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Fixed point theorems for operators on partial metric spaces

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ABSTRACT

Fixed point theorems for operators of a certain type on partial metric spaces are given. Orbitally continuous operators on partial metric spaces and orbitally complete partial metric spaces are defined, and fixed point theorems for these operators are given.

1. Introduction and preliminaries

The existence and uniqueness of fixed points of operators has been a subject of great interest since the work of Banach in 1922 [1]. Results for operators on spaces of various types, for example metric spaces, quasi-metric spaces (see e.g. [2,3]), cone metric spaces (see e.g. [4,5]), Menger (statistical metric) spaces (see e.g. [6]), and fuzzy metric spaces (see e.g [7]), have already been obtained. A new space called a partial metric space (PMS) has been introduced by Matthews [8,9]. Matthews proved a fixed point theorem on this space, which is an analogy of the Banach fixed point theorem. Later, some more results on fixed point theory on PMS were published [10–14].

The definition of a partial metric space is given by Matthews (see [8]) as follows:

Definition 1. Let X be a nonempty set and let $p : X \times X \to \mathbb{R}^+$ satisfy

$$\begin{aligned} &(P1)\ x = y \Leftrightarrow p(x,x) = p(y,y) = p(x,y)\\ &(P2)\ p(x,x) \le p(x,y)\\ &(P3)\ p(x,y) = p(y,x)\\ &(P4)\ p(x,y) \le p(x,z) + p(z,y) - p(z,z) \end{aligned} \tag{1.1}$$

for all x, y and $z \in X$. Then the pair (X, p) is called a partial metric space and p is called a partial metric on X.

One can easily show that the function $d_p : X \times X \to \mathbb{R}^+$ defined as

$$d_p(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.2}$$

satisfies the conditions of a metric on X; therefore it is a (usual) metric on X. Note also that each partial metric p on X generates a T_0 topology τ_p on X, whose base is a family of open p-balls $\{B_p(x,\epsilon) : x \in X, \epsilon > 0\}$ where $B_p(x,\epsilon) = \{y \in X : p(x,y) \le p(x,x) + \epsilon\}$ for all $x \in X$ and $\epsilon > 0$.

On a partial metric space the concepts of convergence, the Cauchy sequence, completeness and continuity as defined as follows [8].

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Definition 2.

(1) A sequence $\{x_n\}$ in the PMS (X, p) converges to the limit x if and only if $p(x, x) = \lim_{n\to\infty} p(x, x_n)$.
(2) A sequence $\{x_n\}$ in the PMS (X, p) is called a Cauchy sequence if $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite.
(3) A PMS (X, p) is called complete if every Cauchy sequence $\{x_n\}$ in X converges with respect to τ_p to a point $x \in X$ such that $p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m)$.
(4) A mapping $f : X \to X$ is said to be continuous at $x_0 \in X$ if for every $\epsilon > 0$, there exists $\delta > 0$ such that $F(B_p(x_0, \delta)) \subseteq B_P(Fx_0, \epsilon)$.

The following lemma on partial metric spaces can be derived easily (see e.g. [8,9,13]).

Lemma 3.

(1) *A sequence $\{x_n\}$ is a Cauchy sequence in the PMS (X, p) if and only if it is a Cauchy sequence in the metric space (X, d_p).*
(2) *A PMS (X, p) is complete if and only if the metric space (X, d_p) is complete. Moreover*

$$\lim_{n\to\infty} d_p(x, x_n) = 0 \Leftrightarrow p(x, x) = \lim_{n\to\infty} p(x, x_n) = \lim_{n,m\to\infty} p(x_n, x_m). \tag{1.3}$$

2. The main results

We start with an easy lemma which has a crucial role in the proof of the main results.

Lemma 4 *(See e.g. [15]).* *Assume that $x_n \to z$ as $n \to \infty$ in a PMS (X, p) such that $p(z, z) = 0$. Then $\lim_{n\to\infty} p(x_n, y) = p(z, y)$ for every $y \in X$.*

Proof. First note that $\lim_{n\to\infty} p(x_n, z) = p(z, z) = 0$. By the triangle inequality we have

$$p(x_n, y) \le p(x_n, z) + p(z, y) - p(z, z) = p(x_n, z) + p(z, y)$$

and

$$p(z, y) \le p(z, x_n) + p(x_n, y) - p(x_n, x_n) \le p(x_n, z) + p(x_n, y).$$

Hence

$$0 \le |p(x_n, y) - p(z, y)| \le p(x_n, z).$$

Letting $n \to \infty$ we conclude our claim. □

In this section we give some fixed point theorems for operators of different types on partial metric spaces. Our first theorem has an analog on metric spaces (see [16]).

