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SYMMETRY AND NOTATION: REGULARITY AND SYMMETRY IN NOTATED COMPUTER GRAPHICS

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Abstract—A comprehensive cognitive method for dealing with shapes and their formal organisation in visual art is derived from the model of movement which finds its expression in Eshkol-Wachman Movement Notation. In this application, shapes are conceived as being swept out by movements which are analysed and recorded in the notation. This method is quantitative and ideally suited for computer input, while the computer is ideally suited for carrying out in detail and with precision the instructions compactly expressed in the symbol code of the notation. Using computer and notation together, regularities and symmetries can be observed in displayed shapes or sequences of shapes, and in their notation; some of these exemplify static symmetry, others dynamic symmetry, and others again, combinations of as-symmetry in individual shapes together with symmetry among variants of the motif, related through the (symmetrical) structure of the process from which they arise. Families of transformations of a motif can be generated, which possess a unity that may be intuitively perceptible and will always be objectively verifiable.

FRAMES OF REFERENCE

Figurative representation in visual art, besides serving ritual, pictorial or anecdotal ends, also provides a unifying framework. By reference to, and comparison with, the "real" physical world, shapes are understood through their degree of conformity with the accepted laws of that world and to the expectations aroused by the evident adoption of a realistic style. The extent and kinds of transformation of a shape are limited to those which allow its identification as a representation of the same physical object wherever variants of the shape occur. The organisation of areas so as to "copy" nature entails rules governing the way they may obscure one another and their relative sizes, and the colour partitions of a given organisation of a picture space are confined to those which suggest natural scenes. This is a well-tried method of preserving unity, but its rules are derived from only a restricted area of visual form and formative processes.

When more "abstract" forms are employed and there is no figurative or specifically associative intention, unity and coherency are often provided through the use of patterns created by the more or less regular organisation of motifs. To ensure intelligibility these are often rather familiar and simple regular shapes of the kind frequently called "geometrical"—i.e. circles, rectangles etc. This imposes limits upon the repertoire of shapes which can serve as motifs. Given these circumstances, any greater subtlety can only be obtained by resorting to improvisation over which there is reduced conscious control.

Whether the approach is realistic or abstract, the formal aspect reveals a greater or less degree of planning, varying from simple decorative repetition to sophisticated conscious design rich in suggestion. This planning is *the way in which the constituent formal elements are related to the whole*, and this will serve, indeed has served, as a first definition of what is meant by symmetry. Without symmetry in this general sense, there can be no design[1].

This rather schematic survey of alternative ways of regarding or ignoring design is perhaps enough to suggest that in visual art there is no generally accessible, comprehensive cognitive method for dealing with shapes and their formal organisation—one in which all intentional forms and transformations are admissable.

The concepts of shape which form the basis for what follows, are derived from the model of movement in three-dimensional space which finds its expression in Eshkol–Wachman Movement Notation. This method, devised for human movement, transcends limitations which had dogged previous attempts at formulating a useful system in that field. An appropriately modified version of this system is perfectly suited to shape description, where too its effectiveness is unhampered by gratuitous extraneous restrictions.

ESHKOL–WACHMAN MOVEMENT NOTATION AND SHAPE DESCRIPTION

In EW movement notation, the movements of a single axis are equated with the easily recognised simple shapes they produce in space, or with rotation about its own length—a movement which produces no shape[2,3]. The movements of chains of two or more articulated axes might appear much harder, or even impossible, to define. But in fact, by treating each axis as if it moved in isolation, the notation enables us to express unambiguously the compound paths of a chain of simultaneously moving axes. Entire three-dimensional *space chords* are defined, using only the three types of movement: plane, conical and rotatory. Symbols are assigned to these types, which together with numerals indicating quantities, constitute the symbolic expression of movements. It is not difficult to conceive intuitively the basic forms, and the corresponding movements of a line which sweeps them out; and it is as easy to pass from shape to movement as it is to pass from movement to shape. This easily grasped *two-way* correspondence is crucial in the present application. Not only is the equivalence effective in the analysis of movement by identifying paths of movements with the shapes of the paths, but the elementary shapes can be identified with the movements which produce them. These *shapes* can therefore be expressed using the associated *movement* symbols (see Fig. 1).

Still using this equivalence, we can, by indicating simultaneous movements of a chain of articulated axes, give unambiguous expression to the compound shape traced by them. Indeed it is possible to define the shape swept out by any line moving in two dimensions or in three. Such a shape will often bear little resemblance to the basic shapes from which it is synthesized and to which it can be reduced. The examples in what follows are all two-dimensional, but the ideas are equally applicable in three dimensions.

