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The forbidden minor characterization of line-search antimatroids of rooted digraphs

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Abstract

An antimatroid is an accessible union-closed family of subsets of a finite set. A number of classes of antimatroids are closed under taking minors such as point-search antimatroids of rooted (di)graphs, line-search antimatroids of rooted (di)graphs, shelling antimatroids of rooted trees, shelling antimatroids of posets, etc. The forbidden minor characterizations are known for point-search antimatroids of rooted (di)graphs, shelling antimatroids of rooted trees and shelling antimatroids of posets. In this paper, we give the forbidden minor characterization of line-search antimatroids of rooted digraphs.

Keywords: Antimatroid; Forbidden minor; Line graph; Line-search antimatroid

1. Introduction

Various kinds of shelling procedures give rise to a class of combinatorial structures called antimatroids, which were introduced by Edelman [2] and Jamison-Walder [5]. Antimatroids can be seen as a combinatorial abstraction of convexity, while matroids can be seen as a combinatorial abstraction of linear independence. Antimatroids are related to matroids in that both can be defined by an apparently similar axioms. This close relationship between antimatroids and matroids provides a lot of interesting properties of antimatroids. For example, antimatroids can be characterized by a greedy algorithm

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like matroids [1]. Note that one of the authors has recently given a greedy-algorithmic characterization of non-simple antimatroids, which is an extension of antimatroids [9].

Both antimatroids and matroids are subclasses of greedoids introduced by Korte–Lovász [6]. See [8] for details and various examples of greedoids. In greedoid theory, some classes are characterized by their forbidden minors: local poset greedoids [7]; undirected branching greedoids [3,13], and poset-shelling antimatroids and point-search antimatroids of rooted (di)graphs [10]. In this paper, we give the forbidden minor characterization for line-search antimatroids of rooted digraphs.

Note that there are still other antimatroids whose forbidden minor characterizations have not been known yet, for example, line-search antimatroids of rooted undirected graphs.

2. Preliminaries

2.1. Antimatroids

Let E be a non-empty finite set, and let $\mathscr{F}$ be a family of subsets of E such that

$$\emptyset \in \mathscr{F}, \quad E \in \mathscr{F}, \tag{1}$$

$$\text{if } X \in \mathscr{F} \setminus \{\emptyset\}, \text{ then there exists an } e \in X \text{ such that } X \setminus \{e\} \in \mathscr{F}, \tag{2}$$

$$\text{if } X, Y \in \mathscr{F}, \text{ then } X \cup Y \in \mathscr{F}. \tag{3}$$

Then we call $(E, \mathscr{F})$ an *antimatroid* on E. When there is no risk of confusion, we use $\mathscr{F}$ instead of $(E, \mathscr{F})$. Each element of $\mathscr{F}$ is called a *feasible set*.

For an antimatroid $\mathscr{F}$, a *minor* $\mathscr{F}[A, B]$ is defined as follows:

$$\mathscr{F}[A, B] = \{X \setminus A \colon X \in \mathscr{F}, A \subseteq X \subseteq B\}, \tag{4}$$

where $A, B \in \mathscr{F}$ and $A \subseteq B$. We can easily check that each minor of an antimatroid is also an antimatroid.

2.2. Point-search antimatroids of rooted digraphs

A *digraph* G is a pair (V, E) such that V is a non-empty finite set of *vertices*, and E is a subset of $\{(x, y)\colon x, y \in V, x \neq y\}$ called a set of *edges*. For simplicity, we write xy instead of (x, y). For an edge $xy \in E$, x is called the *tail*, and y is called the *head*.

A *path* P in $G = (V, E)$ is a sequence of vertices $x_1 x_2 \cdots x_m$ with $x_i x_{i+1} \in E$ for $i = 1, \ldots, m-1$. A path $P = x_1 \cdots x_m$ is also called a path from x_1 to x_m. For a path $P = x_1 \cdots x_m$, if there exists an edge $x_i x_j \in E$ $(i+1 < j)$, then the edge $x_i x_j$ is called a *short cut* of the path P. A path without repeated vertices is called *elementary*. An elementary path without any short cuts is called *straight*.