Theorem 5. *Let (X, p) be a partial metric space. Let $T : X \to X$ be a map for which the inequality*

$$ap(Tx, Ty) + b[p(x, Tx) + p(y, Ty)] + c[p(x, Ty) + p(y, Tx)] \le sp(x, y) + rp(x, T^2x) \tag{2.1}$$

holds for all x, y in X where the constants a, b, c, r and s satisfy

$$0 \le \frac{s-b}{a+b} < 1, \quad a + b \ne 0,\ a + b + c > 0,\ c > 0,\ c - r \ge 0. \tag{2.2}$$

Then T has at least one fixed point in X.

Proof. Take an arbitrary $x_0 \in X$. Define the sequence

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \ldots. \tag{2.3}$$

Substituting $x = x_n$ and $y = x_{n+1}$ in (2.1) we obtain

$$ap(Tx_n, Tx_{n+1}) + b[p(x_n, Tx_n) + p(x_{n+1}, Tx_{n+1})] + c[p(x_n, Tx_{n+1}) + p(x_{n+1}, Tx_n)] \le sp(x_n, x_{n+1}) + rp(x_n, T^2x_n)$$

which implies

$$ap(x_{n+1}, x_{n+2}) + b[p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2})] + c[p(x_n, x_{n+2}) + p(x_{n+1}, x_{n+1})] \le sp(x_n, x_{n+1}) + rp(x_n, x_{n+2}).$$

Rewriting this inequality as

$$(a + b)p(x_{n+1}, x_{n+2}) + (c - r)p(x_n, x_{n+2}) + cp(x_{n+1}, x_{n+1}) \le (s - b)p(x_n, x_{n+1}),$$

and using the fact that

$$(c - r)p(x_n, x_{n+2}) + cp(x_{n+1}, x_{n+1}) \ge 0$$

we obtain

$$p(x_{n+1}, x_{n+2}) \leq kp(x_n, x_{n+1}),$$

where $k = \frac{s-b}{a+b}$ and clearly $0 \leq k < 1$. Thus,

$$p(x_{n+1}, x_{n+2}) \leq kp(x_n, x_{n+1}) \leq k^2 p(x_{n-1}, x_n) \leq \cdots \leq k^{n+1} p(x_0, x_1). \tag{2.4}$$

We will show that $\{x_n\}$ is a Cauchy sequence. Without loss of generality assume that $n > m$. Then, using (2.4) and the triangle inequality for partial metric (P4) we have

$$\begin{aligned} 0 &\leq p(x_n, x_m) \leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) + \cdots + p(x_{m+1}, x_m) \\ &\quad - [p(x_{n-1}, x_{n-1}) + p(x_{n-2}, x_{n-2}) + \cdots p(x_{m+1}, x_{m+1})] \\ &\leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) + \cdots + p(x_{m+1}, x_m) \\ &\leq [k^{n-1} + k^{n-2} + \cdots k^m] p(x_0, x_1) \\ &= k^m \frac{1-k^{n-m}}{1-k} p(x_0, x_1). \end{aligned}$$

Hence, $\lim_{n,m\to\infty} p(x_n, x_m) = 0$, that is, $\{x_n\}$ is a Cauchy sequence in (X, p). By Lemma 3, $\{x_n\}$ is also Cauchy in (X, d_p). In addition, since (X, p) is complete, (X, d_p) is also complete. Thus there exists $z \in X$ such that $x_n \to z$ in (X, d_p); moreover, by Lemma 3,

$$p(z, z) = \lim_{n\to\infty} p(z, x_n) = \lim_{n,m\to\infty} p(x_n, x_m) = 0 \tag{2.5}$$

implies

$$\lim_{n\to\infty} d_p(z, x_n) = 0. \tag{2.6}$$

We will show next that z is the fixed point of T. Notice that due to (2.5), we have $p(z, z) = 0$. Substituting $x = x_n$ and $y = z$ in (2.1) we obtain

$$ap(Tx_n, Tz) + b[p(x_n, Tx_n) + p(z, Tz)] + c[p(z, Tx_n) + p(x_n, Tz)] \leq sp(z, x_n) + rp(x_n, T^2 x_n)$$

which implies

$$ap(x_{n+1}, Tz) + b[p(x_n, x_{n+1}) + p(z, Tz)] + c[p(z, x_{n+1}) + p(x_n, Tz)] \tag{2.7}$$
$$\leq sp(z, x_n) + rp(x_n, x_{n+2}). \tag{2.8}$$