Armed with the concepts expressed in the language of movement notation, it becomes possible to discourse intelligibly about components of visual experience encountered in visual art, such as the shapes of lines and areas, including unfamiliar figures not normally dealt with in geometry. We can define the way in which they vary from place to place and from instant to instant, and the ways in which they interact under varying conditions. The introduction of real movement in explicitly "kinetic" work is indicated in the notation by means of only quite minor differences of writing, and the phenomena which can then be dealt with include observable *processes* of growth and decay, displacement, and arrays of shapes interacting with one another[4–6].

With a proper notation all shapes, including unfamiliar or complex ones, are clear, and it is not necessary to be restricted to obviously regular shapes in order to maintain a firm frame of reference. When shapes and their transformations are defined with precision, our sensitivity to them is greatly increased.

The use of the notation gives practicability to compositional approaches which have not in the past been associated with visual art, but which seem to be aspired to in the more recent systemic and serialist work. The use of movement notation favours an approach wherein processes of growth and transformation are chosen which entail the unfolding of what is latent in the form of a given motif. "Variety within unity" is preserved by a means which constitutes an exploration of what is involved, visually and structurally, in having made an initial choice.

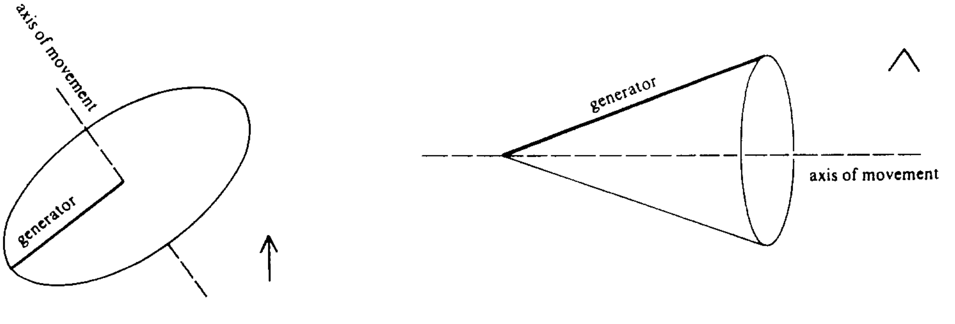


Fig. 1. (a) Movement of the generating axis at an angle of 90° to the axis of movement, producing a plane. The symbol is an arrow. (b) Movement of the generator at an angle of less than 90° to the axis of movement, producing a conical shape. The symbol is a circumflex.

This choice may have been made according to personal preference, a figurative analogy, narrative-pictorial idea, or any other urge or fancy. A particular preference is conscious inasmuch as it is based upon knowledge of the alternatives. The advantage of this way of preserving continuity is that being neither arbitrarily superimposed nor a result of improvisation based upon habit, a formally appropriate procedure can reveal whatever is formally valid in the original idea, untrammelled by preconceived notions of what it *should* be. With even quite a little practical experience, one passes easily from one formal interpretation of the basic idea to another, and at the same time the notation provides a thread which can be confidently retraced through the labyrinth of possible exploratory developments of a motif.

COMPUTER LANGUAGE AND A LANGUAGE FOR SHAPE

EW movement notation is then a tool exact enough to be practical and at the same time flexible enough to be used in all visual media. Complex sequences of shape and movement expressed in it can be carried out intuitively with some accuracy by experienced practioners; this is in fact the way EW notation is used in the performance of dance by human dancers. The precise execution of shapes, using exact measurements and instruments such as compass and straightedge, however, requires calculations which are not necessarily very difficult, but usually very repetitive. This is the lowest (but not insignificant) level at which the microprocessor now provides ideal new instruments for the visual artist.

Instructions formulated in movement notation symbols are primarily humanly oriented, making possible the holistic grasp of basically complex instructions; at the same time, the conciseness and absence of ambiguity of this notation make it an ideal medium for inputting data to a computer. The unfolding in all its detail, of the explicit display of what is implicit in the data, is carried out when a suitably programmed computer processes the compactly expressed symbol code of the notation. The automated realisation of composed shape in a visual display will often be a decisive factor in determining the feasibility of projects which might not be practicable working by hand alone. A computer with high resolution graphics facility can almost instantly display shapes, so that variants of a shape or a sequence can be tried out—at speeds which could never be attained by hand—before embarking on their manual execution in a chosen medium. By using the computer to review systematically the consequences of choosing certain defined conditions, we can explore in detail and in an acceptable length of time, many more strands of development than it would otherwise be reasonable to undertake; and it will always be possible to refer these back to the symbols of the notation through which they were generated[4,7].

Since one of the effects of using a notation is to encompass more extensive structure than the unaided immediate memory could cope with, the combination of movement notation and computer provides an instrument which can serve both as an extension of the hand and of the scope of the mind. As an instrument in its own right, the computer can be treated as a means of performing compositions, using the monitor screen as an active surface upon which real events take place, not a pictorial representation of the world but part of it. The conditions are thus provided for a kinetic visual art composed in a symbolic language adapted specifically for use with visual images[8].

