# New solutions for the color–flavor locked strangelets

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## Abstract

Recent publications rule out the negatively charged beta equilibrium strangelets in ordinary phase, and the color–flavor locked (CFL) strangelets are reported to be also positively charged. This Letter presents new solutions to the system equations where CFL strangelets are slightly negatively charged. If the ratio of the square-root bag constant to the gap parameter is smaller than 170 MeV, the CFL strangelets are more stable than iron and the normal unpaired strangelets. For the same parameters, however, the positively charged CFL strangelets are more stable.
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After the acceptance of quantum chromodynamics as the fundamental theory of strong interactions, it became extremely significant whether a deconfined phase of matter consisting merely of quarks would be possible. Theoretical investigations show that strange quark matter (SQM), which is composed of $u$, $d$, and $s$ quarks, might be absolutely stable [1–3]. Because small lumps of SQM, the so-called strangelets, could be produced in modern relativistic heavy-ion collision experiments, their charge property has attracted a lot of interest [4].

Originally, SQM is believed to show up with some small positive charge [2]. In June 1997, however, Schaffner-Bielich et al. demonstrated that strangelets are most likely heavily negatively charged [5]. In June 1999, it was shown that negative charge can lower the critical density of SQM [6]. In July 1999, Wilczek mentioned an "ice-9"-type transition [7], which was picked up by a British newspaper. Not long ago, in response to public concern, an expert committee published a report [8], which got positive comments [9], as well as criticisms [10]. In fact, the strangelets in Ref. [5] are not in $\beta$ equilibrium which drives the system to flavor equilibrium, and negatively charged

strangelets in normal phase have been ruled out by a recent publication [11].

Much progress has been achieved recently by the introduction of color superconductivity [12,13]. It has been shown that bulk SQM with color–flavor locking is electrically neutral [14]. Immediately, Madsen found a solution to the corresponding system equations of strangelets, where color–flavor locked strangelets are positively charged [15], and they might be a candidate for cosmic rays beyond the GZK cutoff [16].

Very recently, it is shown that CFL phase can exist only when the ratio of the squared strange quark mass to chemical potential, i.e., $m_s^2/\mu$, is smaller than a critical value about 2 times the gap parameter $\Delta$ [17]. It is therefore of interest to study if there is some similar criterion for CFL strangelets given that surface effects become quite important. Fig. 1 explicitly shows the ratio for various parameters. The solid lines are for the CFL strangelets reported in Ref. [15]. These strangelets are positively charged. At the same time, there are new solutions (the dashed lines) which are slightly negatively charged or nearly charge-neutral, to be discussed in detail below. It is found that the stability of the CFL strangelets can be judged by the ratio $\sqrt{B}/\Delta$, i.e., the square-root bag constant to the gap parameter. If the ratio is less than about 170 MeV, these CFL strangelets are more stable than $^{56}$Fe, i.e., the energy per baryon is less than 930 MeV.

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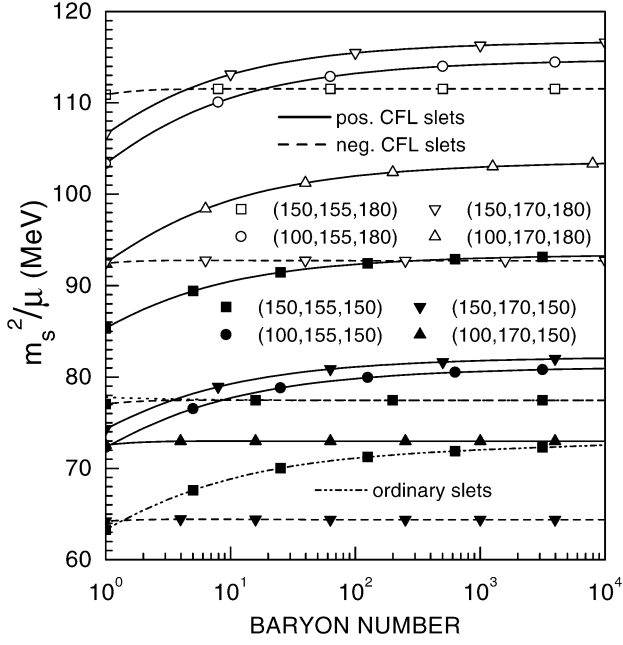  *E-mail address:* gxpeng@ihep.ac.cn (G.X. Peng).