A *rooted digraph* is a triple $G = (V, E, r)$, where $(V \cup \{r\}, E)$ is a digraph and r is a specified vertex called the *root* such that there exists a path from r to every vertex of V. A path from the root r is called a *rooted path*. A vertex v is called an *atom* if $rv \in E$.

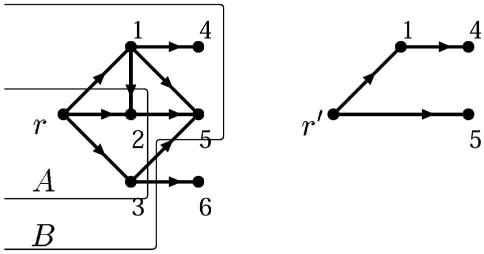

Fig. 1. A rooted digraph and a rooted minor.

For a rooted digraph $G = (V, E, r)$, we consider the following procedure: first we choose one of the atoms, say v; next we shrink v to the root. If we repeat this procedure until all vertices are shrunk to the root, then we will obtain a sequence of vertices selected by the above procedure of shrinking. If we gather all of these sequences, then they form an antimatroid. Formally, for a rooted digraph $G = (V, E, r)$, we define the *point-search antimatroid* $\mathfrak{PS}_D(G)$ as follows:

$$\mathfrak{PS}_D(G) = \{X \subseteq V\colon \text{ every vertex } v \in X \text{ can be reached by a rooted path in the subgraph induced by } X \cup \{r\}\}. \quad (5)$$

Note that the class of point-search antimatroids is closed under taking minors.

In a rooted digraph $G = (V, E, r)$, let $e = xy \in E$ be an edge of G. Suppose $P = ru_1u_2 \cdots u_m$ to be a straight rooted path such that $u_{m-1}u_m = e$. Then we say that *e is supported by P*, or *P supports e*. If there is no path supporting e, then e is called a *redundant* edge. If a rooted digraph contains no redundant edge, then it is called an *irredundant* rooted digraph. Note that redundant edges have no use for defining point-search antimatroids. In particular, irredundant rooted digraphs have no edge whose head is the root r or an atom. For a rooted digraph G, define G_0 as the rooted digraph such that the redundant edges of G are deleted, then the point-search antimatroids of G and G_0 are the same. Therefore, without loss of generality, when we consider point-search antimatroids of rooted digraphs, we only have to handle irredundant ones.

Let $G = (V, E, r)$ be a rooted digraph, and $\mathfrak{PS}_D(G)$ be the point-search antimatroid of G. For $A, B \in \mathfrak{PS}_D(G)$ with $A \subseteq B$, remove $V \setminus B$ and the edges incident to $V \setminus B$ from G, shrink the vertices A to r. Then delete all the redundant edges from the resultant graph. This procedure gives us an irredundant rooted digraph, which we call a *rooted minor* and denote by $G[A, B]$. Fig. 1 shows an example of rooted minors. Note that every rooted minor of an irredundant rooted digraph is also irredundant. Clearly, the point-search antimatroid of $G[A, B]$ is equal to the minor $\mathfrak{PS}_D(G)[A, B]$, namely $\mathfrak{PS}_D(G[A, B]) = \mathfrak{PS}_D(G)[A, B]$. Furthermore, suppose G' to be another irredundant rooted digraph. Then $\mathfrak{PS}_D(G)$ contains a minor isomorphic to $\mathfrak{PS}_D(G')$ if and only if there exists a rooted minor of G which is isomorphic to G'.