Assuming that a microcomputer is the sort of machine most likely to be accessible to the potential user, the remaining discussion and examples will be directly related to the use of a BBC(B) microcomputer programmed in BASIC. The software can of course be adapted and expanded for use with a large computer[9].

As already described, we regard shape as the path swept out by a line segment or a series of articulated segments—a chain, free at one end. The components of a movement generating a shape are then:

(1) *Length.* The links have specified lengths, and these may change during the movement, or while in a position.

(2) *Sense of movement.* A link may move positively, or negatively. Positive movement in a plane is notated $\uparrow$ and for present purposes will be assumed to be always counterclockwise. Negative (clockwise) movement is notated $\downarrow$.

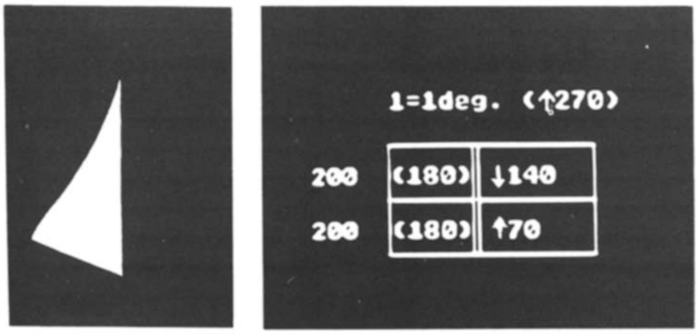


Fig. 2. A shape is generated on the screen, when the data has been keyed in and displayed on the screen as shown.

(3) *Amount of movement.* This is the angular change of position of a link *in relation to the link which carries it.* This magnitude is absolute only in the case of the first link—i.e. the one which "carries" all the others. (The amount can be written in units of a specified number of degrees, but here we shall use one degree as the unit.)
(4) *Time.* When the linkage consists of more than one link, the synchronisation of their movements is vital in determining the total shape swept out; this applies even in the case of a static result. The values are given in terms of time units, seen as a number of vertical columns.

Apart from the positive and negative signs ↑ and ↓ for sense and + and − for length, these components are expressed as numerical quantities.

The program used allows for several modes of interactive operation. The simplest of these, intended for the user familiar with the concepts but not with their symbolic representation in the notation, prompts the supply of data through a dialogue, beginning with the questions:
How many movements? [i.e. the number of movements comprising the sequence.]
Increment?
Number of links? [in the articulated chain of generators]
Length of first link?
. . . and so on, until all the necessary data has been provided, whereupon the screen is cleared and the shapes are produced upon it.

Alternatively, for the reader of movement notation, after the first three questions, the screen is treated as a ready-ruled page upon which movement notation is typed from the terminal keyboard. Again, once the "score" is complete, the display is cleared and the notated shapes are generated on the screen.

Finally, the user can, if he wishes, recall the data entered, facilitating changes or repetitions of the same shapes or sequence. The program can also be used to return points in x and y for plotting when the computer is being used as an adjunct to manually executed work.

Figure 2 shows a simple shape, together with its notation. The two links of length 200 form a linkage; both are initially in position (180)—i.e. vertically upward on the screen. The link which has one end at the origin occupies the lowest space of the score; it is seen to move positively (counterclockwise) through 70 degrees. At the same time, the link which it carries (since the two are articulated), moves in *negative* sense through 140 degrees in relation to the link by which it is carried. The shape shown is the trace swept out by the two generating links in their simultaneous movements[4–6].

REGULARITY, SYMMETRY

In Figure 3 the shape of Fig. 2 is repeated, together with its mirror image, shown this time as a sequence of discrete positions of the two links, demonstrating how the trace is formed. That the two traces are indeed reflectively symmetrical could be seen by physically cutting out the figure and folding it, or using the computer to check as many points x, y as we wish,

Fig. 3.

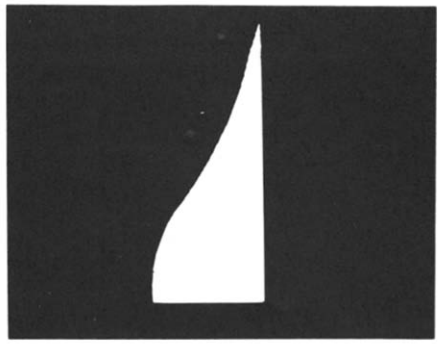


Fig. 4.

