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Single field inflationary models with non-compact Kaluza–Klein theory

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Abstract

We discuss a semiclassical treatment to inflationary models from Kaluza–Klein theory without the cylinder condition. We conclude that the evolution of the early universe could be described by a geodesic trajectory of a cosmological 5D metric here proposed, so that the effective 4D FRW background metric should be a hypersurface on a constant fifth dimension.

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1. Introduction and motivation

In the last years cosmological models with extra dimensions has been studied by many authors [1] with different approaches. One of these is the space–time–matter theory (STM) developed by Wesson and co-workers [2], which is one of the versions of the Kaluza–Klein (KK) theory. There are three versions of the Kaluza's theory. The first one is known as compactified KK theory. In this approach, the Kaluza's cylinder condition is explained through a physical mechanism of compactification for the fifth dimension proposed by Klein. In the second version this condition is explained using projective geometry, in which the fifth dimension is absorbed into ordinary 4D space–time provided the (affine) tensors of general relativity are replaced with projective ones [3]. In the third version the cylinder condition is not imposed and there are no assumptions about the topology of the fifth dimension. This is the usual scenario in non-compact KK theories.

In the STM theory of gravity the 5D metric is an exact solution of the 5D field equations in apparent vacuum [4]. The interesting here, is that matter appears in four dimensions without any dimensional compactification, but induced by the 5D vacuum conditions. In this framework, the study of the early universe has great interest. The

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equivalence between STM theory and brane-world (BW) theories [5] has been studied recently [6]. In BW theories the usual matter in 4D is a consequence of the dependence of 5D metrics on the extra coordinate. If the 5D bulk metric is independent of the extra dimension, then the brane is void of matter. Thus in brane theory, matter and geometry are unified. In particular, in this Letter we are interested in the study of inflationary models from the STM formalism. Inflationary cosmology has been studied from the STM formalism for de Sitter (with a scale factor that evolves as $a(t) \sim e^{H_0 t}$) and power-law inflation (for $a \sim t^p$) [4,8] using respectively the metrics

$$dS^2 = \psi^2\, dt^2 - \psi^2 e^{2\sqrt{\Lambda/(3\psi^2)}t}\, dR^2 - d\psi^2, \tag{1}$$

$$dS^2 = \psi^2\, dt^2 - \psi^{2p/(p-1)} t^{2p}\, dR^2 - \frac{t^2}{(p-1)^2}\, d\psi^2. \tag{2}$$

In metric (1), the Hubble parameter is given by the cosmological constant and the fifth coordinate: $H_0^2 = \Lambda/(3\psi^2)$. As has been demonstrated, both metrics describe inflationary expansions on a 4D space–time embedded in a 5D manifold with ψ constant [9]. These 5D metrics on comoving spatial coordinates and constant ψ has an interval given by $dS = \psi\, dt$ [10]. This should be consistent with 4D particle dynamics, whose corresponding interval or action is defined by $ds = m\, dt$. So, if ψ is constant the rest mass m of a given particle should be constant in this particular frame. However, we could choose a frame in which ψ varies and hence the mass of the 4D particle were variable. For example, as was demonstrated in [10], by means of the 5D geodesic equation

$$\frac{dU^C}{dS} + \Gamma^C_{AB} U^A U^B = 0, \tag{3}$$

we can see that in the metric (2) we obtain the temporal dependence of the fifth coordinate: $\psi(t) = (t/t_0)^{(p-1)^2}$ when the spatial velocities $U^1 = U^2 = U^3 = 0$. Here, Γ^C_{AB} $(A, B, C = 0, 1, 2, 3, 4)$ are the 5D Christoffel symbols and the velocities are given by $U^A = dx^A/dS$. From the point of view of a 5D general relativity theory (which we are working here), it implies that the action is minimized in this particular frame.