A *multi-digraph* H is a quadruple $(N, A; h, t)$, where N is a non-empty finite set of *nodes*, A is a finite set of *arcs*, and h, t are maps from A to N. For $a \in A$, $h(a) \in N$ is called the *head* of a, and $t(a) \in N$ is the *tail* of a. A digraph is a special case of

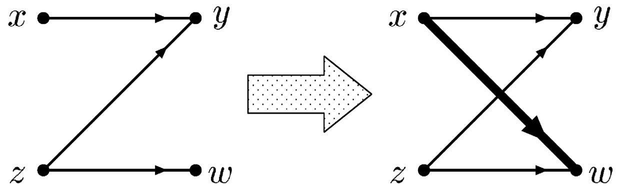

Fig. 2. The Heuchenne condition.

multi-digraphs. A *path* in H is a sequence of arcs $a_1 \cdots a_k$ such that $h(a_i) = t(a_{i+1})$ for $i = 1, \ldots, k-1$. If a path has no repeated arcs, it is called *simple*.

A multi-digraph $H=(N,A;h,t)$ defines a digraph $G=(A,E)$ by $E=\{(a,b)\colon a,b \in A, a \neq b, h(a)=t(b)\}$, which is called the *line graph* of H. A digraph G is a *line graph* if there exists some multi-digraph of which G is the line graph. Syslo [14] gives a polynomial-time algorithm which decides whether the given digraph is a line graph or not. The algorithm is based on the following characterization of line graphs [4,11]:

Proposition 1. *Let $G=(V,E)$ be a digraph. G is a line graph if and only if for every $x,y,z,w \in V$, $(x,y),(z,y),(z,w) \in E$ imply $(x,w) \in E$, as shown in* Fig. 2.

The condition of this proposition is called the *Heuchenne condition*, or the *H-condition*, for short.

A *rooted multi-digraph* is a quintuple $(N,A,r;h,t)$, where $(N \cup \{r\},A;h,t)$ is a multi-digraph and r is a specified node called a *root* such that for every arc there exists a simple path from r which contains it. A rooted multi-digraph $H=(N,A,r';h,t)$ also gives its *rooted line graph* as follows: add a new node r'' and insert an arc $r''r'$ to H, and construct the line graph of this resultant multi-digraph, then we have a digraph G whose vertices are $A \cup \{r\}$, where r is a vertex corresponding to the arc $r''r'$. By assumption, it is obvious that there exists a rooted path to every vertex in G. Hence G is a rooted digraph.

3. The forbidden minor characterization of line-search antimatroids

In analogy to point-search antimatroids, we define the *line-search antimatroid* $\mathfrak{LS}_D(H)$ of a rooted multi-digraph $H=(N,A,r;h,t)$ as follows:

$$\mathfrak{LS}_D(H) = \{X \subseteq A\colon \text{ every arc } a \in X \text{ is contained in a simple path from } r \text{ on the subgraph induced by } X\}. \quad (6)$$

Note that line-search antimatroids of rooted multi-digraphs are also closed under taking their minors.

Let G be the rooted line graph of a rooted multi-digraph H. Then the line-search antimatroid of H coincides with the point-search antimatroid of G. Therefore, the class of point-search antimatroids of rooted digraphs includes that of line-search antimatroids

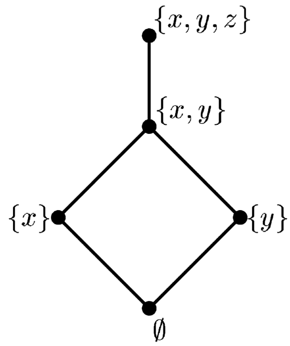

Fig. 3. The forbidden minor D_5 of point-search antimatroids of rooted digraphs.

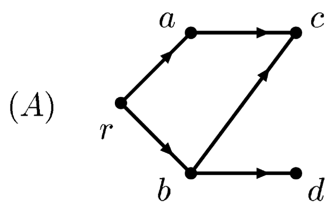

Fig. 4. The rooted digraph A which violates the H-condition.

of rooted multi-digraphs. It is easily checked that there is a one-to-one correspondence between line-search antimatroids of rooted multi-digraphs and irredundant rooted digraphs which satisfy the H-condition.