Taking the limit as $n \to \infty$, using (2.5) and noting Lemma 4, we obtain

$$(a + b + c)p(z, Tz) \leq 0.$$

Since $a + b + c > 0$, we then have

$$0 \leq (a + b + c)p(z, Tz) \leq 0$$

which implies $p(z, Tz) = 0$. Using (1.2), we end up with

$$0 \leq d_p(z, Tz) = 2p(z, Tz) - p(z, z) - p(Tz, Tz) = -p(Tz, Tz) \leq 0,$$

and hence, $d_p(z, Tz) = 0$, that is, $z = Tz$, which completes the proof. □

We next define orbitally continuous maps on partial metric spaces and orbitally complete partial metric spaces. In fact, fixed point theories for certain orbitally continuous maps on metric spaces and orbitally complete metric spaces have been investigated by Ćirić [17]. On partial metric spaces we define these concepts as follows.

Definition 6.

(1) Let (X, p) be a PMS. A map $T : X \to X$ is called orbitally continuous if

$$\lim_{i\to\infty} p(T^{n_i} x, z) = p(z, z) \tag{2.9}$$

implies

$$\lim_{i\to\infty} p(TT^{n_i} x, Tz) = p(Tz, Tz) \tag{2.10}$$

for each $x \in X$.

(2) A PMS (X, p) is called orbitally complete if every Cauchy sequence $\{T^{n_i}x\}_{i=1}^{\infty}$ converges in (X, p), that is, if

$$\lim_{i,j\to\infty} p(T^{n_i}x, T^{n_j}x) = \lim_{i\to\infty} p(T^{n_i}x, z) = p(z, z). \tag{2.11}$$

We need the following lemma in order to proceed with the fixed point theorems of orbitally continuous maps.

Lemma 7 *(See e.g. [15]). Let (X, p) be a complete PMS. Then:*

(A) *If $p(x, y) = 0$, then $x = y$.*
(B) *If $x \neq y$, then $p(x, y) > 0$.*

Proof of (A). Let $p(x, y) = 0$. By (P3), we have $p(x, x) \leq p(x, y) = 0$ and $p(y, y) \leq p(x, y) = 0$. Thus, we have

$$p(x, x) = p(x, y) = p(y, y) = 0.$$

Hence, by (P2), we have $x = y$. □

Proof of (B). Suppose that $x \neq y$. By definition $p(x, y) \geq 0$ for all $x, y \in X$. Assume that $p(x, y) = 0$. By part (A), $x = y$ which is a contradiction. Hence, $p(x, y) > 0$ whenever $x \neq y$. □

In what follows, we state and prove an analog of the fixed point theorem for orbitally continuous maps on orbitally complete metric spaces given by Ćirić in 1974 (see [17]).

Theorem 8. *Let $T : X \to X$ be an orbitally continuous map on an orbitally complete PMS (X, p). If T satisfies*

$$\min\{p(Tx, Ty), p(x, Tx), p(y, Ty)\} \leq ap(x, y) \tag{2.12}$$

for some $a < 1$ and all x, y in X, then the sequence $\{T^n x\}$ converges to a fixed point of T for each x in X.

Proof. Let $x_0 \in X$ be arbitrary. Define the sequence $\{x_n\}$ as follows:

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \ldots.$$

Clearly, if $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then T has a fixed point. Therefore, we assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Hence, by Lemma 7, we have $p(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$. Substituting $x = x_n$ and $y = x_{n+1}$ in (2.12) we obtain

$$\min\{p(Tx_n, Tx_{n+1}), p(x_n, Tx_n), p(x_{n+1}, Tx_{n+1})\} \leq ap(x_n, x_{n+1}) \tag{2.13}$$

which implies

$$\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\} \leq ap(x_n, x_{n+1}). \tag{2.14}$$

