\pm\tau^\pm$ undergoes a minimum when θ varies in the interference band.

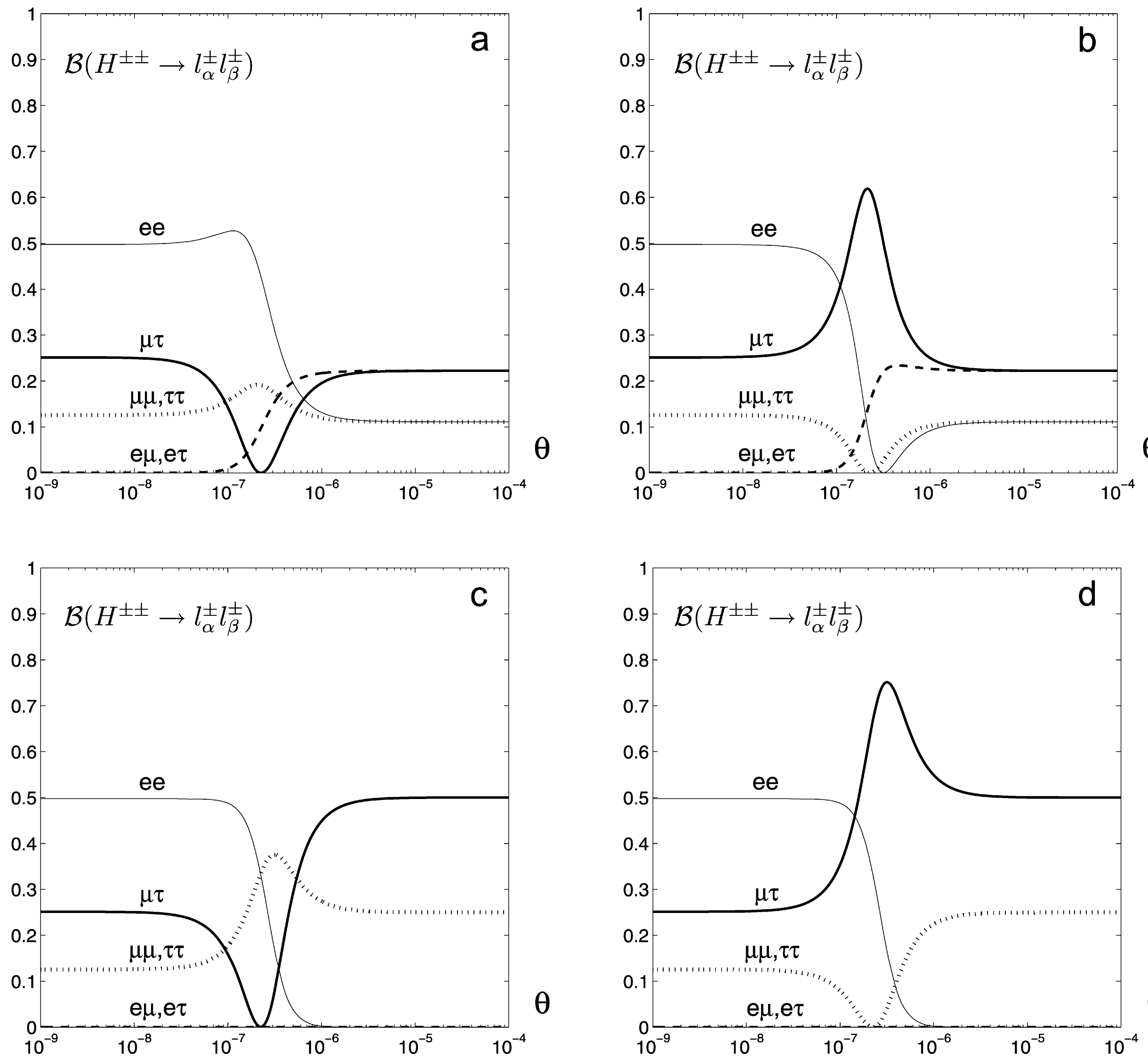

Fig. 2. Branching ratios of $H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm$ decays for the inverted hierarchy of m_i with $m_3 = 0$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$; (d) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$.

