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Heavy quark production by a quasi-classical color field in proton–nucleus collisions

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Abstract

We calculate the inclusive heavy quark production cross section for proton–nucleus collisions at high energies. We perform calculation in a quasi-classical approximation (McLerran–Venugopalan model) neglecting all low-x evolution effects. The derived expression for the differential cross section can be applied for studying the heavy quark production in the central rapidity region at RHIC.

In this Letter we address a problem of heavy quark production in proton–nucleus collisions at very high energy. Heavy quarks are a very important tool for studying the properties of the strong interactions. At not very high energies the heavy quark mass provides a scale which allows one to use the perturbative QCD [1] since the long distance dynamics is effectively decoupled [2]. However, at high energies which correspond to low values of Bjorken x a nucleus becomes a strongly coupled dense partonic system (Color Glass Condensate) with large typical transverse momentum Q_s determined by density of nuclear partons over the nucleus transverse area [3–6]. Experimental data suggest that $Q_s^2 \simeq 2\ \text{GeV}^2$ for the gold nucleus at $x \simeq 0.01$ [7]. It is existence of the strong color field which violates the decoupling of the heavy quark production subprocess from the dynamics of partons in the nucleus wave function [8,9].

The Color Glass Condensate starts to play a significant role in scattering processes at low-x since a coherence length of gluons in a nuclear wave function is of the order $1/(xM_N)$ [10,11] which allows them to coherently interact with all nucleons in a nucleus. It was argued in Refs. [6,12,14] that the color field of a nucleus in a low-x regime is given by the classical solution to the Yang–Mills equations. It includes all multiple rescatterings of a gluons with the color charges of a nucleus [12,14]. However, the quasi-classical approach is not sufficient when $x < e^{-1/\alpha_s}$. In that case quantum evolution effects become important and must be resumed using the low-x evolution equations [3,13–16].

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In this Letter we undertake the first step towards solution of a problem of heavy quark production in pA collisions at high energies by deriving the heavy quark production differential cross section (24) in a quasi-classical approximation which is equivalent to inclusion of all multiple rescatterings of a proton with a nucleus.

The process of heavy quark production at high energies in a quasi-classical approximation has three separated in time stages in the nucleus rest frame. Emission of a gluon g by a proton's valence quark q_v takes much longer time $\tau_{q_v\to q_v g}$ than a subsequent emission of a $q\bar{q}$ pair by a gluon $t_{g\to q\bar{q}} \ll \tau_{q_v\to q_v g}$. In turn, the time of interaction of a $q_v g q\bar{q}$ system with a nucleus is of the order of nuclear length R_A and is negligible as compared to the evolution time of the proton wave function $\tau_{q_v\to q_v g} \gg t_{g\to q\bar{q}} \gg R_A$. Indeed, assume that proton is moving in the "+" light cone direction with four momentum $p = (p^+, 0, \underline{0})$ and nucleus is at rest. Denote the emitted gluon four-momentum by $q = (\varsigma p^+, \underline{p}^2/\varsigma p^+, \underline{p})$. By uncertainty principle emission of a gluon by a valence quark takes time

$$\tau_{q_v\to qg} = \frac{2}{q^+ + q^- + (p-q)^+ + (p-q)^- - p^+ - p^-} = \frac{2\varsigma(1-\varsigma)p_+}{\underline{q}^2} \approx \frac{2q^+}{\underline{q}^2} \tag{1}$$

since the emission of a gluon at high energy is dominated by $\varsigma \ll 1$. The Bjorken x is defined as $x = \underline{q}^2/(2q^+ M_N)$, where M_N is a nucleon mass. Therefore,

$$\tau_{q_v\to qg} = \frac{1}{x\, M_N} \gg R_A \simeq \tau_{\text{int}} \tag{2}$$

for very low x. The same argument applies to the successive emission of a quark–antiquark pair by a gluon in a proton's wave function: a $q_v \to q_v g$ fluctuation spans much longer time than $g \to q\bar{q}$. Therefore, processes in which gluon or heavy quarks are produced in course of the rescatterings in a nucleus are suppressed by powers of energy p^+.