Now, if $\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$, it follows that

$$p(x_n, x_{n+1}) \leq ap(x_n, x_{n+1})$$

which is impossible since $a < 1$. Then we must have

$$\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\} = p(x_{n+1}, x_{n+2})$$

and hence

$$p(x_{n+1}, x_{n+2}) \leq ap(x_n, x_{n+1}). \tag{2.15}$$

Then, for $n > m$ by using (P4) and using (2.15), we have

$$\begin{aligned} 0 &\leq p(x_n, x_m) \leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) + \cdots + p(x_{m+1}, x_m) \\ &\quad - [p(x_{n-1}, x_{n-1}) + p(x_{n-2}, x_{n-2}) + \cdots + p(x_{m+1}, x_{m+1})] \\ &\leq p(x_n, x_{n-1}) + p(x_{n-1}, x_{n-2}) + \cdots p(x_{m+1}, x_m) \\ &\leq [a^{n-1} + a^{n-2} + \cdots a^m]p(x_0, x_1) \\ &= a^m \frac{1 - a^{n-m}}{1 - a} p(x_0, x_1). \end{aligned}$$

Thus,

$$\lim_{m,n\to\infty} p(x_n, x_m) = 0,$$

so we conclude that $\{x_n\} = \{T^n x_0\}$ is Cauchy in PMS (X, p), and since (X, p) is orbitally complete, then $\{T^n x_0\}$ converges to a limit, say $z \in X$, such that

$$\lim_{m,n\to\infty} p(T^n x_0, T^m x_0) = \lim_{n\to\infty} p(T^n x_0, z) = p(z, z) = 0. \tag{2.16}$$

Now, we will show that z is a fixed point of T. Since T is orbitally continuous and noting (2.16) we observe that

$$\lim_{n\to\infty} p(T^n x_0, z) = p(z, z) \Rightarrow \lim_{n\to\infty} p(TT^n x_0, Tz) = p(Tz, Tz). \tag{2.17}$$

Also, taking the limit as $n \to \infty$ and using Lemma 4 with (2.17) we obtain

$$\begin{aligned} p(z, Tz) &\leq p(z, T^{n+1}x_0) + p(T^{n+1}x_0, Tz) - p(T^{n+1}x_0, T^{n+1}x_0) \\ &\leq p(z, x_{n+2}) + p(T^{n+1}x_0, Tz) - p(x_{n+2}, x_{n+2}), \end{aligned} \tag{2.18}$$

which implies that

$$p(z, Tz) \leq p(Tz, Tz).$$

However, from (P2), this is possible only if

$$p(z, Tz) = p(Tz, Tz). \tag{2.19}$$

Using (2.12), (2.19) and (2.16) we have

$$\begin{aligned} &\min\{p(Tz, Tz), p(z, Tz), p(z, Tz)\} \leq ap(z, z) \\ &p(Tz, Tz) = p(z, Tz) \leq ap(z, z) = 0. \end{aligned} \tag{2.20}$$

Thus, (2.20) implies $p(Tz, Tz) = p(z, Tz) = p(z, z) = 0$ and by (P1) we obtain $z = Tz$ which completes the proof. □

Example 9. Let $X = [0, 1]$. Define $p : X \times X \to \mathbb{R}^+$ as

$$p(x, y) = \max\{x, y\}$$

with

$$T : X \to X, \qquad Tx = \frac{x}{2}$$

Clearly, (X, p) is a partial metric space. Now, let $x \leq y$. Then $p(x, Tx) = x$, $p(y, Ty) = y$, $p(Tx, Ty) = \frac{y}{2}$ and $p(x, y) = y$. Hence, we have

$$\min\{p(x, Tx), p(y, Ty), p(Tx, Ty)\} = \min\left\{x, y, \frac{y}{2}\right\}.$$

If $\min\left\{x, y, \frac{y}{2}\right\} = \frac{y}{2}$, then

$$\min\{p(x, Tx), p(y, Ty), p(Tx, Ty)\} = \frac{y}{2} \leq \frac{1}{2}p(x, y).$$

If $\min\left\{x, y, \frac{y}{2}\right\} = x$, then obviously $x \leq \frac{y}{2}$ and

$$\min\{p(x, Tx), p(y, Ty), p(Tx, Ty)\} = x \leq \frac{1}{2}p(x, y).$$

Then T satisfies the conditions of Theorem 8 with $a = \frac{1}{2}$. Therefore, the sequence $\{T^n x\} = \left\{\frac{x}{2^n}\right\}$ converges to the fixed point $z = 0$ of the operator T for every $x \in X$.