Fig. 2(c) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$. In this case, the contributions of N_1 to $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$, $\mathcal{B}(H^{\pm\pm} \to e^\pm \mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm \tau^\pm)$ are vanishing as a consequence of $\theta_{14} = 0$. Hence $\mathcal{B}(H^{\pm\pm} \to e^\pm \mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm \tau^\pm)$ are strongly suppressed in the whole parameter space, so is $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$ for $\theta > 10^{-6}$.

Fig. 2(d) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$. We see that the results of $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$, $\mathcal{B}(H^{\pm\pm} \to e^\pm \mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm \tau^\pm)$ in this case are essentially the same as those in Fig. 2(c). Because the contribution of N_1 flips the sign, now $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \mu^\pm) = \mathcal{B}(H^{\pm\pm} \to \tau^\pm \tau^\pm)$ undergoes a minimum while $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \tau^\pm)$ undergoes a maximum in the interference band.

3.3. Near degeneracy

We assume $m_1 \approx m_2 \approx m_3 \approx 0.1$ eV. Then $m_2 - m_1 \approx 4.0 \times 10^{-4}$ eV and $m_3 - m_2 \approx \pm 1.25 \times 10^{-2}$ eV can be extracted from given values of Δm_{21}^2 and $|\Delta m_{32}^2|$, respectively. For chosen values of θ_{12}, θ_{13}, θ_{23} and δ_{12}, δ_{13}, δ_{23}, Eqs. (10) and (11) can now be simplified to

$$(M_{\rm L})_{ee} \approx m_1 + M_1 \hat{s}_{14}^{*2},$$

$$(M_{\rm L})_{\mu\mu} \approx m_1 + \frac{1}{2}(m_3 - m_2) + M_1 \hat{s}_{24}^{*2},$$

$$(M_{\rm L})_{\tau\tau} \approx m_1 + \frac{1}{2}(m_3 - m_2) + M_1 \hat{s}_{34}^{*2};$$

$$(M_{\rm L})_{e\mu} \approx \frac{1}{3}(m_2 - m_1) + M_1 \hat{s}_{14}^{*} \hat{s}_{24}^{*},$$

$$(M_{\rm L})_{e\tau} \approx \frac{1}{3}(m_1 - m_2) + M_1 \hat{s}_{14}^{*} \hat{s}_{34}^{*},$$