Let us choose the $A_+ = 0$ light cone gauge. In view of the above argument we can separate the process of heavy quark production in nine terms according to the time of gluon emission in the amplitude τ_1 and in the complex conjugate one τ_2, time of quark–antiquark emission in the amplitude t_1 and in the complex conjugate one t_2, and the time of interaction which happens at light-cone time $\tau_{\text{int}} = x_+ = 0$. In Fig. 1 we show all possible cases.

To proceed we need to know the light-cone wave functions of a valence quark and of a virtual gluon in transverse configuration space. The light-cone wave function of a valence quark in momentum space is given by

$$\psi_{q_v\to q_v g}(\underline{q}) = gT^a \frac{\underline{\epsilon}^\lambda \cdot \underline{q}}{\underline{q}^2}, \tag{3}$$

where $\underline{q}$ is the gluon's transverse momentum and ϵ^λ is the gluon's polarization vector. Its Fourier image reads

$$\psi_{q_v\to q_v g}(\underline{z}) = \int \frac{d^2q}{(2\pi)^2}\, e^{-i\,\underline{q}\cdot\underline{z}}\, \psi_{q_v\to q_v g}(\underline{q}) = g\, T^a\, \frac{1}{2\pi i}\, \frac{\underline{\epsilon}^\lambda \cdot \underline{z}}{\underline{z}^2}. \tag{4}$$

Averaging square of Eq. (4) over quantum numbers of the initial quark and summing over quantum numbers of the final quark and gluon we obtain the familiar gluon radiation kernel of a dipole model [17]

$$\Phi_{q_v\to q_v g}(\underline{z}_1, \underline{z}_2) = \frac{1}{2N_c}\sum_{a,\lambda} \psi_{q_v\to q_v g}(\underline{z}_1)\psi^*_{q_v\to q_v g}(\underline{z}_2) = \frac{\alpha_s C_F}{2\pi}\frac{\underline{z}_1\cdot\underline{z}_2}{\underline{z}_1^2\underline{z}_2^2}, \tag{5}$$

where $\underline{z}_1$ and $\underline{z}_2$ are the transverse coordinates of the gluon in the amplitude and in the complex conjugated amplitude correspondingly.

Light-cone wave function of a virtual gluon of momentum q reads, see Fig. 1,

$$\psi_{g\to q\bar{q}}(\underline{k}, \underline{k}-\underline{q}, \alpha) = \frac{gT^a}{(\underline{k}-\alpha\underline{q})^2 + m^2}\big(\delta_{r,r'}(\underline{k}-\alpha\underline{q})\cdot\underline{\epsilon}^\lambda\big[r(1-2\alpha)+\lambda\big] + r\delta_{r,-r'}m(1+r\lambda)\big), \tag{6}$$

where $\underline{k}$ is the produced quark's transverse momentum, m its mass, $\alpha = k^+/q^+$ is the fraction of the gluon's light-cone momentum it carries, r and r' are the quark and the antiquark helicities correspondingly. Eq. (6) can be written in transverse configuration space using modified Bessel functions

$$\begin{aligned}
&\psi_{g\to q\bar{q}}(\underline{z}_1, \underline{x}, \underline{x}_0, \alpha)\\
&\quad= \int \frac{d^2k}{(2\pi)^2} e^{-i\underline{k}\cdot(\underline{x}-\underline{x}_0)} \int \frac{d^2q}{(2\pi)^2} e^{-i\underline{q}\cdot(\underline{x}_0-\underline{z}_1)} \psi_{g\to q\bar{q}}(\underline{k}, \underline{k}-\underline{q}, \alpha)\\
&\quad= \delta\big((\underline{x}_0-\underline{z}_1)+\alpha(\underline{x}-\underline{x}_0)\big)\frac{gT^a}{2\pi}\\
&\qquad\times\left(i\delta_{r,r'}\frac{(\underline{x}-\underline{x}_0)\cdot\epsilon^\lambda}{|\underline{x}-\underline{x}_0|} mK_1\big(|\underline{x}-\underline{x}_0|m\big)\big[r(1-2\alpha)+\lambda\big] + K_0\big(|\underline{x}-\underline{x}_0|m\big) r\delta_{r,-r'} m(1+r\lambda)\right),
\end{aligned} \tag{7}$$

