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CFT adapted gauge invariant formulation of massive arbitrary spin fields in AdS

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ABSTRACT

Using Poincaré parametrization of *AdS* space, we study massive totally symmetric arbitrary spin fields in *AdS* space of dimension greater than or equal to four. CFT adapted gauge invariant formulation for such fields is developed. Gauge symmetries are realized by using Stueckelberg formulation of massive fields. We demonstrate that the mass parameter, curvature and radial coordinate contributions to the gauge transformation and Lagrangian of the *AdS* massive fields can be expressed in terms of ladder operators. Three representations for the Lagrangian are discussed. Realization of the global *AdS* symmetries in the conformal algebra basis is obtained. Modified de Donder gauge leading to simple gauge fixed Lagrangian is found. The modified de Donder gauge leads to decoupled equations of motion which can easily be solved in terms of the Bessel function. New simple representation for gauge invariant Lagrangian of massive $(A)dS$ field in arbitrary coordinates is obtained. Light-cone gauge Lagrangian of massive *AdS* field is also presented.

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1. Introduction

Further progress in understanding AdS/CFT correspondence requires, among other things, better understanding of field dynamics in *AdS* space. Although many interesting approaches to *AdS* fields are known in the literature (for review see Refs. [1–3]), analysis of concrete dynamical aspects of such fields is still a challenging procedure. One of ways to simplify analysis of field and string dynamics in *AdS* space is based on use of the Poincaré parametrization of *AdS* space.[2] Use of the Poincaré coordinates simplifies analysis of many aspect of *AdS* field dynamics and therefore these coordinates have extensively been used for studying the AdS/CFT correspondence. In Ref. [8], we developed a approach which is based on considering of *AdS* field dynamics in the Poincaré coordinates and applied our approach to study of massless *AdS* fields. We think that our approach might be useful for study of *AdS* string massive modes. Therefore it is desirable to generalize our approach to the case of massive *AdS* fields. This is that what we do in this Letter. Namely, using the Poincaré parametrization of *AdS* space we discuss massive totally symmetric arbitrary spin-s, $s \geqslant 1$, bosonic field propagating in AdS_{d+1} space of dimension $d+1 \geqslant 4$. Our results can be summarized as follows.

i) Using the Poincaré parametrization of *AdS*, we obtain gauge invariant Lagrangian for free massive arbitrary spin *AdS* field. The Lagrangian is *explicitly invariant with respect to boundary Poincaré symmetries*, i.e., manifest symmetries of our Lagrangian are adapted to manifest symmetries of boundary CFT. We show that all the mass parameter, curvature and radial coordinate contributions to our Lagrangian and gauge transformation are entirely expressed in terms of ladder operators that depend on the mass parameter, radial coordinate and radial derivative. General structure of the Lagrangian we use is the same as the one for massless *AdS* fields. Lagrangian of massive *AdS* field is distinguished by appropriate ladder operators. We find two new concise expressions for the gauge invariant Lagrangian.

ii) We generalize modified de Donder gauge, found for massless *AdS* fields in Ref. [8], to the case of massive fields. As in the case of massless fields, the modified de Donder gauge leads to simple gauge fixed Lagrangian and *decoupled equations of motion*.[3] Note that the standard de Donder gauge leads to coupled equations of motion whose solutions for $s \geqslant 2$ are not known in closed form even for massless *AdS* fields. In contrast to this, our modified de Donder gauge leads to simple decoupled equations which are easily solved in terms of the Bessel function.

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[2] Studying $AdS_5 \times S^5$ superstring action [4] in Poincaré parametrization may be found in Ref. [5]. Recent interesting application of Poincaré coordinates to studying $AdS_5 \times S^5$ string T-duality may be found in Refs. [6,7].

[3] Our modified de Donder gauge seems to be unique gauge that leads to decoupled equations of motion. Light-cone gauge [9] also leads to decoupled equations of motion, but the light-cone gauge breaks boundary Lorentz symmetries.

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2. Lagrangian and its gauge and global symmetries

We begin with discussion of field content of our approach. In Ref. [10], the massive spin-s field propagating in AdS_{d+1} space is described by double-traceless $so(d,1)$ algebra totally symmetric tensor fields $\Phi^{A_1\ldots A_{s'}}$, $s'=0,1,\ldots,s$.[4] These tensor fields can be decomposed in real-valued scalar, vector, and totally symmetric tensor fields of the $so(d-1,1)$ algebra:

$$\phi_\lambda^{a_1\ldots a_{s'}},\quad \lambda=[s-s']_2,\quad s'=0,1,\ldots,s-1,s. \tag{2.1}$$

Henceforth, the notation $\lambda=[n]_2$ implies that $\lambda=-n,-n+2,-n+4,\ldots,n-4,n-2,n$. To illustrate the field content given in (2.1) we use shortcut $\phi_{(s',\lambda)}$ for the field $\phi_\lambda^{a_1\ldots a_{s'}}$ and note that fields in (2.1) can be represented as

$$\begin{array}{ccccccccc}
 & & & & \phi_{(s,0)} & & & & \\
 & & & \phi_{(s-1,-1)} & & \phi_{(s-1,1)} & & & \\
 & & \cdots & & \cdots & & \cdots & & \\
 & \phi_{(1,1-s)} & & \phi_{(1,3-s)} & \cdots & \phi_{(1,s-3)} & & \phi_{(1,s-1)} & \\
\phi_{(0,-s)} & & \phi_{(0,2-s)} & & \cdots & & \phi_{(0,s-2)} & & \phi_{(0,s)}.
\end{array} \tag{2.2}$$

The fields $\phi_\lambda^{a_1\ldots a_{s'}}$ with $s'\geqslant 4$ are double-traceless,[5]

$$\phi_\lambda^{aabba_5\ldots a_{s'}}=0,\quad \lambda=[s-s']_2,\quad s'=4,5,\ldots,s. \tag{2.3}$$

The fields in (2.1) subject to constraints (2.3) constitute a field content of our approach.

To simplify presentation we use creation operators α^a, α^z, ζ and the respective annihilation operators, $\bar\alpha^a$, $\bar\alpha^z$, $\bar\zeta$. Then, fields (2.1) can be collected into a ket-vector $|\phi\rangle$ defined by[6]

$$|\phi\rangle=\sum_{s'=0}^{s}\sum_{\lambda=[s-s']_2}\frac{\zeta^{\frac{s-s'+\lambda}{2}}\alpha_z^{\frac{s-s'-\lambda}{2}}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-s'+\lambda}{2})!(\frac{s-s'-\lambda}{2})!}}\phi_\lambda^{a_1\ldots a_{s'}}|0\rangle. \tag{2.4}$$

From (2.4) we see that the ket-vector $|\phi\rangle$ is degree-s homogeneous polynomial in α^a, α^z, ζ.[7] In terms of the ket-vector $|\phi\rangle$, double-tracelessness constraint (2.3) takes the form[8]

$$(\bar\alpha^2)^2|\phi\rangle=0. \tag{2.5}$$

Action and Lagrangian we found take the form

$$S=\int d^dx\,dz\,\mathcal{L},\quad \mathcal{L}=\frac{1}{2}\langle\phi|E|\phi\rangle, \tag{2.6}$$

$\langle\phi|\equiv(|\phi\rangle)^\dagger$. We now discuss various representations for operator E and the Lagrangian in turn.

