Fault-Tolerant Backstepping Attitude Control for Autonomous Airship with Sensor Failure

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Abstract

In order to solve the question of attitude stabilization of autonomous airship during spot hovering, this paper proposes a fault-tolerant backstepping control law. The strict-feedback discrete-time dynamics model of airship is obtained via Euler approximation. The discrete control law is designed by using the backstepping method to make the system uniformly stable. By applying two-level strong track filters, which can detect and isolate step bias or time-varying bias sensor fault in both single fault or simultaneous faults scenario, a fault-tolerant control law is constructed. The simulation results show that the method can achieve expected tracking performance and robustness.

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Keywords: Airship; hovering; attitude stabilisation; backstepping; strong track filter

1. Introduction

Autonomous high-altitude airship is a new type of near space vehicle, which has broad application prospects in space development, earth observation and communication platform building, causing extensive research of domestic and foreign scholar. There is a lot of key technology to be realized for autonomous high-altitude airship. One of the most important techniques is flight control system. Some contributions on dynamic modeling and control have been reported in recent years. A dynamic model was built based on analysis of gravity, buoyancy, aerodynamic damp and thrust by use Newton motion law in [1~3]. A backstepping-based controller was designed for attitude and velocity in [4~6], and got good

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control results. In [7, 8], a robust backstepping controller considering disturbance and input saturation is designed. In [9], based on error model of the nonlinear system of airship, a feedback controller is designed, which tracked expected altitude well and was robust for disturbance.

All the above research did not consider the sensor or actuator failures. For this reason, this paper proposes a new fault-tolerant control method for airship attitude tracking problem during hover, which applies two-level strong tracking filters (STF) to detect and isolate the sensors’ faults, and uses backstepping control law to obtain control action by the estimates of the state variables. The approach is simulated in matlab in no-fault case and sensor fault case, respectively.

2. Mathematical model of airship

The airship is constructed as an ellipsoid filling with helium. Two vector thrusters are deployed at stern of an axisymmetric airship hull with 4 equally sized tail fins symmetrically. Based on the research in [1~3], the mathematical model of airship attitude is expressed in state formulation:

\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\psi} \\
\dot{\phi} \\
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} = \begin{bmatrix}
-r \sin \phi + q \cos \phi \\
\frac{1}{\cos \theta} (r \cos \phi + q \sin \phi) \\
p + \tan \theta (r \cos \phi + q \sin \phi) \\
(I_{zz} - I_z (I_y - I_z)) (q - R) - I_{zz} (I_z + I_y - I_z) p q + I_z Z_G \cos \theta \sin \phi + L_x \\
(I_z - I_x) r p - I_{zz} (p^2 - r^2) - Z_G \sin \phi + L_y \\
I_{zz} (I_z + I_y - I_z) r q - (I_{zz} + I_z (I_x - I_y)) p q + I_{zz} Z_G \cos \theta \sin \phi + L_z
\end{bmatrix}
\]

where \( M = \text{diag}\{I_{zz}, -I_x, I_z + I_{zz}, I_x, -I_x, I_z + I_{zz}\} \) is the coefficient matrix, \( I_x, I_y \) and \( I_z \) are moments of inertia about OX, OY and OZ respectively, \( I_{zz} \) is the product of inertia about OY, \( L_x, L_y \) and \( L_z \) are the rolling, pitching and yawing moments.

Define \( x_1 = [\theta \ \psi \ \phi]^T \), \( x_2 = [p \ q \ r]^T \), \( I_1 = -I_x I_z + I_{zz}^2 \), and the model will be shown as

\[
\dot{x}_1 = g_1(x_1)x_2 \\
\dot{x}_2 = f(x_1, x_2) + g_2(x_1, x_2)u
\]

where \( g_1 = \text{diag}\{(-I_x I_z + I_{zz}^2)^{-1}, -I_y, (-I_x I_z + I_{zz}^2)^{-1}\} \), and

\[
g_2 = \begin{bmatrix}
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi & \cos \phi \cos \gamma \\
1 & \tan \gamma \sin \phi & \tan \gamma \cos \phi
\end{bmatrix}
\]

3. Backstepping control law

3.1. Discrete-time backstepping control law

Via Euler approximations, the discrete-time model of airship is given by

\[
\begin{align*}
x_1(k+1) &= x_1(k) + Tg_1(x_1(k), x_2(k)) + v_1(k) \\
x_2(k+1) &= x_2(k) + T(f(x_1(k), x_2(k)) + g_2(x_1(k), x_2(k))u(k)) + v_2(k) \\
y(k+1) &= [x_1(k+1)^T \ \ x_2(k+1)^T]^T + e(k+1)
\end{align*}
\]
where \( x_1(k) \in \mathbb{R}^3 \) and \( x_2(k) \in \mathbb{R}^3 \) are the state vectors of the system, \( u(k) \in \mathbb{R}^3 \) is the input vector, and \( y(k) \in \mathbb{R}^3 \) is the measurement vector. \( v, e \in \mathbb{R}^6 \) are Gaussian white noise sequences of zero-mean and covariance matrices are \( Q_i \) and \( R, T \leq 0.05 \) is the sampling cycle.