where $\underline{x}_1$ and $\underline{x}_0$ are the quark's and antiquark's transverse coordinates in the amplitude correspondingly, see Fig. 1 and $x \equiv |\underline{x}|$. Averaging square of Eq. (7) over quantum numbers of the initial gluon and summing over quantum numbers of the final quark and antiquark [18] we find

$$\begin{aligned}
\Phi_{g\to q\bar{q}}(\underline{z}, \underline{x}, \underline{x}_0, \alpha) = \frac{\alpha_s}{\pi} m^2 \bigg(&\frac{(\underline{x}-\underline{x}_0)\cdot(\underline{y}-\underline{x}_0)}{|\underline{x}-\underline{x}_0||\underline{y}-\underline{x}_0|} K_1\big(|\underline{x}-\underline{x}_0|m\big) K_1\big(|\underline{y}-\underline{x}_0|m\big)\big[\alpha^2+(1-\alpha)^2\big]\\
&+ K_0\big(|\underline{x}-\underline{x}_0|m\big) K_0\big(|\underline{y}-\underline{x}_0|m\big)\bigg),
\end{aligned} \tag{8}$$

where $\underline{y}$ is the quark's transverse coordinate in the complex conjugate amplitude and we do not include two delta functions (see (7)) in definition of $\Phi_{g\to q\bar{q}}$. Eq. (8) is a special case of light-cone wave function of an off-shell gauge boson derived in Refs. [18,19].

Rescatterings of the produced partonic system in a nucleus must be calculated separately for each time ordering [20] as shown in Fig. 1. The result can be written in terms of the Fourier transformation of the normalized gluon–nucleon cross section [20,21]:

$$V(\underline{x}) = \int d^2l\, e^{-i\underline{l}\cdot\underline{x}} \frac{1}{\sigma}\frac{d\sigma}{d^2l}. \tag{9}$$

All diagrams contributing to the time ordering of the diagram F in Fig. 1 are shown in Fig. 2. The sum of diagrams (a)–(j) in Fig. 2 is given by

$$\begin{aligned}
F &= \frac{1}{2N_c}\left(N_c - \frac{2}{N_c}\right)V(\underline{x}) + \frac{1}{N_c^2}V(\underline{x}_0) - \frac{1}{2N_c^2}V(0) - \frac{C_F}{2N_c}V(0) - \frac{C_F}{2N_c}V(0) - \frac{1}{2N_c^2}V(\underline{x}-\underline{x}_0)\\
&\quad - \frac{C_F}{2N_c}V(0) + \frac{1}{N_c^2}V(0) - \frac{C_F}{2N_c}V(\underline{0}) + \frac{1}{2N_c}\left(N_c - \frac{2}{N_c}\right)V(\underline{x}_0)\\
&= \frac{1}{2}\big(V(\underline{x}) - V(0)\big) + \frac{1}{2}\big(V(\underline{x}_0) - V(0)\big) - \frac{1}{2N_c^2}\big(V(\underline{x}-\underline{x}_0) - V(0)\big),
\end{aligned} \tag{10}$$

where $\underline{x}$ and $\underline{y}$ are coordinates of the produced quark in the amplitude and in the complex conjugate one correspondingly, $\underline{x}_0$ is coordinate of antiquark, see Fig. 1. Multiplying expression (10) by the nucleus profile function $T(\underline{b})$, nuclear density ρ and the gluon–nucleon cross section σ [20] we obtain

$$-P(\underline{x}, \underline{x}_0) = -\frac{1}{8}\underline{x}^2 Q_s^2 - \frac{1}{8}\underline{x}_0^2 Q_s^2 + \frac{1}{8N_c^2}(\underline{x}-\underline{x}_0)^2 Q_s^2, \tag{11}$$

where we follow notations of [22]. The saturation scale Q_s^2 in (11) is given by [17,20]

$$Q_s^2(\underline{x}) = \frac{4\pi^2\alpha_s N_c}{N_c^2-1}\rho T(\underline{b}) xG(x, 1/\underline{x}^2), \tag{12}$$