1st representation. This representation is given by

$$E=E_{(2)}+E_{(1)}+E_{(0)}, \tag{2.7}$$

$$E_{(2)}\equiv\Box-\alpha\partial\bar\alpha\partial+\frac{1}{2}(\alpha\partial)^2\bar\alpha^2+\frac{1}{2}\alpha^2(\bar\alpha\partial)^2-\frac{1}{2}\alpha^2\Box\bar\alpha^2 -\frac{1}{4}\alpha^2\alpha\partial\bar\alpha\partial\bar\alpha^2, \tag{2.8}$$

$$E_{(1)}\equiv\bar e_1\mathcal{A}+e_1\bar{\mathcal{A}}, \tag{2.9}$$

$$E_{(0)}\equiv m_1+\alpha^2\bar\alpha^2m_2+\bar m_3\alpha^2+m_3\bar\alpha^2, \tag{2.10}$$

$$\mathcal{A}\equiv\alpha\partial-\alpha^2\bar\alpha\partial+\frac{1}{4}\alpha^2\alpha\partial\bar\alpha^2, \tag{2.11}$$

$$\bar{\mathcal{A}}\equiv\bar\alpha\partial-\alpha\partial\bar\alpha^2+\frac{1}{4}\alpha^2\bar\alpha\partial\bar\alpha^2, \tag{2.12}$$

$$m_1=\bar e_1e_1-2\frac{2s+d-3-2N_z-2N_\zeta}{2s+d-4-2N_z-2N_\zeta}e_1\bar e_1, \tag{2.13}$$

$$m_2=-\frac{1}{2}\bar e_1e_1+\frac{1}{4}\frac{2s+d-2N_z-2N_\zeta}{2s+d-4-2N_z-2N_\zeta}e_1\bar e_1, \tag{2.14}$$

$$m_3=\frac{1}{2}e_1e_1,\qquad \bar m_3=\frac{1}{2}\bar e_1\bar e_1, \tag{2.15}$$

$$-e_1=\zeta r_\zeta\mathcal{T}_{-\nu-\frac{1}{2}}+\alpha^zr_z\mathcal{T}_{\nu-\frac{1}{2}},$$
$$-\bar e_1=\mathcal{T}_{\nu+\frac{1}{2}}r_\zeta\bar\zeta+\mathcal{T}_{-\nu+\frac{1}{2}}r_z\bar\alpha^z, \tag{2.16}$$

$$\mathcal{T}_\nu=\partial_z+\frac{\nu}{z},\quad \nu=\kappa+N_\zeta-N_z,\quad \kappa\equiv E_0-\frac{d}{2}, \tag{2.17}$$

$$r_\zeta=\left(\frac{(s+\frac{d-4}{2}-N_\zeta)(\kappa-s-\frac{d-4}{2}+N_\zeta)(\kappa+1+N_\zeta)}{2(s+\frac{d-4}{2}-N_\zeta-N_z)(\kappa+N_\zeta-N_z)(\kappa+N_\zeta-N_z+1)}\right)^{1/2}, \tag{2.18}$$

$$r_z=\left(\frac{(s+\frac{d-4}{2}-N_z)(\kappa+s+\frac{d-4}{2}-N_z)(\kappa-1-N_z)}{2(s+\frac{d-4}{2}-N_\zeta-N_z)(\kappa+N_\zeta-N_z)(\kappa+N_\zeta-N_z-1)}\right)^{1/2}, \tag{2.19}$$

where subscript n in $E_{(n)}$ (2.7) tells us that $E_{(n)}$ is degree-n homogeneous polynomial in the flat derivative ∂^a. The following remarks are in order.

i) The parameter κ (2.17) is expressed in terms of spin-s massive field lowest energy E_0. Using result in Ref. [24] we can express κ in terms of the standard mass parameter m,

$$\kappa=\sqrt{m^2+\left(s+\frac{d-4}{2}\right)^2}. \tag{2.20}$$

ii) Operator $E_{(2)}$ (2.8) is the symmetrized Fronsdal operator represented in terms of the oscillators. This operator takes the same form as the one of massless field in d-dimensional flat space. Thus, the operator E (2.7) is given by the sum of the standard Fronsdal operator $E_{(2)}$ and new operators $E_{(1)}$, $E_{(0)}$ which depend on the mass parameter m, the radial coordinate and derivative, z, ∂_z.

iii) Dependence of E (2.7) on the mass parameter m, the radial coordinate and derivative, z, ∂_z, is entirely governed by the operators e_1 and $\bar e_1$ (2.16) which we will refer to as ladder operators.[9]

iv) Representation for the Lagrangian in (2.6)–(2.15) is universal and is valid for arbitrary Poincaré invariant theory. Various

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[4] $A,B,C=0,1,\ldots,d$ and $a,b,c=0,1,\ldots,d-1$ are the respective flat vector indices of the $so(d,1)$ and $so(d-1,1)$ algebras. In Poincaré parametrization of AdS_{d+1} space, $ds^2=(dx^adx^a+dzdz)/z^2$. We use the conventions: $\partial_a\equiv\partial/\partial x^a$, $\partial_z\equiv\partial/\partial z$. Vectors of $so(d,1)$ algebra are decomposed as $X^A=(X^a,X^z)$.

[5] Note that $so(d-1,1)$ tensorial components of the Fronsdal–Zinoviev fields $\Phi^{A_1\ldots A_{s'}}$ are not double-traceless. Using appropriate transformation (see (4.28)) those tensorial components can be transformed to our fields in (2.1).

[6] We use oscillator formulation to handle the many indices appearing for tensor fields (for review see Refs. [12,13]). In a proper way, oscillators arise in the framework of world-line approach to higher-spin fields (see e.g. Refs. [14,15]).

[7] Throughout this Letter we use the following notation for operators constructed out the oscillators and derivatives: $N_\alpha\equiv\alpha^a\bar\alpha^a$, $N_z\equiv\alpha^z\bar\alpha^z$, $N_\zeta\equiv\zeta\bar\zeta$, $\alpha^2=\alpha^a\alpha^a$, $\bar\alpha^2=\bar\alpha^a\bar\alpha^a$, $\Box=\partial^a\partial^a$, $\alpha\partial=\alpha^a\partial^a$, $\bar\alpha\partial=\bar\alpha^a\partial^a$.

[8] We adapt the formulation in terms of the double-traceless gauge fields [11] (see also Refs. [10,16]). Discussion of various formulations in terms of unconstrained gauge fields may be found in Refs. [17–21]. Study of other interesting approaches which seem to be most suitable for the theory of interacting fields may be found e.g. in Refs. [22,23].

[9] Interesting application of other ladder operators to studying AdS/QCD correspondence may be found in Ref. [25]. We believe that our approach will also be useful for better understanding of various aspects of AdS/QCD correspondence which are discussed e.g. in Refs. [25,26].

Poincaré invariant theories are distinguished by ladder operators entering the operator E. This is to say that the operators E of massive and conformal fields in flat space depend on the oscillators α^a, $\bar\alpha^a$ and the flat derivative ∂^a in the same way as the operator E of *AdS* fields (2.7). In other words, the operators E for massless and massive *AdS* fields, massive and conformal fields in flat space are distinguished only by the operators e_1 and $\bar e_1$. We note that it is finding the ladder operators that provides real difficulty. Expressions for e_1, $\bar e_1$ appropriate for conformal and massive fields in flat space may be found in Refs. [27,28].