Point-search antimatroids of rooted digraphs are characterized by the forbidden minor [10]:

Proposition 2. *$\mathscr{F}$ is the point-search antimatroid of a rooted digraph if and only if $\mathscr{F}$ does not contain a minor isomorphic to $D_5 = \{\emptyset, \{x\}, \{y\}, \{x, y\}, \{x, y, z\}\}$, as shown in* Fig. 3.

Hence, in order to characterize line-search antimatroids of rooted digraphs, we only need to characterize point-search antimatroids of irredundant rooted digraphs which violate the H-condition.

For example, the irredundant rooted digraph $A = (V(A), E(A), r)$ defined as

$$V(A) = \{a, b, c, d\}, \tag{7}$$

$$E(A) = \{(r, a), (r, b), (a, c), (b, c), (b, d)\}, \tag{8}$$

which is shown in Fig. 4, violates the H-condition.

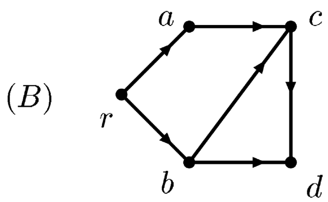

Fig. 5. The rooted digraph B which violates the H-condition.

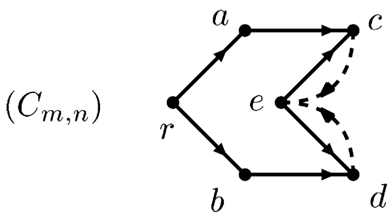

Fig. 6. The rooted digraph $C_{m,n}$ $(m,n \geqslant 1)$ which violates the H-condition, where the broken arrows represent arbitrarily long paths.

Additionally, the following three kinds of irredundant rooted digraphs $B, C_{m,n}, D_{l,m,n}$ also violate the H-condition; $B=(V(B),E(B),r)$ is defined as

$$V(B)=\{a,b,c,d\}, \tag{9}$$

$$E(B)=\{(r,a),(r,b),(a,c),(b,c),(b,d),(c,d)\}, \tag{10}$$

which is shown in Fig. 5; $C_{m,n}=(V(C_{m,n}),E(C_{m,n}),r)$ is defined as

$$V(C_{m,n})=\{a,b,c=x_0,\ d=y_0,e,x_1,\dots,x_{m-1},y_1,\dots,y_{n-1}\}, \tag{11}$$

$$\begin{aligned} E(C_{m,n})=\{&(r,a),(r,b),(a,c),(b,d),(c,x_1),(d,y_1),(e,c),(e,d),\\ &(x_1,x_2),\dots,(x_{m-2},x_{m-1}),(x_{m-1},e),\\ &(y_1,y_2),\dots,(y_{n-2},y_{n-1}),(y_{n-1},e)\}, \end{aligned} \tag{12}$$

where $m,n \geqslant 1$, which is shown in Fig. 6; $D_{l,m,n}=(V(D_{l,m,n}),E(D_{l,m,n}),r)$ is defined as

$$\begin{aligned} V(D_{l,m,n})=\{&a,b,c=x_0,d=y_0,e,f=z_0,\\ &x_1,\dots,x_{l-1},y_1,\dots,y_{m-1},z_1,\dots,z_{n-1}\}, \end{aligned} \tag{13}$$

$$\begin{aligned} E(D_{l,m,n})=\{&(r,a),(r,b),(a,c),(b,d),(c,x_1),(d,y_1),(e,c),(e,d),\\ &(f,z_1),(x_1,x_2),\dots,(x_{l-2},x_{l-1}),(x_{l-1},f),\\ &(y_1,y_2),\dots,(y_{m-2},y_{m-1}),(y_{m-1},f),\\ &(z_1,z_2),\dots,(z_{n-2},z_{n-1}),(z_{n-1},e)\}, \end{aligned} \tag{14}$$

where $l,m,n \geqslant 1$, which is shown in Fig. 7.