The metrics (1) and (2) are the 5D extension of a 4D spatially isotropic, homogeneous and flat Friedmann–Robertson–Walker (FRW) space–time. These can be written in a more general manner [9]

$$dS^2 = -e^{\alpha(\psi,t)}\, dt^2 + e^{\beta(\psi,t)}\, dR^2 + e^{\gamma(\psi,t)}\, d\psi^2, \tag{4}$$

where $dR^2 = dx^2 + dy^2 + dz^2$ and ψ is the fifth coordinate. The equations for the relevant Einstein tensor elements are

$$G^0{}_0 = -e^{-\alpha}\left[\frac{3\dot\beta^2}{4} + \frac{3\dot\beta\dot\gamma}{4}\right] + e^{-\gamma}\left[\frac{3\overset{\star\star}{\beta}}{2} + \frac{3\overset{\star}{\beta}{}^2}{2} - \frac{3\overset{\star}{\gamma}\overset{\star}{\beta}}{4}\right], \tag{5}$$

$$G^0{}_4 = e^{-\alpha}\left[\frac{3\overset{\star}{\beta}}{2} + \frac{3\dot\beta\overset{\star}{\beta}}{4} - \frac{3\dot\beta\overset{\star}{\alpha}}{4} - \frac{3\overset{\star}{\gamma}\dot\gamma}{4}\right], \tag{6}$$

$$G^i{}_i = -e^{-\alpha}\left[\ddot\beta + \frac{3\dot\beta^2}{4} + \frac{\ddot\gamma}{2} + \frac{\dot\gamma^2}{4} + \frac{\dot\beta\dot\gamma}{2} - \frac{\dot\alpha\dot\beta}{2} - \frac{\dot\alpha\dot\gamma}{4}\right] + e^{-\gamma}\left[\overset{\star\star}{\beta} + \frac{3\overset{\star}{\beta}{}^2}{4} + \frac{\overset{\star\star}{\alpha}}{2} + \frac{\overset{\star}{\alpha}{}^2}{4} + \frac{\overset{\star}{\beta}\overset{\star}{\alpha}}{2} - \frac{\overset{\star}{\gamma}\overset{\star}{\beta}}{2} - \frac{\overset{\star}{\alpha}\overset{\star}{\gamma}}{4}\right], \tag{7}$$

$$G^4{}_4 = e^{-\alpha}\left[\frac{3\ddot\beta}{2} + \frac{3\dot\beta^2}{2} - \frac{3\dot\alpha\dot\beta}{4}\right] + e^{-\gamma}\left[\frac{3\overset{\star}{\beta}{}^2}{4} + \frac{3\overset{\star}{\beta}\overset{\star}{\alpha}}{4}\right], \tag{8}$$

where overstars and overdots denote respectively $\frac{\partial}{\partial\psi}$ and $\frac{\partial}{\partial t}$, and $i = 1, 2, 3$. Following the signature $(-, +, +, +)$ for the 4D metric, we define $T^0{}_0 = -\rho$ and $T^1{}_1 = \mathrm{p}$, where ρ is the total energy density and p is the pressure. The

5D-vacuum conditions ($G^A_B = 0$) are given by [10]

$$8\pi G\rho = \frac{3}{4} e^{-\alpha} \dot{\beta}^2, \tag{9}$$

$$8\pi G \mathrm{p} = e^{-\alpha}\left[\frac{\dot{\alpha}\dot{\beta}}{2} - \ddot{\beta} - \frac{3\dot{\beta}^2}{4}\right], \tag{10}$$

$$e^{\alpha}\left[\frac{3\overset{\star}{\beta}{}^2}{4} + \frac{3\overset{\star}{\beta}\overset{\star}{\alpha}}{4}\right] = e^{\gamma}\left[\frac{\ddot{\beta}}{2} + \frac{3\dot{\beta}^2}{2} - \frac{\dot{\alpha}\dot{\beta}}{4}\right]. \tag{11}$$

Hence, from Eqs. (9) and (10) and taking $\dot{\alpha} = 0$, we obtain the equation of state for the induced matter

$$\mathrm{p} = -\left(\frac{4}{3}\frac{\ddot{\beta}}{\dot{\beta}^2} + 1\right)\rho. \tag{12}$$

Notice that for $\ddot{\beta}/\dot{\beta}^2 \leqslant 0$ and $|\ddot{\beta}/\dot{\beta}^2| \ll 1$ (or zero), this equation describes an inflationary universe. If $\dot{\beta} = 2H_c$ (H_c is the classical Hubble parameter), the equality $\ddot{\beta}/\dot{\beta}^2 = 0$ corresponds with a 4D de Sitter expansion for the universe (metric (1)). Inflationary models like a de Sitter expansion or whose in which $H_c(t) \sim t^{-1}$ (metric (2)) can be studied by means of above approach [8]. However, chaotic inflation cannot be studied in this framework. The generalization of this formalism to inflationary models with potentials $V(\varphi) \sim \varphi^n$ is one of the aims of this Letter.