2nd representation for the operator E. Lagrangian can be presented in the form given in (2.6) with the following concise expression for the operator E:

$$E = \boldsymbol{\mu}\big(\Box - \mathcal{M}_\nu^2\big) - C\bar C, \tag{2.21}$$

$$\mathcal{M}_\nu^2 \equiv -\partial_z^2 + \frac{1}{z^2}\left(\nu^2 - \frac{1}{4}\right), \tag{2.22}$$

$$\bar C \equiv \bar\alpha\partial - \frac{1}{2}\alpha\partial\bar\alpha^2 - \bar e_1 \Pi^{[1,2]} + \frac{1}{2}e_1\bar\alpha^2, \tag{2.23}$$

$$C \equiv \alpha\partial - \frac{1}{2}\alpha^2\bar\alpha\partial - e_1 \Pi^{[1,2]} + \frac{1}{2}\bar e_1\alpha^2, \tag{2.24}$$

$$\boldsymbol{\mu} \equiv 1 - \frac{1}{4}\alpha^2\bar\alpha^2, \qquad \Pi^{[1,2]} \equiv 1 - \alpha^2\frac{1}{2(2N_\alpha + d)}\bar\alpha^2, \tag{2.25}$$

where ν is given in (2.17). Operator E in (2.21) differs from the one in (2.7) by terms proportional to $(\alpha^2)^2$ and $(\bar\alpha^2)^2$. Therefore, in view of double-tracelessness constraint (2.5), these two representations for E lead to the same Lagrangian (2.6). We note that operator E in (2.21) respects, in contrast to the one in (2.7), double-tracelessness constraint (2.5). For massless field in d-dimensional *flat* space, $e_1 = \bar e_1 = 0$, operator $\bar C$ (2.23) coincides with the standard de Donder operator. In terms of the ladder operators, mass operator $\mathcal{M}^2$ takes the form

$$\mathcal{M}^2 \equiv -\bar e_1 e_1 + \frac{2s + d - 2 - 2N_z - 2N_\zeta}{2s + d - 4 - 2N_z - 2N_\zeta}e_1\bar e_1, \tag{2.26}$$

while the $\mathcal{M}_\nu^2$ (2.22) is obtained form (2.26) by using ladder operators given in (2.16).

CFT adapted representation of the Lagrangian. Taking into account representation for operator E in (2.21) and noticing the relations $\mathcal{M}_\nu^2 = \mathcal{T}_{\nu-\frac{1}{2}}^\dagger \mathcal{T}_{\nu-\frac{1}{2}}$, $C = -\bar C^\dagger$, where $\mathcal{T}_\nu$ is given in (2.17), we see that Lagrangian (2.6) can be represented as (up to total derivatives)

$$\mathcal{L} = -\frac{1}{2}\langle\partial^a\phi|\boldsymbol{\mu}|\partial^a\phi\rangle - \frac{1}{2}\langle\mathcal{T}_{\nu-\frac{1}{2}}\phi|\boldsymbol{\mu}|\mathcal{T}_{\nu-\frac{1}{2}}\phi\rangle + \frac{1}{2}\langle\bar C\phi||\bar C\phi\rangle. \tag{2.27}$$

This form of the Lagrangian turns out to be very convenient for studying AdS/CFT correspondence.

Gauge symmetries. We now discuss gauge symmetries of Lagrangian (2.6). To this end we introduce the following set of gauge transformation parameters:

$$\xi_\lambda^{a_1\ldots a_{s'}}, \quad \lambda = \big[s - 1 - s'\big]_2, \quad s' = 0, 1, \ldots, s - 1. \tag{2.28}$$

The gauge parameters ξ_λ, ξ_λ^a, and $\xi_\lambda^{a_1\ldots a_{s'}}$, $s' \geqslant 2$ in (2.28), are the respective scalar, vector, and rank-s' totally symmetric tensor fields of the $so(d-1,1)$ algebra. The gauge parameters $\xi_\lambda^{a_1\ldots a_{s'}}$ with $s' \geqslant 2$ are subjected to the tracelessness constraint,

$$\xi_\lambda^{aaa_3\ldots a_{s'}} = 0, \quad \lambda = \big[s - 1 - s'\big]_2, \quad s' = 2, 3, \ldots, s - 1. \tag{2.29}$$

We now, as usually, collect gauge transformation parameters in ket-vector $|\xi\rangle$ defined by

$$|\xi\rangle = \sum_{s'=0}^{s-1}\sum_{\lambda=[s-1-s']_2}\frac{\zeta^{\frac{s-1-s'+\lambda}{2}}\alpha_z^{\frac{s-1-s'-\lambda}{2}}\alpha^{a_1}\ldots\alpha^{a_{s'}}}{s'!\sqrt{(\frac{s-1-s'+\lambda}{2})!(\frac{s-1-s'-\lambda}{2})!}}\xi_\lambda^{a_1\ldots a_{s'}}|0\rangle. \tag{2.30}$$

We note that the ket-vector $|\xi\rangle$ is a degree-$(s-1)$ homogeneous polynomial in the oscillators α^a, α^z, ζ. In terms of the ket-vector $|\xi\rangle$, tracelessness constraint (2.29) takes the form

$$\bar\alpha^2|\xi\rangle = 0. \tag{2.31}$$

Lagrangian (2.6) is invariant under the following gauge transformation:

$$\delta|\phi\rangle = G|\xi\rangle, \quad G = \alpha\partial - e_1 - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar e_1, \tag{2.32}$$

where e_1, $\bar e_1$ are given in (2.16). From (2.32), we see that the mass parameter, radial coordinate and derivative contributions to gauge transformation (2.32) are entirely expressed in terms of the ladder operators e_1 and $\bar e_1$. We note that use of operator G (2.32) allows us to write new representation for the operator E entering Lagrangian (2.6),

$$E = \boldsymbol{\mu}\big(\Box - \mathcal{M}_\nu^2 - G\bar C\big). \tag{2.33}$$

Global so(d,2) symmetries. Relativistic symmetries of AdS_{d+1} space are described by the $so(d,2)$ algebra. In our approach, the massive spin-s AdS_{d+1} field is described by the set of the $so(d-1,1)$ algebra fields (2.1). Therefore it is reasonable to represent the $so(d,2)$ algebra so that to respect manifest $so(d-1,1)$ symmetries. For application to the AdS/CFT correspondence, most convenient form of the $so(d,2)$ algebra that respects the manifest $so(d-1,1)$ symmetries is provided by nomenclature of the conformal algebra. This is to say that the $so(d,2)$ algebra consists of translation generators P^a, conformal boost generators K^a, dilatation generator D, and generators J^{ab} which span $so(d-1,1)$ algebra. Normalization for commutators of the $so(d,2)$ algebra generators we use may be found in formulas (3.1)–(3.4) in Ref. [8].