$(D_{l,m,n})$

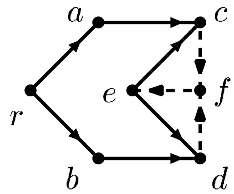

Fig. 7. The rooted digraph $D_{l,m,n}$ $(l,m,n \geqslant 1)$ which violates the H-condition, where the broken arrows represent arbitrarily long paths.

Therefore, it is clear that if G is a rooted line graph then it cannot contain the above rooted digraphs as its rooted minors. Indeed, it turns out to be sufficient to exclude these minors to get a rooted line graph.

Theorem 3. *Let G be an irredundant rooted digraph. Then, G is a rooted line graph if and only if G has no rooted minor isomorphic to A, B,* $C_{m,n}$ *or* $D_{l,m,n}$ $(l,m,n \geqslant 1)$.

Proof. We only need to show the sufficiency. Let $G=(V,E,r)$ be an irredundant rooted digraph containing four vertices x,y,z,w which violate the H-condition and is minor-minimal with respect to this property. Let $\mathscr{W}=\{x,y,z,w\}$.

A vertex $a \in \mathscr{W}$ is the *joint* of a straight path P from r to a vertex of $\mathscr{W}$ if a is the first vertex of $\mathscr{W}$ along the path P from r. Let T be the set of joints for straight paths in G. From the assumption, we have $T \neq \emptyset$ and there must exist a path supporting each of the edges xy, zy, zw, which we denote by P, Q, R, respectively. We consider the following cases according to the size of T.

Case 1. $|T|=1$. It is easily checked that this case leads to a contradiction.

Case 2. $|T|=2$. This has the following six subcases.

Case 2.1: $T=\{x,y\}$. The path Q is not straight since Q must go through x or y. This is a contradiction.

Case 2.2. $T=\{x,z\}$. A path with the joint x supports the edge xy, and a path with the joint z supports the edges zy and zw. From the minimality of G, the vertices of G must be $\{r,x,y,z,w\}$. If we consider all the possible edges among them, then we obtain A and B.

Case 2.3. $T=\{x,w\}$. Suppose that the path Q goes through x, then the edge xy is a short cut. This is a contradiction. Therefore, Q must go through w but not through x. Moreover, Q is $r\cdots w\cdots zy$ since Q does not go through y. If a path with the joint w has no vertex between r and w, then it is a short-cut of the path R. Therefore, it has an extra vertex p between r and w, namely the path is rpw, from the minimality of G. Moreover, the path with the joint x is rx from the minimality of G as a rooted minor. Since the path R does not go through w, it must go through x. We consider the subcases according to whether R goes through the edge xy or not.

Case 2.3.1. *R goes through xy*. R is $r\cdots xy\cdots z$. If there is a common vertex of the part $y\cdots z$ of R and the part $w\cdots z$ of Q except for z, then G must contain $D_{l,m,n}$ as a subgraph. Otherwise, G must contain $C_{m,n}$ as a subgraph.

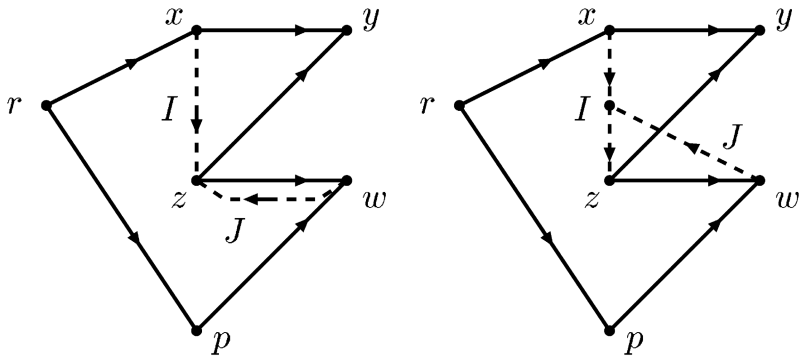

Fig. 8. Case 2.3.2. Broken arrows represent arbitrarily long paths.