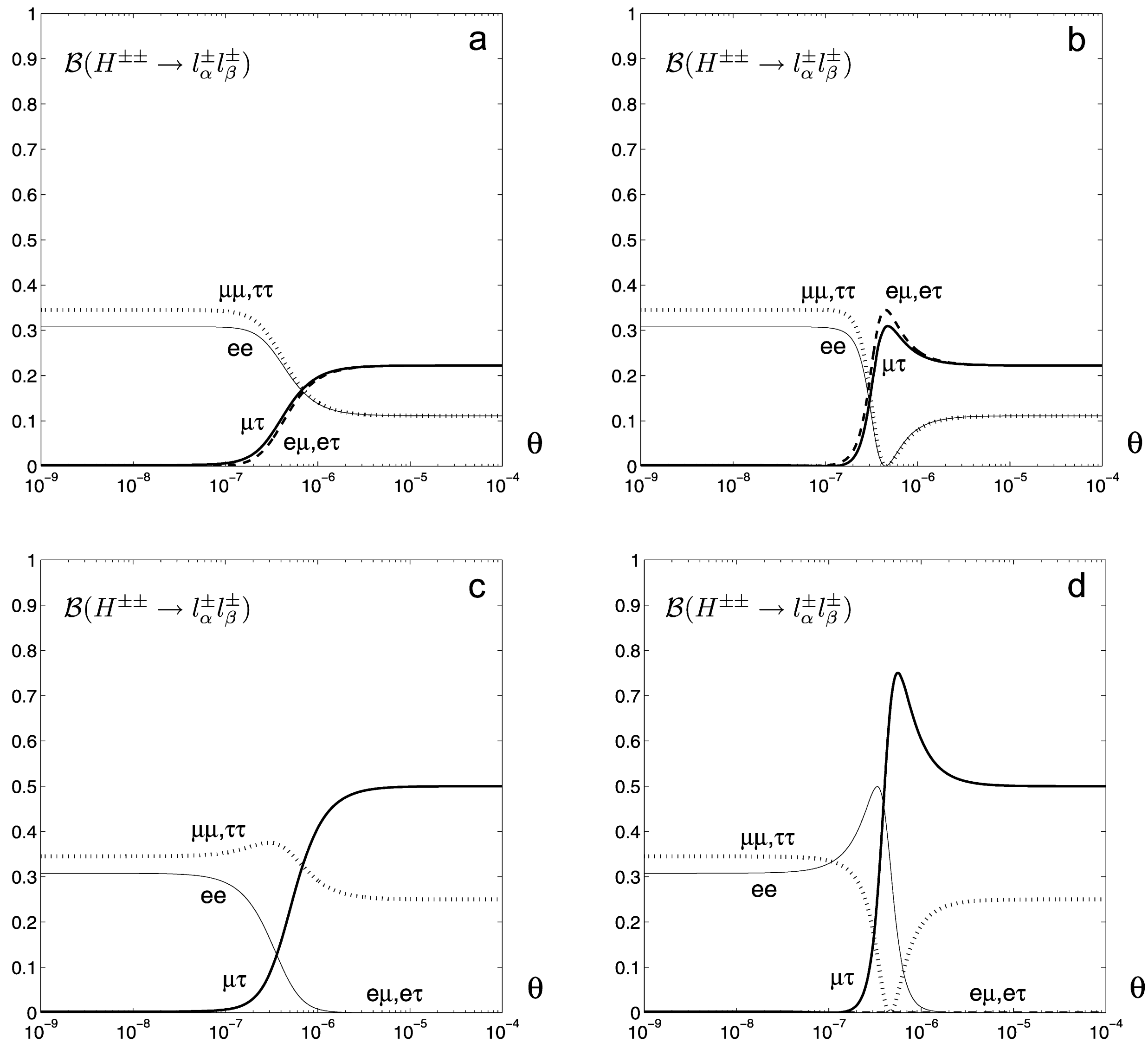

Fig. 3. Branching ratios of $H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm$ decays for the near degeneracy of m_i with $m_3 > m_2$: (a) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$; (b) $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$; (c) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$; (d) $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$.

$$(M_{\rm L})_{\mu\tau} \approx \frac{1}{2}(m_3 - m_2) + M_1 \hat{s}_{24}^* \hat{s}_{34}^*, \tag{18}$$

where we have neglected the small terms proportional to $m_2 - m_1$ in $(M_{\rm L})_{ee}$, $(M_{\rm L})_{\mu\mu}$, $(M_{\rm L})_{\mu\tau}$ and $(M_{\rm L})_{\tau\tau}$. In addition,

$$\sum_{\rho,\sigma} \left|(M_{\rm L})_{\rho\sigma}\right|^2 \approx 3m_1^2 + M_1^2 \left(s_{14}^2 + s_{24}^2 + s_{34}^2\right)^2 + \frac{1}{3} m_1 M_1 \,{\rm Re}\left[(2\hat{s}_{14} - \hat{s}_{24} + \hat{s}_{34})^2 + 2(\hat{s}_{14} + \hat{s}_{24} - \hat{s}_{34})^2 + 3(\hat{s}_{24} + \hat{s}_{34})^2\right], \tag{19}$$

where we have omitted the small mass differences of ν_i. We fix $m_3 > m_2$ in our numerical calculations. The results for the branching ratios $\mathcal{B}(H^{\pm\pm} \to l_\alpha^\pm l_\beta^\pm)$ are shown in Fig. 3. Some comments and discussions are in order.

Fig. 3(a) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = 0$. In this case, the near degeneracy of $\mathcal{B}(H^{\pm\pm} \to e^\pm \mu^\pm)$, $\mathcal{B}(H^{\pm\pm} \to e^\pm \tau^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \tau^\pm)$ is just because of the smallness of $m_2 - m_1$ and $m_3 - m_2$. A small discrepancy between $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \mu^\pm) = \mathcal{B}(H^{\pm\pm} \to \tau^\pm \tau^\pm)$ for $\theta < 7 \times 10^{-7}$ is due to the small terms proportional to $m_3 - m_2$ in $(M_{\rm L})_{\mu\mu}$ and $(M_{\rm L})_{\tau\tau}$.

Fig. 3(b) is obtained by taking $\theta_{14} = \theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{14} = \delta_{24} = \delta_{34} = \pi/2$. We see some mild interference effects in all the decay channels. Among them, the branching ratio of $H^{\pm\pm} \to e^\pm \mu^\pm$, $e^\pm \tau^\pm$ or $\mu^\pm \tau^\pm$ undergoes a maximum, while the branching ratio of $H^{\pm\pm} \to e^\pm e^\pm$, $\mu^\pm \mu^\pm$ or $\tau^\pm \tau^\pm$ undergoes a minimum.