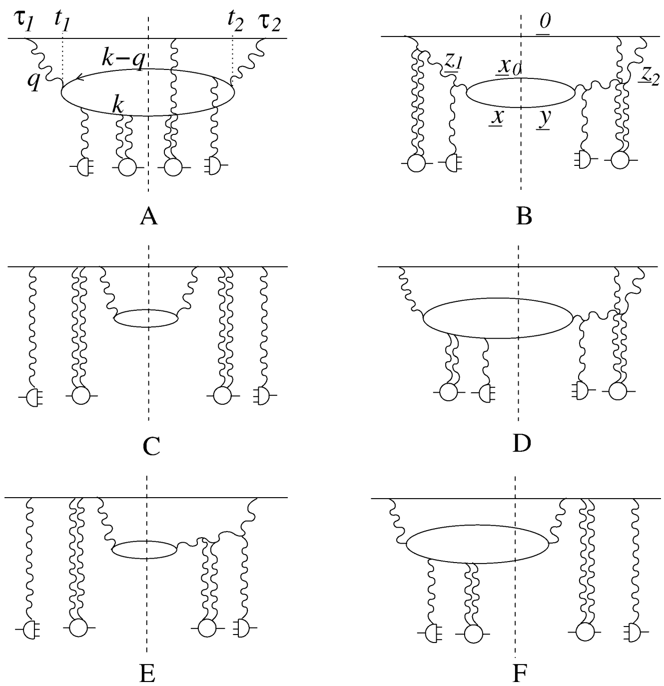

Fig. 1. Diagrams which contribute to the heavy quark production in the light-cone perturbation theory. A: $\tau_1 < 0$, $t_1 < 0$, $\tau_2 < 0$, $t_2 < 0$, B: $\tau_1 < 0$, $\tau_2 < 0$, $t_1 > 0$, $t_2 > 0$, C: $\tau_1 > 0$, $t_1 > 0$, $\tau_2 > 0$, $t_2 > 0$, D: $\tau_1 < 0$, $t_1 < 0$, $\tau_2 < 0$, $t_2 > 0$, E: $\tau_1 > 0$, $t_1 > 0$, $\tau_2 < 0$, $t_2 > 0$, F: $\tau_1 < 0$, $t_1 < 0$, $\tau_2 > 0$, $t_2 > 0$. Not shown are the complex conjugates D*: $\tau_1 < 0$, $t_1 > 0$, $\tau_2 < 0$, $t_2 < 0$, E*: $\tau_1 < 0$, $t_1 > 0$, $\tau_2 > 0$, $t_2 > 0$, F*: $\tau_1 > 0$, $t_1 > 0$, $\tau_2 < 0$, $t_2 < 0$. Instantaneous interaction of a $q_v g q \bar{q}$ system with the nucleus happens at light-cone time $\tau_{\text{int}} = 0$. The final state is denoted by the vertical dashed line at $\tau = \infty$.