Fig. 3. The shape shown in Fig. 2, with its mirror image displayed as a sequence of positions. Fig. 4. The notation of this shape expresses its similarity to, and difference from, that of Fig. 2.

for distance from the starting position. Or, by observing that the notation for the right-hand trace is

200	(180)	$\uparrow$140
200	(180)	$\downarrow$70

(a)

That is to say that the movements, starting from the same positions, are different only in *sense* of movement; amount of movement and all else, remain the same. The notation thus not only provides the instructions for the generation of the shapes, and conversely an exact description of them, but also reflects the fact of their symmetry.

Figure 4 shows a figure whose similarity to, and difference from, the first, can be seen in the notation:

200	(180)	$\downarrow$180
200	(180)	$\uparrow$90

(b)

A trace which is reflectively symmetrical will be generated as before, by inverting the senses. Reflective symmetry about the *end* position will be produced by continuing the movements in the same sense and through the same amount of movement. The presence of reflective symmetry is revealed if we write this as two separate traces in opposite senses away from the "middle" position (axis of reflective symmetry):

200	(270)	$\uparrow$180
200	(90)	$\downarrow$90

(c)

200	(270)	$\downarrow$180
200	(90)	$\uparrow$90

(d)

If the movement is continued in the same sense through a complete cycle (See Figure 5.), i.e.:

200	(180)	↑720
200	(180)	↓360

(e)

then resulting trace is found to have reflective symmetry about two axes, and also two-fold rotational symmetry in the plane. This can be verified by moving the completed figure as a rigid shape ↑180 and ↑360 about its central origin. (A completed path of movement may be treated as a "frozen trace" which can be moved in ways which are notated in the score[4].) These symmetries could of course be classified in terms of type and degree of symmetry without using EW notation, but this would indicate nothing about the specific way in which they were generated.

The notation can be used to indicate, and the computer to execute, transfer of the relation between the linkage and the fixed origin, resulting in displacement of the "frozen trace". See Figs. 6–9, which exhibit three-, four-, six- and eight-fold rotational symmetry. By the addition of an "empty" ("pen up") carrying link, any displacement can be expressed. The "carrying" link in Fig. 6 has the movement sequence ↑120 ↑120 ↑120; in Fig. 7, ↓90 ↓90 ↓90 ↓90 . . . and similarly with the others.

Orthogonal translation is effected by change of length of the "empty" carrying link, as in Figs. 10–12. The motif shown in Fig. 10(a) allows for close packing, and an infinitely extendable tessellation is obtained by regularly adding carrying links and increasing the number of repetitions of the motif; see Fig. 10(b). This is presented in horizontally and vertically alternating black and white. Figures 11 and 12 show orthogonal translation of another motif; in Fig. 12 this is produced by the simultaneous movements of *two* empty carrying links:

200	(270)	↑360
200	(270)	↓180

(f)

But here the displacements are not equally spaced. Neither must regular angular displacements in simple plane movement of a single link necessarily be of equal magnitude, or necessarily produce self-coincidence. In Fig. 13 the seven appearances of the motif are placed by movements of unequal (but related) size, of the empty carrying link. In Fig. 14 the successive repetitions of the motif are at unequally increasing distances from the origin. In Fig. 15 both distances and movements are determined by this unequal series, which is the well-known Fibonacci series 1, 1, 2, 3, 5, 8, 13 . . . Here we have passed from instances of "static" symmetry, which is dependent upon division of the whole into even multiple parts, to dynamic symmetry; we describe both in terms of shape and movement, without need of changing the adopted frame of reference. Static symmetry may be seen as a special case of dynamic symmetry; but while the first can be used either consciously or intuitively, dynamic symmetry can hardly be used intuitively. "The world," said Hambidge, "cannot always regard the artist as a mere medium who reacts blindly, unintelligently, to a productive yearning. There must come a time when instinct will work with, but be subservient to, intelligence."[1].

We saw earlier how a shape is the result of a given amount of positive or negative movement

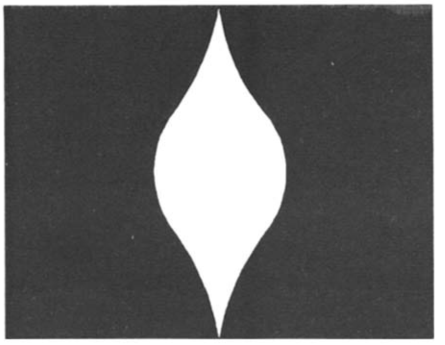


Fig. 5. The preceding trace is continued through a complete cycle.

of a generating link (or links) of constant or changing length about the origin. The specific shape is determined by the attribution of values to these components. A patterned sequence of values may be uniform, or it may have the regularity of a non-monomial progression such as we saw in Figs. 13 to 15; or a more complex regularity may be generated, as Joseph Schillinger suggested[10], by the combination of simple types of periodicity. For example, a sequence such as 2 1 1 2 may be formed by the synchronisation of the two monomial periodicities 2 and 3—the simplest relation which yields a non-monomial result. By distributive involution, a more extended series is developed: 4 2 2 4 2 1 1 2 2 1 1 2 4 2 2 4, preserving the same ratios between the groups as obtained between the single values of the original motif[10].