Requiring $so(d,2)$ symmetries implies that the action is invariant with respect to transformation $\delta_{\hat G}|\phi\rangle = \hat G|\phi\rangle$, where the realization of $so(d,2)$ algebra generators $\hat G$ in terms of differential operators acting on the ket-vector $|\phi\rangle$ takes the form

$$P^a = \partial^a, \qquad J^{ab} = x^a\partial^b - x^b\partial^a + M^{ab}, \tag{2.34}$$

$$D = x\partial + \Delta, \quad \Delta \equiv z\partial_z + \frac{d-1}{2}, \tag{2.35}$$

$$K^a = -\frac{1}{2}x^2\partial^a + x^a D + M^{ab}x^b + R^a, \tag{2.36}$$

$x\partial \equiv x^a\partial^a$, $x^2 \equiv x^a x^a$. In (2.34), (2.36), M^{ab} is spin operator of the $so(d-1,1)$ algebra. Representation of M^{ab} and operator R^a (2.36) on space of ket-vector $|\phi\rangle$ (2.4) takes the form

$$M^{ab} = \alpha^a\bar\alpha^b - \alpha^b\bar\alpha^a, \tag{2.37}$$

$$R^a = z\tilde I^a\big(r_\zeta\bar\zeta + r_z\bar\alpha^z\big) - z\big(\zeta r_\zeta + \alpha^z r_z\big)\bar\alpha^a - \frac{1}{2}z^2\partial^a, \tag{2.38}$$

$$\tilde I^a \equiv \alpha^a - \alpha^2\frac{1}{2N_\alpha + d - 2}\bar\alpha^a, \tag{2.39}$$

where r_ζ, r_z are given in (2.18), (2.19). We see that realization of Poincaré symmetries on bulk *AdS* fields (2.34) coincide with realization of Poincaré symmetries on boundary CFT operators. Note that realization of D- and K^a-symmetries on bulk *AdS* fields (2.35), (2.36) coincides, by module of contributions of operators Δ and R^a, with the realization of D- and K^a-symmetries on boundary CFT operators. Realizations of the $so(d,2)$ algebra on bulk *AdS* fields and boundary CFT operators are distinguished by Δ and R^a. The realization of the $so(d,2)$ symmetries given in (2.34)–(2.36) turns out to be very convenient for studying AdS/CFT correspondence [28].

3. Modified de Donder gauge

To discuss modified de Donder gauge we use representation for Lagrangian given in (2.6), (2.21). It is easy to see that use of the following *modified de Donder* gauge-fixing term

$$\mathcal{L}_{\rm g.fix} = \frac{1}{2}\langle\phi|C\bar{C}|\phi\rangle, \qquad (3.1)$$

leads to the surprisingly simple gauge fixed Lagrangian $\mathcal{L}_{\rm total}$, $\mathcal{L}_{\rm total} \equiv \mathcal{L} + \mathcal{L}_{\rm g.fix}$,

$$\mathcal{L}_{\rm total} = \frac{1}{2}\langle\phi|E_{\rm total}|\phi\rangle,$$

$$E_{\rm total} = \left(1 - \frac{1}{4}\alpha^2\bar{\alpha}^2\right)\left(\Box - \mathcal{M}_\nu^2\right), \qquad (3.2)$$

where $\mathcal{M}_\nu^2$ is given in (2.22). We note that our gauge-fixing term (3.1) respects the Poincaré and dilatation symmetries but breaks the conformal boost K^a-symmetries, i.e., the simple form of gauge fixed Lagrangian (3.2) is achieved at the cost of the K^a-symmetries. In terms of tensorial components, gauge fixed Lagrangian (3.2) takes the form

$$\mathcal{L}_{\rm total} = \sum_{s'=0}^{s}\sum_{\lambda=[s-s']_2}\mathcal{L}_{s',\lambda}, \qquad (3.3)$$

$$\mathcal{L}_{s',\lambda} = \frac{1}{2s'!}\left(\phi_\lambda^{a_1\ldots a_{s'}}\Box_{\kappa+\lambda}\phi_\lambda^{a_1\ldots a_{s'}} - \frac{s'(s'-1)}{4}\phi_\lambda^{aaa_3\ldots a_{s'}}\Box_{\kappa+\lambda}\phi_\lambda^{bba_3\ldots a_{s'}}\right), \qquad (3.4)$$

$$\Box_{\kappa+\lambda} \equiv \Box + \partial_z^2 - \frac{1}{z^2}\left((\kappa+\lambda)^2 - \frac{1}{4}\right). \qquad (3.5)$$

We see that the modified de Donder gauge fixing leads to simple gauge fixed Lagrangian.

We now discuss gauge-fixing procedure at the level of equations of motion. Representation for the operator E in (2.33) turns out to be convenient for this purpose.[10] This is to say that Lagrangian with E in (2.33) leads to the following gauge invariant equations of motion

$$\left(\Box - \mathcal{M}_\nu^2 - G\bar{C}\right)|\phi\rangle = 0. \qquad (3.6)$$

Modified de Donder gauge condition is then defined to be

$$\bar{C}|\phi\rangle = 0, \qquad (3.7)$$

where $\bar{C}$ is given in (2.23). The fact that this gauge is accessible with gauge transformation (2.32) can be proved as follows. i) By virtue of (2.5), we have the relation $\bar{\alpha}^2\bar{C}|\phi\rangle = 0$ which implies that gauge condition (3.7) respects constraint for gauge transformation parameter $|\xi\rangle$, (2.31); ii) Gauge variation of $\bar{C}|\phi\rangle$ is given by $\delta(\bar{C}|\phi\rangle) = (\Box - \mathcal{M}_\nu^2)|\xi\rangle$. Making standard assumption that the operator $\Box - \mathcal{M}_\nu^2$ is invertible, we see that gauge condition (3.7) is indeed accessible.

[10] Operators E in (2.21), (2.33) respect, in contrast to operator E in (2.7), double-tracelessness constraint (2.5).

Using the modified de Donder gauge condition in gauge invariant equations of motion (3.6) leads to the following gauge fixed equations of motion:

$$\left(\Box - \mathcal{M}_\nu^2\right)|\phi\rangle = 0, \qquad (3.8)$$

where $\mathcal{M}_\nu^2$ is defined in (2.22). In terms of fields (2.1), Eqs. (3.8) can be represented as

$$\Box_{\kappa+\lambda}\phi_\lambda^{a_1\ldots a_{s'}} = 0, \quad \lambda = [s-s']_2, \quad s' = 0,1,\ldots,s, \qquad (3.9)$$

where $\Box_{\kappa+\lambda}$ is given in (3.5). Thus, our *modified de Donder gauge condition* (3.7) *leads to decoupled equations of motion* (3.9) which can easily be solved in terms of the Bessel function.[11] For spin-1 field, gauge condition (3.7) turns out to be a modification of the Lorentz gauge.

4. Comparison of standard and modified de Donder gauges

Our approach to the massive spin-s field in AdS_{d+1} is based on use of double-traceless $so(d-1,1)$ algebra fields (2.1). One of popular approaches to the massive spin-s field in AdS_{d+1} is based on use of double-traceless $so(d,1)$ algebra fields $\Phi^{A_1\ldots A_{s'}}$, $s' = 0,1,\ldots,s$, [10]. In this section, our aims are as follows. i) Using the fields $\Phi^{A_1\ldots A_{s'}}$ and arbitrary parametrization of $(A)dS$ space, we find new representation for gauge invariant Lagrangian of massive $(A)dS$ field and standard de Donder gauge condition.[12] ii) We explain how our modified de Donder gauge (3.7) is represented in terms of the fields $\Phi^{A_1\ldots A_{s'}}$. iii) We show explicitly how our fields (2.1) are related to the fields $\Phi^{A_1\ldots A_{s'}}$.