Backstepping control is based on Lyapunov theory and can be used for strict feedback systems. It is recursive procedure, and is derived in a vectorial setting through two steps.

**Step 1:** Define the tracking error variable \( z_1(k) = x_1(k) - x_{id}(k) \). Equations (3) give

\[
 z_1(k + 1) = x_1(k) + Tg_1(x_1(k), \xi(k))x_2(k) - x_{id}(k + 1)
\]

(4)

The Backstepping idea is to choose \( x_2 \) as the virtue input vector:

\[
 z_2(k + 1) = T^{-1}g_1^{-1}(x_1(k), \xi(k))(-x_1(k) + k_1z_1(k) + x_{id}(k + 1))
\]

(5)

with \( 0 < k_1 < 1 \).

**Step 2:** Define the tracking error variable \( z_2(k) = x_2(k) - x_{id}(k) \). Equations (3) give

\[
 z_2(k + 1) = x_2(k) + T[f(x_1(k), x_2(k), \xi(k)) + g_2(x_1(k), x_2(k), \xi(k))u(k)] - x_{id}(k + 1)
\]

(6)

Now the input vector can be expressed as:

\[
 u(k) = [Tg_2(x_1(k), x_2(k), \xi(k))]^{-1}[-x_1(k) - TF(x_1(k), x_2(k), \xi(k)) + k_2z_2(k) + x_{id}(k + 1)]
\]

(7)

where

\[
 x_{id}(k + 1) = [Tg_1(x_1(k + 1), \xi(k + 1))]^{-1}(-x_1(k + 1) + k_1z_1(k + 1) + x_{id}(k + 2))
\]

(8)

which can be predicted by (2).

3.2. Stability analysis

**Theorem 1** Assume that \( g_i \) is bounded. Under the above control law (5), (7) and (8), \( k_1, k_2 \) and \( \varepsilon \) satisfy inequalities (9):

\[
\begin{align*}
 k_1^2 - 1 + k_1 T \varepsilon &< 0 \\
 k_2^2 - 1 + k_2 T \lambda_m \frac{1}{\varepsilon} + T^2 \lambda_m^2 &< 0
\end{align*}
\]

(9)

is asymptotically stable equilibrium of closed-loop nonlinear discrete-time system (3).

**Proof:**

Construct Lyapunov function

\[
 V(k) = z_1^T(k)z_1(k) + z_2^T(k)z_2(k)
\]

(10)

The difference between \( V(k + 1) \) and \( V(k) \) is

\[
 \Delta V = V(k + 1) - V(k) = z_1^T(k + 1)z_1(k + 1) - z_1^T(k)z_1(k) + z_2^T(k + 1)z_2(k + 1) - z_2^T(k)z_2(k)
\]

(11)

By (4), (6), (7) and (8), (11) becomes

\[
 \Delta V = (k_1^2 - 1)z_1^T(k)z_1(k) + (k_2^2 - 1)z_2^T(k)z_2(k) + 2k_1 T z_1^T(k)g_1z_1(k) + T^2 z_1^T(k)g_1^T g_1 z_1(k) + T z_2^T(k)g_2^T g_2 z_2(k)
\]

(12)

Because \( g_1^T g_1 > 0 \), and by inequality \( 2z_1^T(k)z_1(k) \leq \varepsilon z_1^T(k)z_1(k) + \frac{1}{\varepsilon} z_2^T(k)z_2(k) \) ( \( \varepsilon \) is a proper constant), (11) becomes

\[
 \Delta V \leq (k_1^2 - 1)z_1^T(k)z_1(k) + (k_2^2 - 1)z_2^T(k)z_2(k)
\]

\[
 + k_1 T \varepsilon z_1^T(k)z_1(k) + k_2 T \frac{1}{\varepsilon} z_2^T(k)g_1^T g_1 z_1(k) + T^2 z_1^T(k)g_1^T g_2^T g_2 z_1(k)
\]

\[
 \leq (k_1^2 - 1 + k_1 T \varepsilon)z_1^T(k)z_1(k) + (k_2^2 - 1 + k_2 T \lambda_m \frac{1}{\varepsilon} + T^2 \lambda_m^2)z_2^T(k)z_2(k)
\]

(13)
where $k_1$, $k_2$ and $\varepsilon$ are unknown coefficients, $\lambda_\text{m}$ is the maximal eigenvalue of $g_i^T g_i$. Because $T$ is very small, if $k_1$, $k_2$ and $\varepsilon$ satisfy (9), $\Delta V \leq 0$. Applying Lyapunov stability theorem, asymptotical stability is proved.

4. STF-based FDI method

4.1. Strong Track Filter (STF)

STF has the following good properties: 1) low sensitivity to the statistics of the initial states and the statistics of the system or measurement noise, 2) strong tracking ability to the suddenly changing states, 3) acceptable computational complexity. The detailed steps for STF algorithm are described in [10].