We now interpret the series as the (total) *length* of the generator at the start of each movement, and as *amount of movement* of a single generating axis, taking 10° as the unit; thus:

Length of generator: 4 2 2 4 . . . etc.
Movement of generator: ↑ 4 2 2 4 . . . etc.

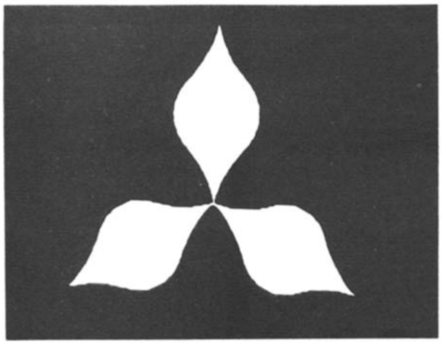


Fig. 6.

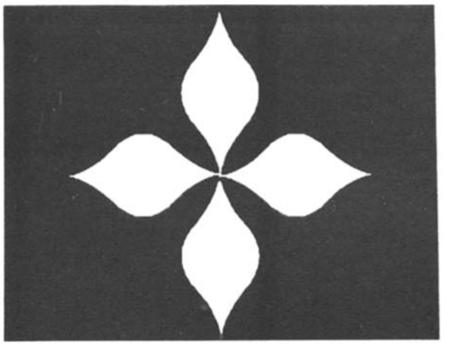


Fig. 7.

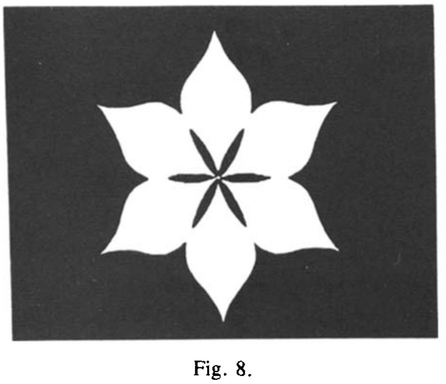


Fig. 8.

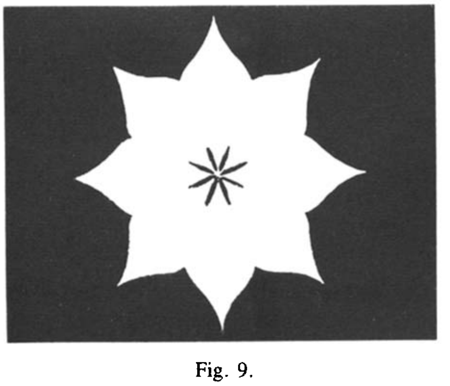
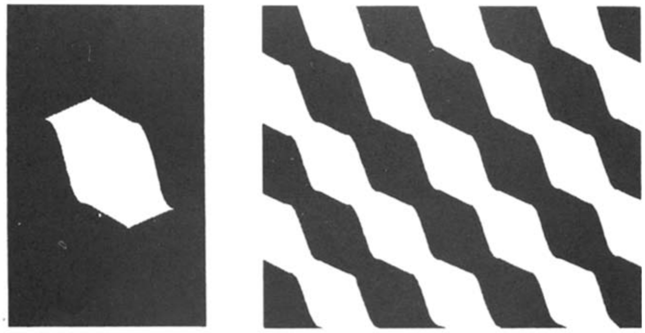


Fig. 9.

Fig. 6–9. These four figures exhibit displacements of the motif of Fig. 5, resulting in three-, four-, six- and eight-fold symmetry.



(a) (b)

Fig. 10. The motif (a) is shown in (b) translated by the changing lengths of (unseen) additional carrying links, and displayed alternately as black and as white.

If we next apply circular permutation to the series of lengths, a family of shapes is obtained—schematically:

Lengths (constant):	4 2 2 4 2 1 1 2 2 1 1 2 4 2 2 4
Positive movement:	4 2 2 4 2 1 1 2 2 1 1 2 4 2 2 4
	2 2 4 2 1 1 2 2 1 1 2 4 2 2 4 4
	2 4 2 1 1 2 2 1 1 2 4 2 2 4 4 2
	⋮
	4 4 2 2 4 2 1 1 2 2 1 1 2 4 2 2
	4 2 2 4 2 1 1 2 2 1 1 2 4 2 2 4

(See Figure 16, progressing from left to right, top to bottom.)