New representation for gauge invariant Lagrangian of massive field in $(A)dS_{d+1}$. We begin with discussion of gauge invariant Lagrangian using arbitrary coordinates of $(A)dS$. To simplify the presentation we introduce ket-vector $|\Phi\rangle$,

$$|\Phi\rangle \equiv \sum_{s'=0}^{s}\frac{\zeta^{s-s'}\alpha^{A_1}\ldots\alpha^{A_{s'}}}{s'!\sqrt{(s-s')!}}\Phi^{A_1\ldots A_{s'}}|0\rangle, \qquad (4.1)$$

$$\left(\bar{\boldsymbol{\alpha}}^2\right)^2|\Phi\rangle = 0, \qquad (4.2)$$

$\boldsymbol{\alpha}^2 \equiv \alpha^A\alpha^A$, $\bar{\boldsymbol{\alpha}}^2 \equiv \bar{\alpha}^A\bar{\alpha}^A$, where (4.2) tells us that the $\Phi^{A_1\ldots A_{s'}}$ are double-traceless. We find the following concise expression for gauge invariant Lagrangian of massive spin-s field in $(A)dS_{d+1}$:

$$\mathcal{L} = \frac{1}{2}e\langle\Phi|E|\Phi\rangle, \qquad (4.3)$$

$$E = \left(1 - \frac{1}{4}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2\right)\left(\Box_{(A)dS} + \mathbf{m}_1 + \rho\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}^2\right) - \mathbf{C}_{\rm st}\bar{\mathbf{C}}_{\rm st}, \qquad (4.4)$$

$$\mathbf{m}_1 = -m^2 + \rho\left(s(s+d-5) - 2d + 4 + N_\zeta(2s+d-1-N_\zeta)\right), \qquad (4.5)$$

$$\bar{\mathbf{C}}_{\rm st} \equiv \bar{\boldsymbol{\alpha}}\mathbf{D} - \frac{1}{2}\boldsymbol{\alpha}\mathbf{D}\bar{\boldsymbol{\alpha}}^2 - \bar{\mathbf{e}}_1\boldsymbol{\Pi}^{[1,2]} + \frac{1}{2}\mathbf{e}_1\bar{\boldsymbol{\alpha}}^2, \qquad (4.6)$$

$$\mathbf{C}_{\rm st} \equiv \boldsymbol{\alpha}\mathbf{D} - \frac{1}{2}\boldsymbol{\alpha}^2\bar{\boldsymbol{\alpha}}\mathbf{D} - \mathbf{e}_1\boldsymbol{\Pi}^{[1,2]} + \frac{1}{2}\bar{\mathbf{e}}_1\boldsymbol{\alpha}^2, \qquad (4.7)$$

[11] Interesting method of solving AdS field equations of motion which is based on star algebra products in auxiliary spinor variables is discussed in Refs. [29,30]. As a side of remark we note that our modified de Donder gauge can be generalized to conformal flat spaces (see Appendix D in Ref. [28]).

[12] To our knowledge, the standard de Donder gauge for arbitrary spin massive $(A)dS$ fields has not been discussed in earlier literature. Study of the standard de Donder gauge for flat arbitrary spin massive fields may be found in Ref. [28]. Recent applications of the *standard* de Donder gauge to the various problems of massless fields may be found in Refs. [31,32].

$$\boldsymbol{\Pi}^{[1,2]} = 1 - \boldsymbol{\alpha}^2 \frac{1}{2(2N_{\boldsymbol{\alpha}} + d + 1)} \bar{\boldsymbol{\alpha}}^2, \quad N_{\boldsymbol{\alpha}} \equiv \alpha^A \bar{\alpha}^A, \tag{4.8}$$

$$\mathbf{e}_1 = \zeta \tilde{\mathbf{e}}_1, \qquad \bar{\mathbf{e}}_1 = -\tilde{\mathbf{e}}_1 \bar{\zeta}, \tag{4.9}$$

$$\tilde{\mathbf{e}}_1 = \left(\frac{2s + d - 3 - N_\zeta}{2s + d - 3 - 2N_\zeta} \right)^{1/2} \left(m^2 - \rho N_\zeta (2s + d - 4 - N_\zeta) \right)^{1/2}, \tag{4.10}$$

$$\Box_{(A)dS} \equiv D^A D^A + \omega^{AAB} D^B,$$

$$\bar{\boldsymbol{\alpha}} \mathbf{D} \equiv \bar{\alpha}^A D^A, \qquad \boldsymbol{\alpha} \mathbf{D} \equiv \alpha^A D^A, \tag{4.11}$$

where $e = \det e_\mu^A$, e_μ^A stands for vielbein of $(A)dS_{d+1}$ space, and D^A is covariant derivative (for details of notation, see Appendix A). We use $\rho = -1$ for AdS space, $\rho = 0$ for flat space, and $\rho = 1$ for dS space. It is the use of operators $\bar{\mathbf{C}}_{\rm st}$, $\mathbf{C}_{\rm st}$ (4.6), (4.7) that allows us to write down the concise expression for operator E in (4.4). For massless field in $(A)dS$, $\mathbf{e}_1 = \bar{\mathbf{e}}_1 = 0$, our operator $\bar{\mathbf{C}}_{\rm st}$ (4.6) coincides with the standard de Donder operator in $(A)dS$ background.

Lagrangian (4.3) is invariant under gauge transformation

$$\delta|\Phi\rangle = \mathbf{G}|\Xi\rangle, \quad \mathbf{G} \equiv \boldsymbol{\alpha}\mathbf{D} - \mathbf{e}_1 - \frac{\boldsymbol{\alpha}^2}{2s + d - 5 - 2N_\zeta} \bar{\mathbf{e}}_1, \tag{4.12}$$

$$|\Xi\rangle \equiv \sum_{s'=0}^{s-1} \frac{\zeta^{s-1-s'} \alpha^{A_1} \ldots \alpha^{A_{s'}}}{s'! \sqrt{(s-1-s')!}} \Xi^{A_1 \ldots A_{s'}} |0\rangle, \tag{4.13}$$

where gauge transformation parameters $\Xi^{A_1 \ldots A_{s'}}$ are traceless, $\Xi^{AAA_3 \ldots A_{s'}} = 0$, i.e., $\bar{\boldsymbol{\alpha}}^2 |\Xi\rangle = 0$. Also we note that Lagrangian (4.3) can alternatively be represented as

$$\mathcal{L} = \frac{1}{2} e \langle \Phi | \left(1 - \frac{1}{4} \boldsymbol{\alpha}^2 \bar{\boldsymbol{\alpha}}^2 \right) \mathcal{E} |\Phi\rangle, \tag{4.14}$$

$$\mathcal{E} = \Box_{(A)dS} + \mathbf{m}_1 + \rho \boldsymbol{\alpha}^2 \bar{\boldsymbol{\alpha}}^2 - \mathbf{G} \bar{\mathbf{C}}_{\rm st}. \tag{4.15}$$

Using (4.10) and denoting eigenvalues of N_ζ by k, we find the critical values of the mass parameter, $m_k^2 = \rho k(2s + d - 4 - k)$, $k = 0, 1, \ldots, s-1$. The case $k = 0$ corresponds to massless field, while $k = 1, 2, \ldots, s-1$ correspond to the partial massless fields [33,34] (see also [10,16,35]).