4.2. FDI Design

Consider the sensor faults and define a new state vector $x_e = [x_1^T \ x_2^T \ y_0^T]^T$, system (3) is denoted as

$$
\begin{align*}
    x_e(k+1) &= \begin{bmatrix} T[g_i(x_1(k), x_2(k)) x_2(k) + T[f_2(x_1(k), x_2(k))] + & v_1(k) \\
    g_2(x_1(k), x_2(k))(u(k) + \delta(k)) & v_2(k) \\
    y_0(k) & 0 \end{bmatrix} \\
    y(k+1) &= [I \ 0]^T x_e(k+1) + [e(k+1)^T \ 0]^T
\end{align*}
$$

(14)

For (15), this paper proposed a FDI architecture shown in Fig.1.

![Fig. 1. The architecture of FDI](image)

Based on (3), STF1 is used to estimate the state vector. For (16), STF2 estimates the state variables and sensors’ faults. The supervisor module use the weighted sum of squares residuals (WSSR) to detect fault. The FDI method can be stated as:

**Step 1:** STF1 outputs the residuals of state variables, and supervisor module uses them to compute WSSR. If $\text{WSSR} < \sigma$, go to step 3;

**Step 2:** STF2 outputs the estimates of state variables and faults $\hat{y}_0$. If $\hat{y}_0 < \gamma$, $\hat{x}(k+1 | k+1) = \hat{x}(k+1 | k)$; If $\hat{y}_0(i) > \gamma$, the $i$th sensor is faulty. Replace the faulty sensor’s measurement value $y(i)$ with corresponding prior estimate of STF1 $\hat{x}(k+1 | k)$, and re-compute the posterior estimate of STF1.

**Step 3:** the supervisor module outputs posterior estimate of STF1— $\hat{x}(k+1) = \hat{x}(k+1 | k+1)$. 


In this way, the fault-tolerant backstepping controller can be denoted as follows:

$$u(k) = [T g_z (\hat{x}_1(k), \hat{x}_2(k))]^{-1}[-\dot{\hat{x}}_1(k) - T f_z (\hat{x}_1(k), \hat{x}_2(k)) + k_2z_2(k) + x_{id}(k+1)]_{k_{i1}(k)}$$  \hspace{1cm} (15)

5. Simulation

For an experimental airship, the values of moment of inertia are

$$I_x = 833.222 \text{kg m}^2; I_y = 13229.521 \text{kg m}^2; I_z = 12856.753 \text{kg m}^2; I_{xz} = 1047.665 \text{kg m}^2$$

The initial values of state variables are:

$$\begin{bmatrix} \theta \\ \psi \\ \phi \end{bmatrix} = [0.1, 0.1, 0.1] \text{ rad}, \quad \begin{bmatrix} p \\ q \\ r \end{bmatrix} = [0.2, 0.2, 0.2] \text{ rad/s}$$

The desired attitude vector is denoted as:

$$x_{id} = [1 - e^{-0.372t}(\sin(0.372t) + \cos(0.372t))] [0.5, 0.7, 0.8]$$

The controller and STF parameters used in the simulations are given in table 2.

**Table 1. Parameters of Controller and STF**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$k_1$</th>
<th>$k_2$</th>
<th>$\varepsilon$</th>
<th>$\rho$</th>
<th>$\beta$</th>
<th>$\sigma$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>0.95</td>
<td>1</td>
<td>0.0015</td>
<td>[0.01, 0.01, 0.01]</td>
</tr>
</tbody>
</table>

The process noise is modeled by using white noise with a variance of 0.0001^2 rad^2, and the measurement noise is the same as process noise.

In the simulations, two scenarios are considered. One is done for the airship without sensor fault, and the other is for the simultaneous faults scenario.
The attitude response is shown in Fig. 2 (a). Clearly, the tracking performance of the controller is very good. The tracking error is limited to about 0.00015 rad, and the controller responds quickly. In the sensor fault scenario, a time-varying sensor bias fault about $t/10000T$ rad/s in output channel $q$ and a step bias fault about 0.3 rad/s in output channel $r$ are added between $t=10$ and $t=16$ seconds. The attitude response without FDI module is shown in fig. 2 (b), and with FDI module in fig. 2 (c). By comparing the results, it can be observed that the tracking performance of backstepping control with FDI module is better, when sensors are faulty simultaneously. The fault estimates are shown in Fig. 2. (d). One can easily detect and isolate the fault according to estimations.

6. Conclusion

Based on the dynamic equations of airship, the mathematics model of attitude under hover mode is built. The backstepping control law is designed for the discrete-time nonlinear system, and the uniform stability is proved by Lyapunov theory. By applying two-level STF to diagnose and isolate sensors’ fault, a fault-tolerant control is constructed. The simulation results show the control method can track desired attitude, and is robust for both step bias and time-varying bias sensor fault.

References