We may now make the following observations about this sequence:

(1) The parts of the series of values, as already remarked, are interrelated according to a defined ratio—i.e. the 2 1 1 2 motif is preserved, and the extended series too has symmetry about its mid-point:

4 2 2 4 2 1 1 2 | 2 1 1 2 4 2 2 4
axis

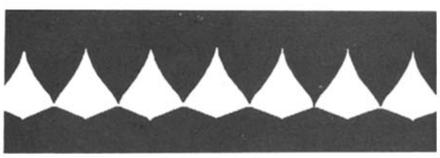


Fig. 11. Orthogonal translation of a motif.

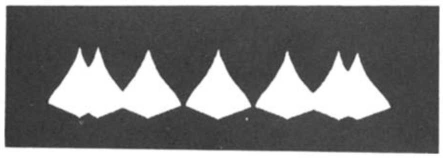


Fig. 12. Orthogonal translation of the motif of Fig. 11, produced by the simultaneous movement of two (unseen) carrying links.

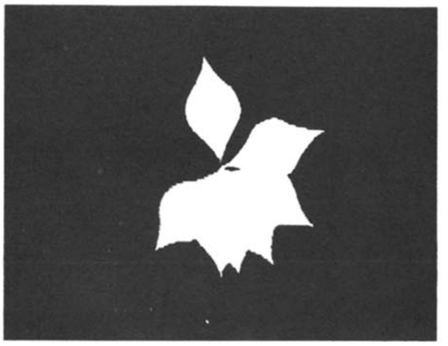


Fig. 13. The motif is displaced by movements of unequal angular size.

(2) The series is also interpreted as initial lengths of the moving generator—therefore also as the final lengths arrived at with the close of each movement. Thus the *movement* is concurrent with a *change of length* from one term of the series to the next. Synchronising the two series in this specific manner, we obtain shapes none of which is itself symmetrical in the sense of being in any way capable of being brought into self-coincidence by movement as a rigid form.

(3) Considering the whole sequence of shapes, we find the following: Two adjacent pairs (only) in the sequence are symmetrical in relation to each other, in that reflective symmetry is exhibited by the two middle shapes, and the first and last—which we may call adjacent in view of the cyclic nature of the sequence. Other pairs of reflectively symmetrical shapes are distributed (symmetrically) on either side of the two middle terms. Thus, the successive shapes are phases in a process which produces reflective symmetry over 16 terms.

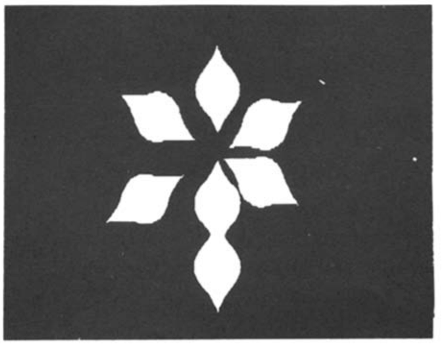


Fig. 14. The motif is displaced by movements of equal size but unequally increased distances from the origin.

Fig. 15. Distances and movements are unequal.