Now we should check that if G has no rooted minor isomorphic to $C_{m,n}$ and $D_{l,m,n}$, then G must have A or B as its rooted minor, or it leads to a contradiction.

Case 2.3.1.1: *$C_{m,n}$ has extra edges*. Refer Definition (11,12) of $C_{m,n}$.

Case 2.3.1.1.1: *The edge cd exists*. If we shrink a to r and we set $a=c$ and $c=x_1$, then we can reduce this case to A or B.

Case 2.3.1.1.2: *The edge x_iy_j exists* $(0<i<m, 0<j<n)$. If we shrink a, b, c, $x_1,\dots,x_{i-1}$, $y_0,\dots,y_{j-2}$ to r and we set $a=x_i$, $b=y_{j-1}$, $c=x_{i+1}$ and $d=y_j$, then we reduce this case to A or B.

Case 2.3.1.1.3: *The edge x_ie exists.* A contradiction since the edge $x_{m-1}e$ is redundant.

Case 2.3.1.2: *$D_{l,m,n}$ has extra edges.* We can check similarly to Case 2.3.1.1.

Case 2.3.2: *R does not go through xy*. Then, we obtain the graphs shown in Fig. 8, where I is a path from x to z and J is a path from w to z. In the left case, I and J have a unique common vertex z, and in the right case they have at least two common vertices.

Now we show that these graphs have A or B as a rooted minor. We consider the left case. The right case is shown similarly.

Case 2.3.2.1: *The length of I is one*, *and the length of J is also one*. If we shrink p to r, then it is reduced to B.

Case 2.3.2.2: *The length of I is one*, *and the length of J is more than one.* Let $J=wj_1j_2\dots j_hz$ for $h\geqslant 1$. If we shrink p, w, $j_1,\dots,j_{h-1}$ to r, then it is reduced to B.

Case 2.3.2.3: *The length of I is more than two*, *and the length of J is one.* If we shrink p and w to r, then it is reduced to A.

Case 2.3.2.4: *The length of I is more than two*, *and the length of J is one.* Let $I=xi_1i_2\dots i_kz$ for $k\geqslant 2$. If we delete $i_2,\dots,i_k$ and shrink p and w to r, then it is reduced to A.

Case 2.3.2.5: *The lengths of both I and J are more than one.* Let $I=xi_1i_2\dots i_kz$ for $k\geqslant 1$, and $J=wj_1\dots j_hz$ for $h\geqslant 1$. If we delete $i_2,\dots,i_k$ and shrink $p,w,j_1,\dots,j_h$ to r, then it is reduced to A.

Case 2.4: $T=\{y,w\}$. From the minimality and the irredundancy of G, the length of a path with the joint y is two, and let it be rpy. Similarly, the length of a path

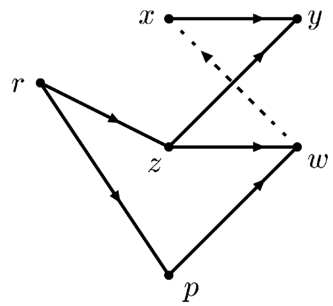

Fig. 9. Case 2–6.

with the joint w is two, and let it be rqw. If $p = q$, then the three edges xy, zy and zw are always redundant. Therefore, we have $p \neq q$.

The path Q goes through neither x nor y. Therefore, Q is $rqw \cdots zy$.

The path R does not go through w. Hence, it must go through y. If we delete x, then it is reduced to $C_{m,n}$ or $D_{l,m,n}$.

Case 2.5: $T = \{y, z\}$. The path P does not go through y. Therefore, it must go through z. Then, it is a contradiction since the edge zy is a short cut.