2. Formalism

In order to develop a different approach to the reviewed in the last section, we can propose the following metric to describe the universe

$$dS^2 = \psi^2\, dN^2 - \psi^2 e^{2N}\, dr^2 - d\psi^2. \tag{13}$$

Here, the parameters (N, r) are dimensionless and the fifth coordinate ψ has spatial unities. As can be demonstrated, the metric (13) describes a flat 5D manifold in apparent vacuum ($G_{AB} = 0$). In the metric (13) the parameter N could be a general function of t, r and ψ (and perhaps of additional coordinates χ_i with $i = 5, \ldots, n$ and $d\chi_i = 0$), but in this Letter we are going to study the particular case where N only depends on the cosmic time t: $N = N(t)$. Using Eqs. (9) and (10), we can calculate the vacuum solutions of the metric (13). We obtain the following expressions for the 4D induced pressure (p) and radiation energy density (ρ)

$$8\pi G \mathrm{p} = -3\psi^{-2}, \tag{14}$$

$$8\pi G\rho = 3\psi^{-2}. \tag{15}$$

It implies that all the matter (here described by ρ) is given by ψ. More exactly, as the metric (13) with $N = N(t)$ describes a extended spatially flat FRW metric, the results (14) and (15) indicate that $\psi^{-1}(N) = H_c(N)$, where $H_c(N)$ is the classical Hubble parameter (see Section 3). Note that the induced 4D equation of state give us a vacuum one $\mathrm{p} = -\rho$.

Before study some inflationary example we can discuss the properties of the metric (13). We consider the geodesic equations for the metric (13) in a comoving frame $U^r = \partial r/\partial S = 0$. The relevant Christoffel symbols are

$$\Gamma^N_{\psi\psi} = 0, \qquad \Gamma^N_{\psi N} = 1/\psi, \qquad \Gamma^{\psi}_{NN} = \psi, \qquad \Gamma^{\psi}_{N\psi} = 0, \tag{16}$$

so that the geodesic dynamics $\frac{dU^C}{dS} = \Gamma^C_{AB} U^A U^B$ is described by the following equations of motion for the velocities U^A

$$\frac{dU^\psi}{dS} = -\frac{2}{\psi} U^N U^\psi, \tag{17}$$

$$\frac{dU^N}{dS} = -\psi U^N U^N, \tag{18}$$

$$\psi^2 U^N U^N - U^\psi U^\psi = 1, \tag{19}$$

where Eq. (19) describes the constraint condition $g_{AB} U^A U^B = 1$. From the general solution $\psi U^N = \cosh[S(N)]$, $U^\psi = -\sinh[S(N)]$ (where $S(N) = N$), we obtain the equation that describes the geodesic evolution for ψ

$$\frac{d\psi}{dN} = \frac{U^\psi}{U^N} = -\psi \tanh\big[S(N)\big]. \tag{20}$$

If we define $\tanh[S(N)] = -1/p(N)$, we obtain

$$\psi(N) = \psi_0 e^{\int dN/p(N)} \tag{21}$$

for the velocities

$$U^\psi = -\frac{1}{\sqrt{p^2(N) - 1}}, \qquad U^N = \frac{p(N)}{\psi\sqrt{p^2(N) - 1}}, \tag{22}$$

where ψ_0 in Eq. (21) is a constant of integration. The resulting 5D metric is given by

$$dS^2 = dt^2 - e^{2\int H_c(t)\,dt}\, dR^2 - dL^2, \tag{23}$$

with $t = \int \psi(N)\, dN$, $R = r\psi$ and $L = \psi_0$ for $H_c(t) = 1/\psi(t)$. With this representation, we obtain the following velocities U^A:

$$U^T = \frac{2p(t)}{\sqrt{p^2(t) - 1}}, \qquad U^R = \frac{r}{\sqrt{p^2(t) - 1}}, \qquad U^L = 0. \tag{24}$$

The solution $|S| = \mathrm{arctanh}[1/p(t)]$ corresponds to a power-law expanding universe with time dependent power $p(t)$ for a scale factor $a \sim t^{p(t)}$. Since $H_c(t) = \dot{a}/a$, the resulting Hubble parameter is

$$H_c(t) = \dot{p} \ln(t/t_0) + p(t)/t, \tag{25}$$

where t_0 is the initial time.