In the next theorem we consider an orbitally continuous operator satisfying a condition given by a rational expression. This theorem also has an analog in metric spaces (see [18]).

Theorem 10. *Let $T : X \to X$ be an orbitally continuous map on an orbitally complete PMS (X, p). If T satisfies*

$$\frac{\min\{p(Tx, Ty)p(x, y), p(x, Tx)p(y, Ty)\}}{\min\{p(x, Tx), p(y, Ty)\}} \leq ap(x, y) \tag{2.21}$$

for some $a < 1$ and all x, y in X such that $p(x, Tx) \neq 0$ and $p(y, Ty) \neq 0$, then the sequence $\{T^n x\}$ converges to a fixed point of T for each x in X.

Proof. Let $x_0 \in X$ be arbitrary. Define the sequence $\{x_n\}$ as follows:

$$x_{n+1} = Tx_n, \quad n = 0, 1, 2, \ldots$$

It is clear that if $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then T has a fixed point. Thus, we suppose that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N}$. Hence, by Lemma 7, we have $p(x_n, x_{n+1}) > 0$ for all $n \in \mathbb{N}$. We substitute $x = x_n$ and $y = x_{n+1}$ in (2.21) and obtain

$$\frac{\min\{p(Tx_n, Tx_{n+1})p(x_n, x_{n+1}), p(x_n, Tx_n)p(x_{n+1}, Tx_{n+1})\}}{\min\{p(x_n, Tx_n), p(x_{n+1}, Tx_{n+1})\}} \leq ap(x_n, x_{n+1}) \tag{2.22}$$

which implies

$$\frac{p(x_{n+1}, x_{n+2})p(x_n, x_{n+1})}{\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\}} \leq ap(x_n, x_{n+1}). \tag{2.23}$$

Now, if $\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\} = p(x_{n+1}, x_{n+2})$, it follows that

$$p(x_n, x_{n+1}) \leq ap(x_n, x_{n+1})$$

which is impossible since $a < 1$. Then we must have $\min\{p(x_{n+1}, x_{n+2}), p(x_n, x_{n+1})\} = p(x_n, x_{n+1})$ and hence

$$p(x_{n+1}, x_{n+2}) \leq ap(x_n, x_{n+1}). \tag{2.24}$$

Regarding the analogy with Theorem 8, we get

$$\lim_{n\to\infty} p(x_n, z) = p(z, z) = 0, \quad \text{and} \quad p(Tz, z) = p(Tz, Tz). \tag{2.25}$$

If $p(Tz, z) = p(Tz, Tz) = 0$, then by (P1) we have $Tz = z$. Suppose $p(Tz, z) = p(Tz, Tz) > 0$. Consider (2.21) for $x = Tz$ and $y = z$:

$$\frac{\min\{p(T^2z, Tz)p(Tz, z), p(Tz, T^2z)p(z, Tz)\}}{\min\{p(Tz, T^2z), p(z, Tz)\}} \leq ap(Tz, z) \tag{2.26}$$

Then we have

$$p(T^2z, Tz)p(Tz, z) \leq ap(Tz, z) \min\{p(Tz, T^2z), p(z, Tz)\}. \tag{2.27}$$

If $\min\{p(Tz, T^2z), p(z, Tz)\} = p(Tz, T^2z)$, then since $a < 1$, we have a contradiction. Thus, $\min\{p(Tz, T^2z), p(z, Tz)\} = p(z, Tz)$ and hence by (2.27)

$$p(T^2z, Tz)p(Tz, z) \leq ap(z, Tz)p(z, Tz) \Leftrightarrow p(T^2z, Tz) \leq ap(z, Tz). \tag{2.28}$$

Notice that due to (P2) we always have

$$p(Tz, Tz) \leq p(T^2z, Tz). \tag{2.29}$$

Combining (2.28), (2.29) and (2.25), we get

$$p(Tz, z) = p(Tz, Tz) \leq p(T^2z, Tz) \leq ap(z, Tz)$$

which is possible only if $p(Tz, z) = 0$. Hence, by Lemma 7, we get $Tz = z$. □

The fixed point theorems presented in this work give conditions only for the existence of fixed points and not for the uniqueness. In the case of (usual) metric space, these theorems give similar results, that is, the conditions only imply the existence of fixed points.

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