Standard de Donder gauge. We proceed with discussion of standard de Donder gauge for $(A)dS$ massive field. Representation for Lagrangian given in (4.3), (4.4) is well adopted for this purpose. This is to say that use of the following *de Donder* gauge-fixing term

$$\mathcal{L}_{\rm g.fix} = \frac{1}{2} e \langle \Phi | \mathbf{C}_{\rm st} \bar{\mathbf{C}}_{\rm st} |\Phi\rangle, \tag{4.16}$$

leads to de Donder gauge fixed Lagrangian $\mathcal{L}_{\rm total}$, $\mathcal{L}_{\rm total} \equiv \mathcal{L} + \mathcal{L}_{\rm g.fix}$,

$$\mathcal{L}_{\rm total} = \frac{1}{2} e \langle \Phi | \left(1 - \frac{1}{4} \boldsymbol{\alpha}^2 \bar{\boldsymbol{\alpha}}^2 \right) \mathcal{E}_{\rm total} |\Phi\rangle, \tag{4.17}$$

$$\mathcal{E}_{\rm total} = \Box_{(A)dS} + \mathbf{m}_1 + \rho \boldsymbol{\alpha}^2 \bar{\boldsymbol{\alpha}}^2. \tag{4.18}$$

Note that Lagrangian (4.14) leads to the following gauge invariant equations of motion $\mathcal{E}|\Phi\rangle = 0$, where $\mathcal{E}$ is given in (4.15). It easy to see that imposing standard de Donder gauge $\bar{\mathbf{C}}_{\rm st}|\Phi\rangle = 0$ we obtain gauge fixed equations of motion $\mathcal{E}_{\rm total}|\Phi\rangle = 0$, where $\mathcal{E}_{\rm total}$ is given in (4.18).

Modified de Donder gauge. We now discuss modified de Donder gauge. From now on we consider fields in AdS, i.e. we set $\rho = -1$, and use Poincaré parametrization of AdS. The modified de Donder gauge fixing is defined to be

$$\mathcal{L}_{\rm g.fix} = \frac{1}{2} e \langle \Phi | \mathbf{C} \bar{\mathbf{C}} |\Phi\rangle, \tag{4.19}$$

$$\mathbf{C} \equiv \mathbf{C}_{\rm st} - 2\mathbf{C}_\perp^z, \qquad \bar{\mathbf{C}} \equiv \bar{\mathbf{C}}_{\rm st} + 2\bar{\mathbf{C}}_\perp^z, \tag{4.20}$$

$$\mathbf{C}_\perp^z \equiv \alpha^z - \frac{1}{2} \boldsymbol{\alpha}^2 \bar{\alpha}^z, \qquad \bar{\mathbf{C}}_\perp^z \equiv \bar{\alpha}^z - \frac{1}{2} \alpha^z \bar{\boldsymbol{\alpha}}^2. \tag{4.21}$$

We now make sure that gauge fixed Lagrangian $\mathcal{L}_{\rm total}$, $\mathcal{L}_{\rm total} \equiv \mathcal{L} + \mathcal{L}_{\rm g.fix}$, takes the form

$$\mathcal{L}_{\rm total} = \frac{1}{2} e \langle \Phi | \left(1 - \frac{1}{4} \boldsymbol{\alpha}^2 \bar{\boldsymbol{\alpha}}^2 \right) \mathcal{E}_{\rm total} |\Phi\rangle, \tag{4.22}$$

$$\mathcal{E}_{\rm total} = \Box_{0\,AdS} - m^2 - \alpha^2 \bar{\alpha}^z \bar{\alpha}^z - \left(s + \frac{d-4}{2} - N_z \right)^2 - N_\zeta (2s + d - 2 + 2N_z - N_\zeta) + \frac{d^2}{4} + 2\tilde{\mathbf{I}}^z \bar{\mathbf{e}}_1 - 2\mathbf{e}_1 \bar{\alpha}^z, \tag{4.23}$$

$$\Box_{0\,AdS} \equiv z^2 \left(\Box + \partial_z^2 \right) + (1-d) z \partial_z,$$

$$\tilde{\mathbf{I}}^z = \alpha^z - \boldsymbol{\alpha}^2 \frac{1}{2N_{\boldsymbol{\alpha}} + d - 1} \bar{\alpha}^z. \tag{4.24}$$

We proceed with discussion of gauge-fixing procedure at the level of equations of motion. To this end we note that gauge invariant Lagrangian (4.14) leads to the following equations of motion:

$$\mathcal{E}|\Phi\rangle = 0, \tag{4.25}$$

where $\mathcal{E}$ is given in (4.15). We now define modified de Donder gauge conditions as

$$\bar{\mathbf{C}}|\Phi\rangle = 0, \tag{4.26}$$

where $\bar{\mathbf{C}}$ is given in (4.20). Using (4.26) in (4.25) we get gauge fixed equations of motion

$$\mathcal{E}_{\rm total}|\Phi\rangle = 0, \tag{4.27}$$

where $\mathcal{E}_{\rm total}$ is given in (4.23). We note that, because of $\mathbf{C}_\perp^z$- and $\bar{\mathbf{C}}_\perp^z$-terms, the modified de Donder gauge breaks some of the $so(d,2)$ symmetries. In the conformal algebra nomenclature, these broken symmetries correspond to broken conformal boost K^a-symmetries.

From $\mathcal{E}_{\rm total}$ (4.23), we see that, because of terms like $\alpha^2 \bar{\alpha}^z \bar{\alpha}^z$, $\tilde{\mathbf{I}}^z \bar{\mathbf{e}}_1$, and $\mathbf{e}_1 \bar{\alpha}^z$ the modified de Donder gauge itself does not lead automatically to decoupled gauge fixed equations for the ket-vector $|\Phi\rangle$. It turns out that in order to obtain decoupled gauge fixed equations of motion we should introduce our fields in (2.1). We remind that $|\Phi\rangle$ is a double-traceless field (4.2) of the $so(d,1)$ algebra, while $|\phi\rangle$ describes double-traceless fields (2.5) of the $so(d-1,1)$ algebra. This is to say that to get decoupled equations of motion we have to make transformation from the $so(d,1)$ ket-vector $|\Phi\rangle$ to $so(d-1,1)$ ket-vector $|\phi\rangle$. We find a transformation from the ket-vector $|\Phi\rangle$ to our ket-vector $|\phi\rangle$ and the corresponding inverse transformation,

$$|\phi\rangle = z^{\frac{1-d}{2}} V^\dagger \mathcal{N} \Pi^{\phi\Phi} |\Phi\rangle, \tag{4.28}$$

$$|\Phi\rangle = z^{\frac{d-1}{2}} \Pi^{\Phi\phi} \mathcal{N} V |\phi\rangle, \tag{4.29}$$

where V is unitary operator, $V^\dagger V = 1$, and we introduce the z-factor in r.h.s. of (4.28) to obtain canonically normalized ket-vector $|\phi\rangle$. Operators $\Pi^{\Phi\phi}$, $\Pi^{\phi\Phi}$, $\mathcal{N}$, and V are defined in Appendix A.