From the above results we can propose that the universe was born in a state with $S \simeq 0$ (i.e., in a vacuum state $\mathrm{p} \simeq -\rho$) and evolved through the geodesic $|S| = \mathrm{arctanh}[1/p(t)]$ in a comoving frame $dr = 0$, such that the effective 4D space–time is a FRW metric

$$dS^2 = dt^2 - e^{2\int H_c(t)\,dt}\, dR^2 - dL^2 \to ds^2 = dt^2 - e^{2\int H_c(t)\,dt}\, dR^2. \tag{26}$$

Note that L depends on the initial value of ψ: $L = \psi_0$. In this framework we can define the 5D Lagrangian

$$\mathcal{L}(\varphi, \varphi_{,A}) = -\sqrt{-{}^{(5)}g}\left[\frac{1}{2} g^{AB} \varphi_{,A}\varphi_{,B} + V(\varphi)\right], \tag{27}$$

for the scalar field $\varphi(N, r, \psi)$ with the metric (13). Here, ${}^{(5)}g$ is the determinant of the 5D metric tensor in (13) and $V(\varphi)$ is the potential. On the geodesic $|S| = \mathrm{arctanh}[1/p(t)]$ in the comoving frame $dr = 0$, the effective Lagrangian for the metric (26) is

$$\mathcal{L}(\varphi, \varphi_{,A}) \to \mathcal{L}(\varphi, \varphi_{,\mu}) = -\sqrt{-{}^{(4)}g}\left[\frac{1}{2} g^{\mu\nu} \varphi_{,\mu}\varphi_{,\nu} + V(\varphi)\right], \tag{28}$$

where ${}^{(4)}g$ is the determinant of the metric tensor in the 4D effective FRW background metric (26) and $\varphi \equiv \varphi(t, R)$. In this frame the energy density and the pressure, are

$$8\pi G\rho = 3H_c^2, \tag{29}$$

$$8\pi G\mathrm{p} = -\big(3H_c^2 + 2\dot{H}_c\big), \tag{30}$$

with $H_c(t) = \dot{a}/a$ for a given scale factor $a(t) \sim t^{p(t)}$.

3. An application: semiclassical chaotic inflation

The inflationary universe scenario asserts that, at some very early time, the universe went through a superluminical expansion with a scale factor growing as $a \sim t^{p(t)}$ (with $p \gg 1$). Inflation is needed because it solves the horizon, flatness and monopole problems of the very early universe and also provides a mechanism for the creation of primordial density fluctuations. For these reasons it is an integral part of standard cosmological model.

To illustrate the results of the last section we can develop a semiclassical treatment [13] to a chaotic inflationary model [11] with a potential

$$V(\varphi) = \frac{m^2}{2}\varphi^2 + \frac{\lambda^2}{24}\varphi^4, \tag{31}$$

where m is the mass of the inflaton field and $\lambda \ll 1$ describes the self-interaction. The equation of motion for φ and the Friedmann equation (in an effective 4D FRW metric (26)), are

$$\ddot{\varphi} + 3H\dot{\varphi} - \frac{1}{a^2}\nabla^2\varphi + V'(\varphi) = 0, \tag{32}$$

$$H^2 = \frac{8\pi}{3M_p^2}\left\langle \frac{\dot{\varphi}^2}{2} + \frac{1}{2a^2}(\nabla\varphi)^2 + V(\varphi)\right\rangle. \tag{33}$$