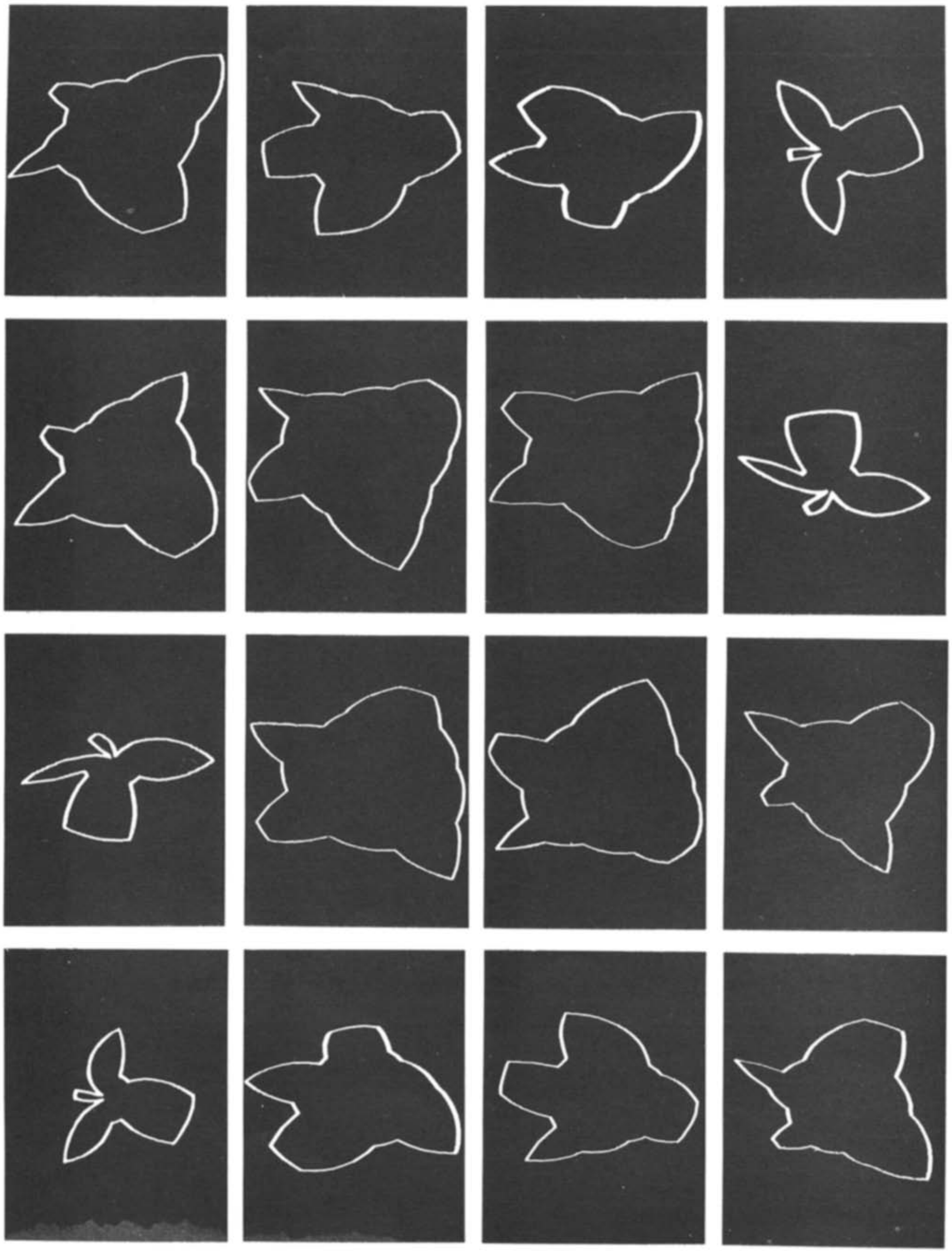


Fig. 16. Applying synchronisation of periodicities, distributive involution and circular permutation, a sequence of sixteen cycles is developed (above). The order is from left to right, top to bottom.

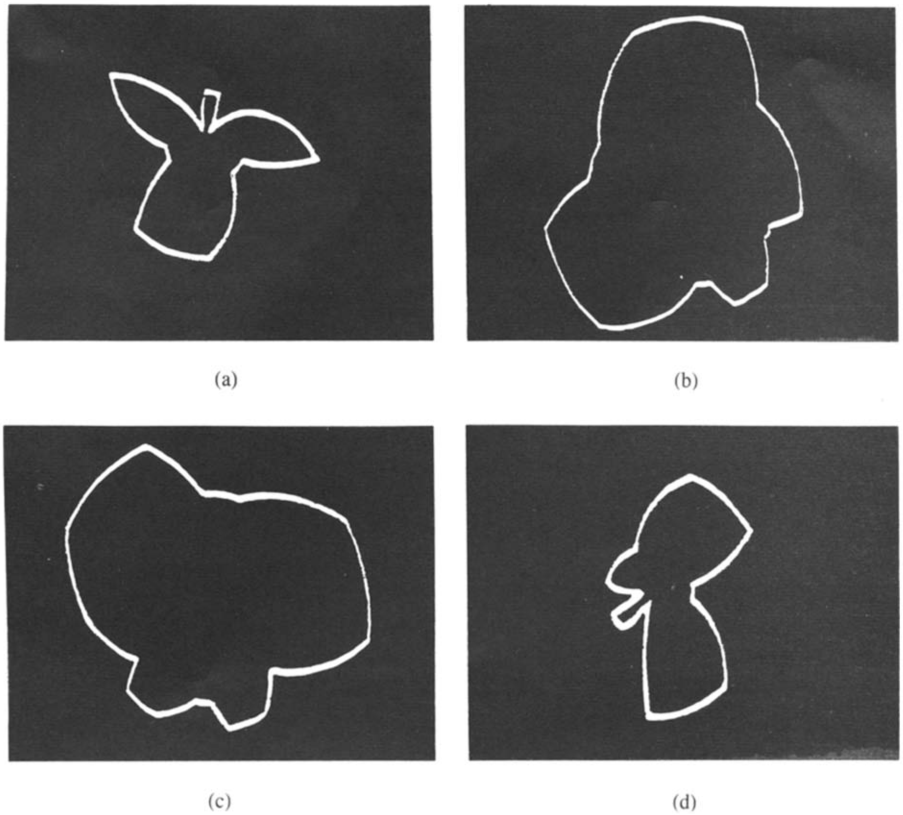


Fig. 17. Four variations resulting from raising to the second power circular permutations of the basic motif of Fig. 16.