We now ready to compare modified de Donder gauges for $|\phi\rangle$ (3.7) and $|\Phi\rangle$ (4.26). Inserting (4.29) in (4.26), we make sure that modified de Donder gauge for $|\Phi\rangle$ (4.26) amounts to one for $|\phi\rangle$ (3.7) i.e., modified de Donder gauges for $|\phi\rangle$ (3.7) and $|\Phi\rangle$ (4.26) match. Also one can make sure that gauge invariant Lagrangian for $|\Phi\rangle$ (4.3) and the one for $|\phi\rangle$ (2.6) match.

We now compare gauge transformation of $|\phi\rangle$ (2.32) and gauge transformation of $|\Phi\rangle$ given in (4.12). To this end we note that gauge transformation parameters $|\xi\rangle$ and $|\Xi\rangle$ are related as

$$|\xi\rangle = z^{\frac{3-d}{2}} V^\dagger \mathcal{N}' \Pi^{[1]}_\alpha |\Xi\rangle, \quad |\Xi\rangle = z^{\frac{d-3}{2}} \Pi^{[1]}_{\boldsymbol{\alpha}} \mathcal{N}' V|\xi\rangle, \tag{4.30}$$

$$\mathcal{N}' \equiv \mathcal{N}|_{N_\alpha \to N_\alpha+1}, \tag{4.31}$$

where $\Pi^{[1]}_\alpha$, $\Pi^{[1]}_{\boldsymbol{\alpha}}$ are defined in Appendix A. Using (4.29), (4.30), we make sure that gauge transformations (2.32) and (4.12) match.

Finally we compare realization of $so(d,2)$ symmetries on the ket-vectors $|\phi\rangle$ and $|\Phi\rangle$. To this end we note that on space of $|\Phi\rangle$ realization of the $so(d,2)$ algebra transformations takes the form

$$\delta_{P^a}|\Phi\rangle = \partial^a|\Phi\rangle,$$

$$\delta_{J^{ab}}|\Phi\rangle = \left(x^a\partial^b - x^b\partial^a + M^{ab}\right)|\Phi\rangle, \tag{4.32}$$

$$\delta_D|\Phi\rangle = x^B\partial^B|\Phi\rangle,$$

$$\delta_{K^a}|\Phi\rangle = \left(-\frac{1}{2}x^Bx^B\partial^a + x^ax^B\partial^B + M^{aB}x^B\right)|\Phi\rangle, \tag{4.33}$$

where $x^Bx^B = x^bx^b + z^2$, $x^B\partial^B = x^b\partial^b + z\partial_z$, $M^{aB}x^B = M^{ab}x^b - M^{za}z$. Comparing (2.34) and (4.32), we see that the realizations of Poincaré symmetries on $|\phi\rangle$ and $|\Phi\rangle$ match from the very beginning. Taking into account z-factor in (4.29), it is easily seen that D-transformations for $|\phi\rangle$ (2.35) and $|\Phi\rangle$ (4.33) match. After this, we make sure that realizations of the operator K^a on $|\phi\rangle$ (2.36) and on $|\Phi\rangle$ (4.33) also match.

Light-cone Lagrangian. Using gauge invariant action (2.6) and imposing light-cone gauge, we find the light-cone Lagrangian

$$\mathcal{L}_{\rm l.c.} = \sum_{s'=0}^{s} \sum_{\lambda=[s-s']_2} \frac{1}{2s'!}\phi_\lambda^{i_1\ldots i_{s'}} \Box_{\kappa+\lambda}\phi_\lambda^{i_1\ldots i_{s'}}, \tag{4.34}$$

where $\Box_{\kappa+\lambda}$ is defined in (3.5) and transverse indices take values $i = 1,2,\ldots,d-2$. As usually, the light-cone fields $\phi_\lambda^{i_1\ldots i_{s'}} = 0$ are traceless, $\phi_\lambda^{iii_3\ldots i_{s'}} = 0$.

To summarize, using the Poincaré parametrization of *AdS* space, we have developed the CFT adapted formulation of massive totally symmetric arbitrary spin *AdS* field. In recent years, mixed symmetry fields have attracted considerable interest (see e.g. Refs. [36–40]). We think that generalization of our approach to the case of mixed symmetry massless and massive *AdS* fields might be useful for study of dynamical aspects of such fields. In this respect, it would be interesting to find generalization of the modified de Donder gauge to the case of mixed symmetry fields.

Appendix A. Notation

Vector indices of the $so(d-1,1)$ algebra take the values $a,b,c = 0,1,\ldots,d-1$, while vector indices of the $so(d,1)$ algebra take the values $A,B,C = 0,1,\ldots,d-1,d$. We use mostly positive flat metric tensors η^{ab}, η^{AB}. To simplify our expressions we drop η_{ab}, η_{AB} in the respective scalar products, i.e., we use $X^aY^a \equiv \eta_{ab}X^aY^b$, $X^AY^A \equiv \eta_{AB}X^AY^B$, $\eta^{AB} = (\eta^{ab},1)$. Using the identification $X^d \equiv X^z$ gives the following decomposition of the $so(d,1)$ algebra vector: $X^A = X^a, X^z$. This implies $X^AY^A = X^aY^a + X^zY^z$.

We use the creation operators α^a, α^z, ζ and the respective annihilation operators $\bar\alpha^a$, $\bar\alpha^z$, $\bar\zeta$

$$[\bar\alpha^a,\alpha^b] = \eta^{ab}, \qquad [\bar\alpha^z,\alpha^z] = 1, \qquad [\bar\zeta,\zeta] = 1,$$

$$\bar\alpha^a|0\rangle = 0, \qquad \bar\alpha^z|0\rangle = 0, \qquad \bar\zeta|0\rangle = 0. \tag{A.1}$$

These operators are referred to as oscillators in this Letter. The oscillators α^a, $\bar\alpha^a$ and α^z, ζ, $\bar\alpha^z$, $\bar\zeta$, transform in the respective vector and scalar representations of the $so(d-1,1)$ algebra and satisfy the hermitian conjugation rules, $\alpha^{a\dagger} = \bar\alpha^a$, $\alpha^{z\dagger} = \bar\alpha^z$, $\zeta^\dagger = \bar\zeta$. Oscillators α^a, α^z and $\bar\alpha^a$, $\bar\alpha^z$ are collected into the respective $so(d,1)$ algebra oscillators $\alpha^A = \alpha^a,\alpha^z$ and $\bar\alpha^A = \bar\alpha^a,\bar\alpha^z$.

$x^A = x^a$, z denote coordinates in $d+1$-dimensional AdS_{d+1} space,

$$ds^2 = \frac{1}{z^2}\left(dx^a dx^a + dz\,dz\right), \tag{A.2}$$

while $\partial_A = \partial_a, \partial_z$ denote the respective derivatives, $\partial_a \equiv \partial/\partial x^a$, $\partial_z \equiv \partial/\partial z$. We use the notation $\Box = \partial^a\partial^a$, $\alpha\partial = \alpha^a\partial^a$, $\bar\alpha\partial = \bar\alpha^a\partial^a$, $\alpha^2 = \alpha^a\alpha^a$, $\bar\alpha^2 = \bar\alpha^a\bar\alpha^a$, $N_\alpha = \alpha^a\bar\alpha^a$, $N_z = \alpha^z\bar\alpha^z$, $N_\zeta = \zeta\bar\zeta$. The covariant derivative D^A is given by $D^A = \eta^{AB}D_B$,

$$D_A \equiv e^\mu_A D_\mu, \quad D_\mu \equiv \partial_\mu + \frac{1}{2}\omega_\mu^{AB}M^{AB},$$