We can make a semiclassical treatment [7] for the scalar field $\varphi = \phi_c(t) + \phi(\vec{R}, t)$, where $\phi_c(t) = \langle\varphi\rangle$ and the small inflaton fluctuations are zero-mean-valued $\langle\phi\rangle = 0$. If the cosmological constant Λ is given by

$$\Lambda = \frac{2m^2}{9} + \frac{4m^4\pi}{\lambda^2 M_p^2} + \frac{\lambda^2 M_p^2}{18^2\pi}, \tag{34}$$

the classical Hubble parameter will be related with the classical potential through the Einstein equation [12]

$$V(\phi_c) = \frac{3M_p^2}{8\pi}\left[H_c^2 - \frac{M_p^2}{12\pi}(H_c')^2 - \Lambda\right],$$

where $M_p = G^{-1/2}$ is the Planckian mass and $H_c = \frac{8\pi}{3M_p^2}[\dot{\phi}_c^2/2 + V(\phi_c)]$. The classical Hubble parameter for the potential (31) is given by

$$H_c(\phi_c) = \frac{\lambda}{3M_p}\sqrt{\pi}\phi_c^2 + \frac{2m^2\pi^{1/2}}{\lambda M_p} + \frac{\lambda M_p}{18\pi^{1/2}}. \tag{35}$$

However, the effective Hubble parameter H, is given by the expression [12] (there is a little mistake in [12]—the correct expression is the following)

$$H(t) = H_c\left[1 + \frac{4\pi}{3H_c^2}\left\langle \frac{\dot{\phi}^2}{2} + \frac{1}{2a^2}(\nabla\phi)^2 + \sum_{n=1}\frac{V^{(n)}(\phi_c)}{n!}\phi^n\right\rangle\right], \tag{36}$$

where we denote $H_c \equiv H_c(\phi_c) = \dot{a}/a$ and $V^{(n)}(\phi_c) \equiv \frac{dV(\varphi)}{d\varphi}|_{\phi_c}$. If the inflaton fluctuations are small, we can make a first order expansion on ϕ for $V(\varphi)$, and the following approximation is valid

$$H = H_c\left[1 + \frac{4\pi}{3H_c^2}\left\langle\frac{\dot{\phi}^2}{2} + \frac{1}{2a^2}(\nabla\phi)^2\right\rangle\right]. \tag{37}$$

The treatment of H in the context of semiclassical inflation is very problematic because the terms inside the brackets include back-reaction effects [14]. As was demonstrated by Nambu, back-reaction effects are different on super-Hubble and sub-Hubble scales. On sub-Hubble scales such that effects are important and the effective curvature is increases, but on super-Hubble scales the consequences of back-reaction are no very important. For this reason, the standard approximation that appears in the literature (see, for example, [12,13,15]) consists on making $H = H_c$, because $(\nabla\phi)^2/a^2$ and $\langle\dot{\phi}^2\rangle$ become negligible on cosmological scales at the end of inflation. For simplicity, in this Letter we adopt this approximation.

Since $\dot{\phi}_c = -\frac{4\pi}{M_p^2}H_c'$, we can describe the temporal evolution for the spatially homogeneous component of the inflaton field

$$\phi_c(t) = \phi_0 e^{-\frac{\lambda M_p}{6\sqrt{\pi}}t}, \tag{38}$$

where ϕ_0 is $\phi_c(t_0)$. If we replace (38) in the expression for the Hubble parameter (35), we obtain its temporal dependence

$$H_c(t) = \frac{\lambda\sqrt{\pi}\phi_0^2 e^{-\frac{\lambda M_p}{3\sqrt{\pi}}t}}{3M_p} + \frac{2m^2\sqrt{\pi}}{\lambda M_p} + \frac{\lambda M_p}{18\sqrt{\pi}}, \tag{39}$$

such that, for a scale factor that evolves as $a(t) \sim t^{p(t)}$ (i.e., $a(N) \sim e^N$ in the representation (13)), we obtain the differential equation