Fig. 1. The ratio of squared strange quark mass to chemical potential for different strangelets with various parameters. The solid lines are for the CFL strangelets in Ref. [15]. The dashed lines give the new solutions of CFL strangelets reported in this Letter. Unlike the previous stranglets, which are positively charged, these new strangelets are slightly negatively charged, or nearly neutral. Parameters are indicated as $(\Delta, B^{1/4}, m_s)$ in MeV.

As done in Ref. [15], the thermodynamic potential density is written as $\Omega = \Omega_f + \Omega_{pair} + B$. The paring contribution is $\Omega_{pair} = -3\Delta^2 \bar{\mu}^2/\pi^2$ with $\Delta$ being the paring gap and $\bar{\mu} = (\mu_u + \mu_d + \mu_s)/3$ being the average chemical potential of quarks. The normal quark contribution is

$$\Omega_f = \sum_{i=u,d,s} \int_0^\nu \left( \sqrt{p^2 + m_i^2} - \mu_i \right) n'(p, m_i, R)\, dp \qquad (1)$$

with the density of state $n'(p, m_i, R)$ given in the multi-expansion approach [18] by

$$n'(p, m_i, R) = g\left[ \frac{p^2}{2\pi^2} + \frac{3}{R} f_S\left(\frac{m_i}{p}\right) p + \frac{6}{R^2} f_C\left(\frac{m_i}{p}\right) \right], \quad (2)$$

where $g = 6$ is the degeneracy factor for quarks, and the functions $f_S$ [19] and $f_C$ [20] are given, respectively, by

$$f_S(x) = -\frac{\arctan(x)}{4\pi^2}, \qquad f_C(x) = \frac{1}{12\pi^2}\left[1 - \frac{3}{2}\frac{\arctan(x)}{x}\right].$$

The common Fermi momentum $\nu$ is a fictional intermediate parameter. It does not fully specify the quark number density, as it does in the unpaired case. The basic requirement is that it cannot be negative. As a general practice, it is determined by minimizing $\Omega$ at fixed radius $R$, i.e.,

$$\frac{\partial \Omega}{\partial \nu} = \sum_{i=u,d,s} n'(\nu, m_i, R)\left[\sqrt{\nu^2 + m_i^2} - \mu_i\right] = 0. \qquad (3)$$

The number densities $n_i$ ($i = u, d, s$) for quarks are

$$n_i = -\frac{d\Omega}{d\mu_i}\bigg|_R = -\frac{\partial \Omega}{\partial \mu_i} - \frac{\partial \Omega}{\partial \nu}\frac{\partial \nu}{\partial \mu_i}. \qquad (4)$$

Because of Eq. (3), the second term vanishes, while the first term gives

$$n_i = \frac{g\nu^3}{6\pi^2} + \frac{3n_{i,S}}{R} + \frac{6n_{i,C}}{R^2} + \frac{2\Delta^2\bar{\mu}}{\pi^2},$$

where

$$n_{i,S} = \frac{gm_i^2}{8\pi^2}\left[\phi_i - \tan\phi_i - \left(\frac{\pi}{2} - \phi_i\right)\tan^2\phi_i\right], \qquad (5)$$

$$n_{i,C} = \frac{gm_i}{16\pi^2}\left[\phi_i + \frac{1}{3}\tan\phi_i - \left(\frac{\pi}{2} - \phi_i\right)\tan^2\phi_i\right], \qquad (6)$$

with $\phi_i \equiv \arctan(\nu/m_i)$.

The chemical potentials $\mu_i$ and the radius $R$ are the independent state variables. For a given baryon number $A$, one should give these quantity to fix a strangelet. To have chemical/weak equilibrium, the chemical potentials $\mu_i$ satisfy $\mu_d = \mu_s = \mu_u + \mu_e$, maintained by reactions such as $u + d \leftrightarrow s + u$, $u + e^- \leftrightarrow d + \nu_e$. Because the strangelet radius is much smaller than the Compton wave length of electrons, the electron's number density, and accordingly the chemical potential $\mu_e$, must be zero. Therefore, strangelets in perfect $\beta$ equilibrium always have $\bar{\mu} = \mu_u = \mu_d = \mu_s \equiv \mu$. Consequently, Eq. (3) gives

$$\mu = \sum_i n'(\nu, m_i, R)\sqrt{\nu^2 + m_i^2}\left[\sum_i n'(\nu, m_i, R)\right]^{-1}. \qquad (7)$$