Case 2.6: $T = \{z, w\}$. Since the path P does not go through z, it must go through w. From the minimality of G, the length of a path with the joint w is two, and the length of a path with the joint z is one. Now, we obtain the graph shown in Fig. 9. Then, if we delete the vertices of the path $w \cdots x$ except for w, then it is reduced to A.

Case 3: $|T| = 3$. This has the following four subcases.

Case 3.1: $T = \{x, y, z\}$. The path P has the joint x. Moreover, the paths Q and R have the joint z. Suppose that the length of a path Y with the joint y is one. Then the edges xy and zy are redundant. Therefore, the length of Y is more than one, that is, $Y = ry_1 \cdots y_k py$ for $k \geqslant 0$. Note that p is contained neither in P nor in Q.

Let $P = ru_1 \cdots u_l x$ and $Q = rv_1 \cdots v_m z$ for $l, m \geqslant 0$. If we delete p and shrink $u_1, \ldots, u_l$, $v_1, \ldots, v_m$, $y_1, \ldots, y_k$ to r, then it is reduced to A or B.

Case 3.2: $T = \{x, y, w\}$. Suppose that the length of a path Y with the joint y is one. Then the edges xy and zy are redundant. Therefore, the length of Y is more than one, that is, $Y = ry_1 \cdots y_k py$ for $k \geqslant 0$. If we delete x, then $\{p, y, z, w\}$ is the set of vertices which violates the H-condition. Therefore, it is reduced to Case 2.3.

Case 3.3: $T = \{x, z, w\}$. The path P has the joint x. Moreover, the paths Q and R have the joint z. Suppose that the length of a path Y with the joint w is one. Then the edge zw is redundant. Therefore, the length of Y is more than one, that is, $Y = ry_1 \cdots y_k pw$ for $k \geqslant 0$. Note that p is contained neither in P nor in Q.

Let $P = ru_1 \cdots u_l x$ and $Q = rv_1 \cdots v_m z$ for $l, m \geq 0$. If we delete p, and shrink $u_1, \ldots, u_l$, $v_1, \ldots, v_m$, $y_1, \ldots, y_k$ to r, then it is reduced to A or B.

Case 3.4: $T = \{y, z, w\}$. The paths Q and R have the joint z. Let Y be the path with the joint y. Note that the length of Y is more than one. Similarly, let W be the path with the joint w, then its length is more than one. The path P supporting the edge xy

has the joint w. Let p be the vertex of Y which precedes y and q be the vertex of W which precedes w. Suppose that $p=q$, and consider the path P supporting the edge xy. The joint of P is not y. If the joint of P is z, then the edge zy is a short-cut of P. If the joint of P is w, then the edge py is a short-cut of P. Therefore, we have $p \neq q$.

Let $Y = ry_1 \cdots y_l py$, $W = rw_1 \cdots w_m qw$ and $Q = rq_1 \cdots q_n z$ for $l,m,n \geqslant 0$. If we delete p and x, and shrink $y_1,\ldots,y_l$, $w_1,\ldots,w_m$, $q_1,\ldots,q_n$ to r, then it is reduced to A or B.

Case 4: $|T|=4$. It is easily checked that this case is reduced to Case 3.1 or Case 3.3.

Theorem 3 directly gives the forbidden minor characterization of line-search antimatroids of rooted digraphs as below.

Corollary 4. *Let $\mathscr{F}$ be an antimatroid. Then, $\mathscr{F}$ is a line-search antimatroid of a rooted digraph if and only if $\mathscr{F}$ has no minor isomorphic to D_5 or the point-search antimatroids of A, B, $C_{m,n}$ or $D_{l,m,n}$ $(l,m,n \geqslant 1)$.*

Robertson–Seymour [12] have shown the Graph Minor Theorem, that is, in every infinite set of graphs there are two graphs such that one is a minor of the other. From this theorem, we conclude that every minor-closed property of graphs can be characterized by finitely many forbidden minors. But for antimatroids, Theorem 3 implies that there exists an infinite set of antimatroids such that any of them is not a proper minor of the other one.

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