$$\dot{p}(t)\ln(t/t_0) + p(t)/t = H_c(t). \tag{40}$$

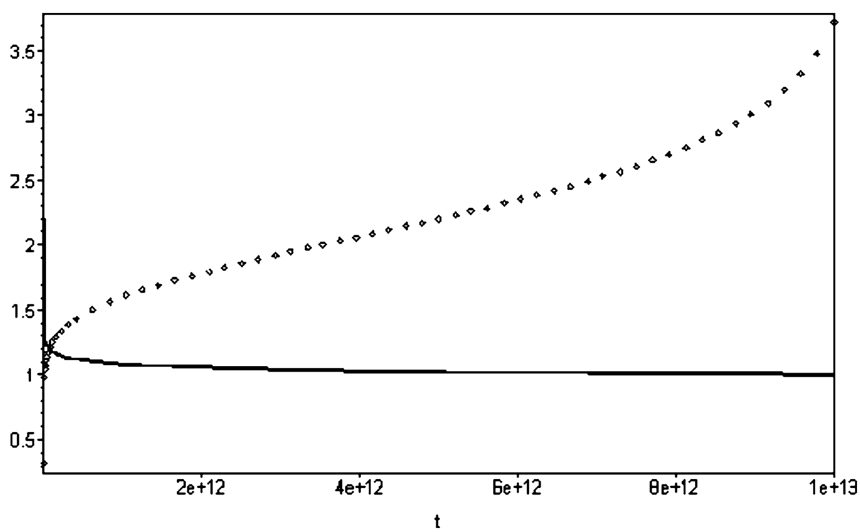

Fig. 1. Evolution of $|S(t)|$ (dotted line) and $p(t)$ (continuous line).

Here, $H_c(t)$ is given by Eq. (39). If we replace Eqs. (39) in (40), we obtain the temporal evolution for $p(t)$

$$p(t) = \frac{18CM_p^2\lambda\sqrt{\pi} + t(36m^2\pi M_p + \lambda^2 M_p^3) - 18\pi^{3/2}\phi_0^2\lambda e^{-\frac{\lambda M_p}{3\sqrt{\pi}}t}}{18M_p^2\lambda\sqrt{\pi}\ln(t/t_0)}, \tag{41}$$

where C is a dimensionless constant of integration. Note that the last term in the numerator approaches to zero before inflation ends.

In Fig. 1 we show $|S(t)|$ (dotted line) and $p(t)$ (continuous line) for $m = 0.8 \times 10^{-17} M_p$ (i.e., 1.5×10^2 GeV), $\lambda = 10^{-15}$, $C = 30$ and $\phi_0 = 0.1 M_p$. Note that $p(t) \to 1$ at the end of inflation (i.e., for $t \simeq 10^{13} M_p^{-1}$), but $|S(t)|$ increases from its initial value $S(t_0) = 0$. The interesting here is that the mass value of the inflaton field agrees with the expected for the Higgs mass: $M_{\mathrm{Higgs}} \simeq 150$ GeV [16].

4. Final comments

We have developed a cosmological model from non-compact Kaluza–Klein theory, in which the evolution of the early universe is described by a geodesic trajectory $|S(N)| = \mathrm{arctanh}[1/p(N)]$ in a comoving frame $dr = 0$ of a 5D metric

$$dS^2 = \psi^2\,dN^2 - \psi^2 e^{2N}\,dr^2 - d\psi^2,$$

such that, by means of the transformation $t = \int \psi\,dN$, $R = r\psi$ and $L = \psi_0$, the resulting 5D background metric for $\psi = H_c^{-1}$ is described by

$$dS^2 = dt^2 - e^{2\int H_c(t)\,dt}\,dR^2 - dL^2,$$

which give us an effective 4D FRW background metric

$$ds^2 = dt^2 - e^{2\int H_c(t)\,dt}\,dR^2,$$

on the hypersurface $L = \psi_0$. In this model, the 4D effective dynamics is governed by the temporal evolution of the fifth dimension. Physical properties such as the mean energy density and pressure of matter are well-defined consequences of how the extra coordinate enters the metric. That is, matter is explained as the consequence of geometry in five dimensions.

To illustrate the model we have studied a chaotic inflationary model with $p(N) > 1$ for a massive inflaton field which is self-interacting. An interesting result is that the mass of the inflaton field here obtained ($m = 1.5 \times 10^2$ GeV), agrees quite well with the expected value for the Higgs mass [16]. Of course, the method could be applied to other inflationary models with potentials $V(\varphi) \sim \varphi^n$. Moreover, the formalism also could be developed for more general cosmological models where $|S(N)| = \mathrm{arctanh}[1/p(N)]$ would give us the evolution of the universe from its creation to the present epoch. For example, a cosmological model in which the universe evolves from a "big bounce" was considered in [17]. However, this issue go beyond the scope of this Letter.

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