Fig. 3(c) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = 0$. In this case, $\mathcal{B}(H^{\pm\pm} \to e^\pm \mu^\pm)$ and $\mathcal{B}(H^{\pm\pm} \to e^\pm \tau^\pm)$ are strongly suppressed in the whole parameter space. We see no obvious interference in other decay modes.

Fig. 3(d) is obtained by taking $\theta_{14} = 0$, $\theta_{24} = \theta_{34} \equiv \theta$ and $\delta_{24} = \delta_{34} = \pi/2$. One can see that $\mathcal{B}(H^{\pm\pm} \to e^\pm e^\pm)$ undergoes a maximum in the interference band, so does $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \tau^\pm)$. In comparison, $\mathcal{B}(H^{\pm\pm} \to \mu^\pm \mu^\pm) = \mathcal{B}(H^{\pm\pm} \to \tau^\pm \tau^\pm)$ undergoes a minimum. The interference effects in this case are more significant than those in Fig. 3(b).

4. Summary

We have studied the dileptonic decays of doubly-charged Higgs bosons $H^{\pm\pm}$ in the minimal type-II seesaw model with only one heavy Majorana neutrino and one Higgs triplet. Their branching ratios $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$ depend not only on the masses, flavor mixing angles and CP-violating phases of three light neutrinos ν_i (for $i = 1, 2, 3$), but also on the mass (M_1) and mixing parameters (θ_{i4} and δ_{i4}) of the heavy Majorana neutrino N_1. We have focused our attention on the interference bands of $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$, in which the contributions of ν_i and N_1 are comparable in magnitude. Assuming $M_1 \sim 200$ GeV–1 TeV and taking three possible mass patterns of ν_i as allowed by current neutrino oscillation data, we have figured out the generous interference bands $\sqrt{|\sin\theta_{i4} \sin\theta_{j4}|} \sim 10^{-8}$–$10^{-5}$ (for $i, j = 1, 2, 3$) and presented a detailed numerical analysis of $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$.

We stress that both constructive and destructive interference effects are possible in the interference bands of $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$, and thus it is very difficult to distinguish the (minimal) type-II seesaw model from the triplet seesaw model in this parameter space. Although our numerical results are subject to a simplified type-II seesaw scenario, they can serve as a good example to illustrate the interplay between light and heavy Majorana neutrinos in a generic type-II seesaw framework. The latter involves more free parameters, so the corresponding interference bands of $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$ will be in a mess.

It is worth pointing out that the lepton-number-violating decays of singly-charged Higgs bosons $H^{\pm}$ are also important for testing the gauge triplet nature of the Higgs field. For example, the observation of $H^{+} \to l^{+}_{\alpha} \bar{\nu}_{\alpha}$ and $H^{-} \to l^{-}_{\alpha} \nu_{\alpha}$ (for $\alpha = e, \mu, \tau$) decays will be particularly useful to determine the mass spectrum of three light Majorana neutrinos [10] because these processes are independent of the unknown Majorana phases in the triplet seesaw model. A similar study of the lepton-number-violating $H^{\pm}$ decays can be done in the type-II seesaw model, where heavy Majorana neutrinos exist, although the interference bands of $\mathcal{B}(H^{+} \to l^{+}_{\alpha} \bar{\nu}_{\alpha})$ and $\mathcal{B}(H^{-} \to l^{-}_{\alpha} \nu_{\alpha})$ are expected to be different from those of $\mathcal{B}(H^{\pm\pm} \to l^{\pm}_{\alpha} l^{\pm}_{\beta})$. We shall carry out a systematic analysis of both $H^{\pm\pm}$ decays and $H^{\pm}$ decays in the minimal type-II seesaw scenario elsewhere [17].

It is certainly a big challenge to identify the unique or correct seesaw mechanism of neutrino mass generation, if such a mechanism really exists, at the upcoming LHC and the future International Linear Collider. In particular, the collider signatures of both the Higgs triplet and heavy Majorana neutrinos will have to be experimentally established before a claim of having verified the type-II seesaw mechanism can be made. While the running of the LHC itself might be very difficult to help us pin down the true flavor dynamics of leptons and quarks, we hope that it would at least shed light on what this dynamics looks like at the TeV energy scale.

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