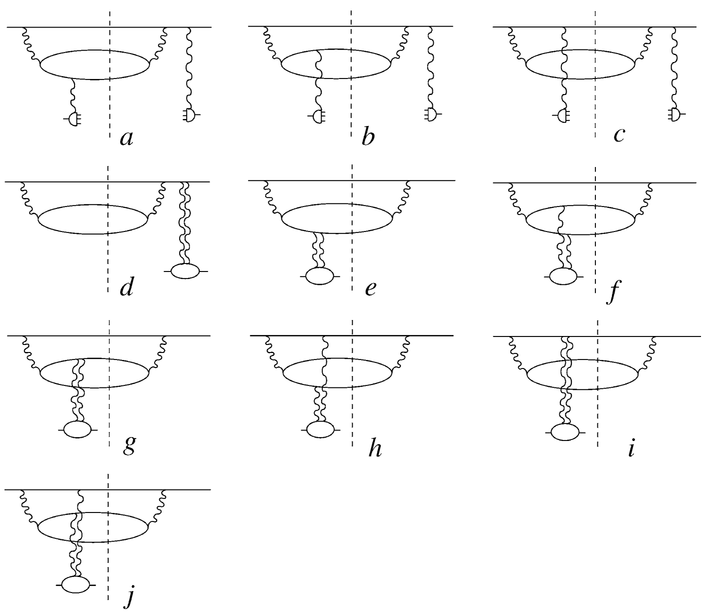

Fig. 2. Diagrams contributing to the time ordering of the diagram F in Fig. 1.

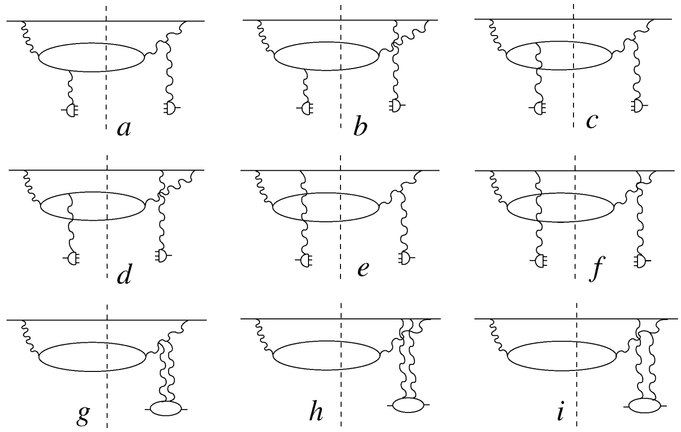

Fig. 3. Diagrams contributing to the time ordering of the diagram D in Fig. 1.

where the gluon distribution function in a nucleon reads

$$xG(x,1/\underline{x}^2)=\frac{\alpha_s C_F}{\pi}\ln\frac{1}{\underline{x}^2\mu^2}, \tag{13}$$

with μ some infrared cutoff. For spherical nucleus $T(\underline{b})=2\sqrt{R^2-\underline{b}^2}$. Assuming that the interactions of a proton with individual nucleons are independent [17] we can exponentiate the formula (11) to obtain for the diagram F on Fig. 1

$$F=\exp\{-P(\underline{x},\underline{x}_0)\}=\exp\left\{-\frac{1}{8}\underline{x}^2Q_s^2-\frac{1}{8}\underline{x}_0^2Q_s^2+\frac{1}{8N_c^2}(\underline{x}-\underline{x}_0)^2Q_s^2\right\}. \tag{14}$$

This formula coincides with the $q\bar{q}g$ "propagator" derived in Refs. [22–24].

Analogously, all diagrams contributing to the time ordering of the diagram D in Fig. 1 are shown in Fig. 3 and in Fig. 2: (e)–(j). The sum of diagrams (a)–(i) in Fig. 3 and (e)–(j) in Fig. 2 yields

$$\begin{aligned}
D&=\frac{1}{2}V(\underline{x}-\underline{z}_2)-\frac{1}{N_c^2}V(\underline{x})+\frac{1}{2}V(\underline{x}_0-z_2)-\frac{1}{2N_c}\left(N_c-\frac{2}{N_c}\right)V(\underline{x}_0)\\
&\quad-\frac{1}{2}V(\underline{z}_2)+\frac{C_F}{N_c}V(0)-\frac{1}{2}V(0)-\frac{C_F}{2N_c}V(0)+\frac{1}{2}V(\underline{z}_2)-\frac{C_F}{2N_c}V(0)-\frac{1}{2N_c^2}V(\underline{x}-\underline{x}_0)\\
&\quad-\frac{C_F}{2N_c}V(0)+\frac{1}{N_c^2}V(0)-\frac{C_F}{2N_c}V(0)-\frac{1}{2N_c}\left(N_c-\frac{2}{N_c}\right)V(\underline{x}_0)\\
&=\frac{1}{2}\big(V(\underline{x}-\underline{z}_2)-V(0)\big)+\frac{1}{2}\big(V(\underline{x}_0-\underline{z}_2)-V(0)\big)-\frac{1}{2N_c^2}\big(V(\underline{x}-\underline{x}_0)-V(0)\big).
\end{aligned} \tag{15}$$