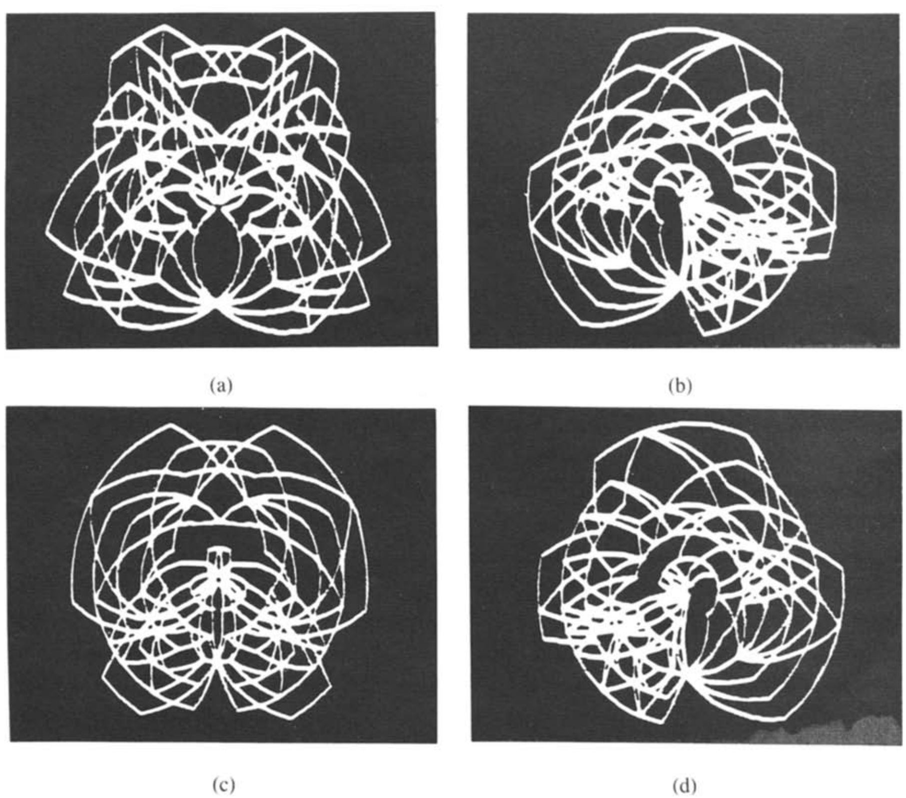


Fig. 18. When the "family" of each variant of Fig. 17 is displayed in superimposition, the symmetrical relations between and within them are clearly perceptible.

Circular permutation of the basic motif 2 1 1 2, when raised to the second power, yields four related series including the one we have just discussed:

2 1 1 2, raised to the second power =
4 2 2 4 2 1 1 2 2 1 1 2 4 2 2 4 . . . a (Fig. 17(a)),
1 1 2 2, raised to the second power =
1 1 2 2 1 1 2 2 2 2 4 4 2 2 4 4 . . . b (Fig. 17(b)),
1 2 2 1, raised to the second power =
1 2 2 1 2 4 4 2 2 4 4 2 1 2 2 1 . . . c (Fig. 17(c)),
2 2 1 1, raised to the second power =
4 4 2 2 4 4 2 2 2 2 1 1 2 2 1 1 . . . d (Fig. 17(d)).

These four series can be interpreted as we have described, to generate four clearly different shapes including the original: Fig. 17; and each can be developed into a distinct "family" like that of Fig. 16. It will be noticed that the series of values (c) has symmetry similar to that of (a) (the original form), whereas (b) and (d) have not; (b) and (d), however, are seen to be symmetries of each other. These characteristics of the four series are very clearly seen if all the cycles of each are retained on the screen, as in Fig. 18. They can be further verified by obtaining points in x and y as mentioned previously.

Thus not only is each cycle in such a sequence of permutations a stage arrived at and departed from; each sequence is in turn also the starting point for new departures.

SUMMARY

We have found that regularities of and between shapes can be explored by means of the computer and movement notation, by virtue of the detailed quantitative approach which has thereby been made possible. Symmetries are uncovered not all of which are conveniently classifiable in terms of static (intuitive) symmetry. Families of transformations of a motif can be generated as successive cycles in a sequence which is itself regular, and in some cases [like (a) and (c) above] symmetrical. Neither the individual variants of the motif nor the relations between them necessarily possess symmetry in themselves, but the regularity of the whole, and the proportions preserved in the passage from shape to shape bestow a unity which may be intuitively perceptible, and is always objectively verifiable.

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