$$M^{AB} \equiv \alpha^A\bar\alpha^B - \alpha^B\bar\alpha^A, \tag{A.3}$$

$\partial_\mu = \partial/\partial x^\mu$, where e^μ_A is inverse vielbein of AdS_{d+1} space, D_μ is the Lorentz covariant derivative and the base manifold index takes values $\mu = 0,1,\ldots,d$. The ω_μ^{AB} is the Lorentz connection of AdS_{d+1} space, while M^{AB} is a spin operator of the Lorentz algebra $so(d,1)$. Note that AdS_{d+1} coordinates x^μ carrying the base manifold indices are identified with coordinates x^A carrying the flat vectors indices of the $so(d,1)$ algebra, i.e., we assume $x^\mu = \delta^\mu_A x^A$, where δ^μ_A is Kronecker delta symbol. AdS_{d+1} space contravariant tensor field, $\Phi^{\mu_1\ldots\mu_s}$, is related with field carrying the flat indices, $\Phi^{A_1\ldots A_s}$, in a standard way $\Phi^{A_1\ldots A_s} \equiv e^{A_1}_{\mu_1}\ldots e^{A_s}_{\mu_s}\Phi^{\mu_1\ldots\mu_s}$. Helpful commutators involving the covariant derivative D^A and the oscillators α^A, $\bar\alpha^A$ may be found in Appendix in Ref. [8].

For the Poincaré parametrization of AdS_{d+1} space, vielbein $e^A = e^A_\mu dx^\mu$, Lorentz connection $\omega^{AB} = \omega_\mu^{AB}dx^\mu$, and $\omega^{ABC} = e^{A\mu}\omega_\mu^{BC}$ are given by

$$e^A_\mu = \frac{1}{z}\delta^A_\mu, \qquad \omega_\mu^{AB} = \frac{1}{z}\left(\delta^A_z\delta^B_\mu - \delta^B_z\delta^A_\mu\right),$$

$$\omega^{ABC} = \eta^{AC}\delta^B_z - \eta^{AB}\delta^C_z. \tag{A.4}$$

With choice made in (A.4), the covariant derivative takes the form $D^A = z\partial^A + M^{zA}$, $\partial^A = \eta^{AB}\partial_B$.

The operators $\Pi^{\phi\Phi}$, $\Pi^{\Phi\phi}$ used in Section 4 are defined by relations

$$\Pi^{\phi\Phi} \equiv \Pi^{[1]}_\alpha + \alpha^2\frac{1}{2(2N_\alpha+d)}\Pi^{[1]}_\alpha\left(\bar\alpha^2 + \frac{2N_\alpha+d}{2N_\alpha+d-2}\bar\alpha^z\bar\alpha^z\right), \tag{A.5}$$

$$\Pi^{[1]}_\alpha \equiv \Pi^{[1]}(\alpha,0,N_\alpha,\bar\alpha,0,d), \tag{A.6}$$

$$\mathcal{N} \equiv \left(\frac{2^{N_z}\Gamma(N_\alpha+N_z+\frac{d-3}{2})\Gamma(2N_\alpha+d-3)}{\Gamma(N_\alpha+\frac{d-3}{2})\Gamma(2N_\alpha+N_z+d-3)}\right)^{1/2}, \tag{A.7}$$

$$\Pi^{\Phi\phi} \equiv \Pi^{[1]}_{\boldsymbol{\alpha}} + \boldsymbol{\alpha}^2\frac{1}{2(2N_{\boldsymbol{\alpha}}+d+1)}\Pi^{[1]}_{\boldsymbol{\alpha}} \times\left(\bar\alpha^2 - \frac{2}{2N_{\boldsymbol{\alpha}}+d-1}\bar\alpha^z\bar\alpha^z\right), \tag{A.8}$$

$$\Pi^{[1]}_{\boldsymbol{\alpha}} \equiv \Pi^{[1]}\left(\alpha,\alpha^z,N_{\boldsymbol{\alpha}},\bar\alpha,\bar\alpha^z,d+1\right), \tag{A.9}$$

$$\Pi^{[1]}\left(\alpha,\alpha^z,X,\bar\alpha,\bar\alpha^z,Y\right) \equiv \sum_{n=0}^{\infty}\left(\alpha^2+\alpha^z\alpha^z\right)^n\frac{(-)^n\Gamma(X+\frac{Y-2}{2}+n)}{4^n n!\,\Gamma(X+\frac{Y-2}{2}+2n)}\left(\bar\alpha^2+\bar\alpha^z\bar\alpha^z\right)^n, \tag{A.10}$$

where $N_{\boldsymbol{\alpha}} = N_\alpha + N_z$, $\boldsymbol{\alpha}^2 = \alpha^2 + \alpha^z\alpha^z$, and Γ is Euler gamma function. We note that the $\Pi^{[1]}_\alpha$ in (A.6) is obtained from (A.10) by equating $\alpha^z = \bar\alpha^z = 0$, $X = N_\alpha$, $Y = d$, while $\Pi^{[1]}_{\boldsymbol{\alpha}}$ in (A.9) is obtained from (A.10) by equating $X = N_{\boldsymbol{\alpha}}$, $Y = d+1$.

The operator V used in Section 4 is defined by relations

$$V = \sum_{N=0}^{s} V^{(N)}, \quad V^{(N)} = \sum_{l,n=0,1,\ldots,N} V_{ln}^{(N)}, \tag{A.11}$$

$$V_{ln}^{(N)} = v_{ln}^{(N)} \alpha_z^{N-l} \zeta^l |0\rangle\langle 0| \bar{\alpha}_z^{N-n} \bar{\zeta}^n, \quad v_{nl}^{(N)} = \mathcal{N}_{nl}^{(N)} X_{nl}^{(N)}, \tag{A.12}$$

$$X_{nl}^{(N)} = \sum_{t=0}^{l} \frac{(-)^t (\kappa - N + l)_t}{t!(l-t)!(\kappa+1)_t} X_{nt}, \tag{A.13}$$

$$X_{nt} = \sum_{p=0}^{\min n,t} \frac{(n+1-p)_p (t+1-p)_p (2s+d-2-n-p)_p}{p!(s+\frac{d-2}{2}-p)_p (\kappa - s - \frac{d-4}{2})_p}, \tag{A.14}$$

$$\mathcal{N}_{nl}^{(N)} = \frac{(-)^n}{n!(N-n)!} \left(\frac{\kappa - N + 2l}{\kappa}\right)^{1/2} \left(\frac{2s+d-3-2n}{2s+d-3-n}\right)^{1/2} \times \left(\frac{(s+\frac{d-2}{2}-l)_l (s+\frac{d-2}{2}-N+l)_{N-l}}{(2s+d-3-n-N)_N}\right)^{1/2} \times \left(\frac{(\kappa - s - \frac{d-4}{2})_n (\kappa - s - \frac{d-4}{2})_l (\kappa + s + \frac{d-2}{2} - N + l)_{N-l} (\kappa+1)_l}{(\kappa + s + \frac{d-2}{2} - n)_n (\kappa - N + l)_{N-l}}\right)^{1/2}, \tag{A.15}$$

where in (A.13)–(A.15) we use the notation $(a)_b$ for the Pochhammer symbol, $(a)_b \equiv \frac{\Gamma(a+b)}{\Gamma(a)}$.

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