Multiplying (15) by $T(\underline{b})\rho\sigma$ and exponentiating we derive

$$D=\exp\{-P(\underline{x}-\underline{z}_2,\underline{x}_0-\underline{z}_2)\}, \tag{16}$$

where we used definition (14). Complex conjugated of F and D can be obtained by replacing $\underline{x}\leftrightarrow\underline{y}$, and $\underline{z}_1\leftrightarrow\underline{z}_2$:

$$D^*=\exp\{-P(\underline{y},\underline{x}_0)\}, \tag{17}$$

$$F^*=\exp\{-P(\underline{y}-\underline{z}_1,\underline{x}_0-\underline{z}_1)\}. \tag{18}$$

Diagrams B, C and E, E* have been calculated in [20]. The only difference is additional color factor $1/2$ emerging due to fluctuation of a virtual gluon into quark–antiquark pair. We included this factor in the definition of the wave function (8). We have

$$B = \exp\left\{-\frac{1}{4}(\underline{z}_1 - \underline{z}_2)^2 Q_s^2\right\}, \tag{19}$$

$$E = \exp\left\{-\frac{1}{4}\underline{z}_2^2 Q_s^2\right\}, \tag{20}$$

$$E^* = \exp\left\{-\frac{1}{4}\underline{z}_1^2 Q_s^2\right\}. \tag{21}$$

Finally, it is easy to see that the diagram A in Fig. 1 is equal to

$$A = \exp\left\{-\frac{1}{4}\frac{C_F}{N_c}(\underline{x} - \underline{y})^2 Q_s^2\right\}. \tag{22}$$

Summing up diagrams A–E and their complex conjugates results in the following rescatterings factor

$$\begin{aligned} &\Xi(\underline{x}, \underline{y}, \underline{x}_0, \underline{z}_1, \underline{z}_2) \\ &\quad = \exp\left\{-\frac{1}{4}(\underline{z}_1 - \underline{z}_2)^2 Q_s^2\right\} - \exp\left\{-\frac{1}{4}\underline{z}_1^2 Q_s^2\right\} - \exp\left\{-\frac{1}{4}\underline{z}_2^2 Q_s^2\right\} + \exp\left\{-\frac{1}{4}\frac{C_F}{N_c}(\underline{x} - \underline{y})^2 Q_s^2\right\} \\ &\qquad + e^{-P(\underline{x},\underline{x}_0)} + e^{-P(\underline{y},\underline{x}_0)} - e^{-P(\underline{x}-\underline{z}_2,\underline{x}_0-\underline{z}_2)} - e^{-P(\underline{y}-\underline{z}_1,\underline{x}_0-\underline{z}_1)}. \end{aligned} \tag{23}$$

Using (5), (8) and (23) we can write down the inclusive quark production cross section

$$\begin{aligned} \frac{d\sigma}{d^2k\,dy} = &\int d^2b\, d^2z_1\, d^2z_2 \frac{\alpha_s C_F}{\pi^2} \frac{\underline{z}_1 \cdot \underline{z}_2}{\underline{z}_1^2 \underline{z}_2^2} \int d^2x_0 \int d\alpha \int \frac{d^2x\, d^2y}{(2\pi)^3} \Phi_{g\to q\bar{q}}(\underline{x} - \underline{x}_0, \underline{y} - \underline{x}_0, \alpha) \\ &\times e^{-i\underline{k}\cdot(\underline{x}-\underline{y})} \Xi(\underline{x}, \underline{y}, \underline{x}_0, \underline{z}_1, \underline{z}_2) \delta\big((\underline{x}_0 - \underline{z}_1) + \alpha(\underline{x} - \underline{x}_0)\big) \delta\big((\underline{x}_0 - \underline{z}_2) + \alpha(\underline{y} - \underline{x}_0)\big), \end{aligned} \tag{24}$$

where $\underline{b}$ is an impact parameter. This formula is a generalization of result obtained by Kopeliovich and Tarasov in Ref. [25].

Formula (24) is the main result of our Letter. It resums all higher twist effects in the quasi-classical approximation which means that we keep all terms proportional to $\alpha_s^2 A^{1/3} \sim 1$ and neglect terms suppressed by powers of $\alpha_s \ll 1$. We explicitly neglected the low-x quantum evolution effects assuming that $\alpha_s \ln(1/x) \ll 1$. Therefore formula (24) is applicable when $e^{-1/\alpha_s} \lesssim x \ll 1$. As resent experimental data on dA collisions at RHIC show, this corresponds to the central rapidity region at $\sqrt{s} = 200$ GeV [43].

Formula (24) has been used in Ref. [9] for numerical calculations of charm production at RHIC. It was shown that the charm spectrum obtained according to (24) is much harder than in naive parton model approach. This is attributed to the presence of a hard 'intrinsic' scale Q_s^2. It is clear that the dependence of a heavy quark yield on A is closely related to the relation between Q_s and m. In the strong color field $Q_s \gg m$ the total cross section of heavy quark production in pA collisions is proportional to the transverse size of a nucleus $\sigma_{\text{tot}} \sim A^{2/3}$ due to saturation in a nucleus wave function. In the opposite limit $Q_s \ll m$ the color field of a nucleus is not able to produce heavy quarks from the vacuum in which case $\sigma_{\text{tot}} \sim A$. Therefore, at high energies one expects suppression of the heavy quark yield. In the case of charm quark production numerical calculations in [9] show that at $y = 0$ at RHIC the charm quark yield is not suppressed. However, at forward rapidities it gets suppressed since the nuclear color field strength increases at small x due to quantum evolution. I refer the reader interested in phenomenological applications of (24) to Ref. [9] for more detailed discussion.

Dynamics of saturated quasi-classical color fields dominates the total multiplicities of AA and dA collisions in the central rapidity region at RHIC [7,26–29]. The Cronin enhancement seen in the data [30] is produced by multiple rescatterings of a proton [34,35] in a saturated wave function of a nucleus [31–33]. These multiple rescatterings produce particle correlations which give a substantial contribution to the elliptic flow phenomenon in AA collisions [36]. Eq. (24) can be used for analysis of the heavy quark production in pA collisions at the central rapidity region at RHIC. In particular, one can address the question of whether formula (24) yields the Cronin enhancement of charm production analogously to the case of gluon production [31–33].

High energy quantum evolution has been neglected throughout this Letter. However, as energy/rapidity increases the quantum evolution becomes important [14–16,37]. It gives rise to a number of spectacular effects [9, 31,33,38] associated with the extended geometric scaling phenomenon [39–42]. Recent results of BRAHMS Collaboration [43] at RHIC indicate onset of the high energy evolution at rapidities close to the proton fragmentation region in agreement with theoretical predictions. Therefore, generalization of (24) to include low-x quantum evolution is an important task which will be pursued in our forthcoming publications.

Finally, we would like to mention an important theoretical question which have not been touched in this Letter. It is whether Eq. (24) can be reduced to the k_T-factorized form. The k_T-factorization was proved for *dilute* target regime in [8,44,45] and have been recently rederived in a Color Glass Condensate framework in Ref. [46]. It was also proved in Ref. [47] that the inclusive gluon production cross section in pA collisions can be reduced to the k_T-factorized form even if the quantum evolution is included. So far all phenomenological studies of the heavy quark hadroproduction at high partonic densities [9,48] have been based on k_T-factorization. Therefore, it is important to understand to what extend it can be realized at high energies and/or for heavy nuclei. We are going to address this problem elsewhere.

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