When $R \to \infty$ and $m_u = m_d = 0$, this equation gives $\mu = (2\nu + \sqrt{\nu^2 + m_s^2})/3$ or $\nu = 2\mu - \sqrt{\mu^2 + m_s^2/3}$, which is the same as in Refs. [14,21] for bulk CFL quark matter. For a given baryon number $A$, one naturally has

$$n_b \equiv \frac{1}{3}\sum_{i=u,d,s} n_i = \frac{3A}{4\pi R^3}. \qquad (8)$$

To maintain mechanical equilibrium, one must require that the pressure is zero, i.e.,

$$P = -\Omega - \frac{R}{3}\frac{\partial \Omega}{\partial R} = 0. \qquad (9)$$

Please note, there is an extra term when it is compared to the normal case $P = -\Omega$. This is because of the direct radius (or volume) dependence of the thermodynamic potential density.

For a given baryon number $A$, one can solve the three equations (7), (8), and (9) for $\mu$, $\nu$, and $R$. Then the overall electric charge is $Z = V(2n_u/3 - n_d/3 - n_s/3)$ with $V = 4\pi R^3/3$ being the volume. Numerical results are given in Fig. 2 for parameters $\Delta = 150$ MeV, $B = (155$ MeV$)^4$, and $m_s = 150$ MeV. It is found that there are three solutions for each given baryon number $A$. The strangelet corresponding to the first solution (dashed line) is positively charged. It is just the one that has been previously found in Ref. [15]. The strangelet corresponding to the second solution is negatively charged (solid line), and the third solution is nearly neutral (dotted line). For convenience, these three solutions are marked, respectively, with CFL slet-1, slet-2 and slet-3. The ordinary strangelets without color–flavor locking have also been plotted in the same figure for comparison purpose. It can be seen that the charge of the CFL slet-1 is approximately proportional to $A^{2/3}$, while that of the CFL slet-2 or slet-3 is nearly proportional to $A^{1/3}$.
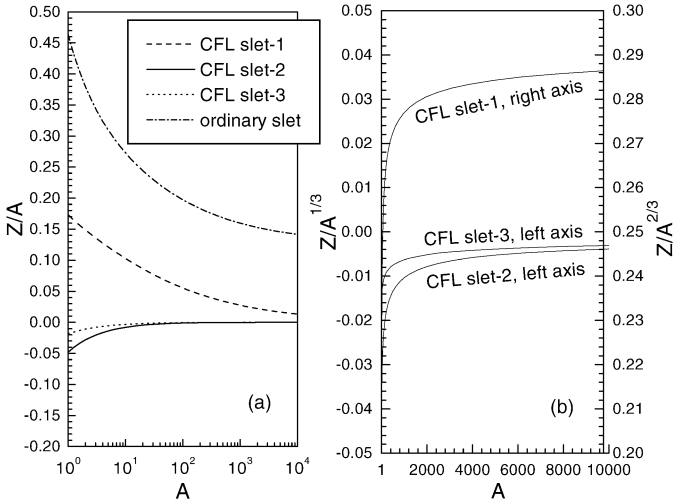
Fig. 2. Charge of strangelets. The horizontal axis is the baryon number $A$. The vertical axis is the electric charge $Z$ to $A^{2/3}$. There are three kinds of CFL strangelets marked with CFL slet-1 (dashed line), CFL slet-2 (solid line), and CFL slet-3 (dotted line). They are, respectively, charge-positive, negative, and nearly neutral. The ordinary strangelets are also plotted (dot-dashed line).

To have a better understanding of the three solutions, let us take some mathematical analysis.

First, assume the common Fermi momentum $\nu$ is much bigger than the strange quark mass, i.e., $m_s/\nu \ll 1$. In this case, one can take the limit of $m_{u,d} \to 0$ first, and then expand to a Taylor series with respect to $m_s$ on all the above expressions, to get simple expressions. The expansion of the pressure is

$$P \approx \frac{g m_s}{16\pi^2}\left[\frac{\pi\mu}{R^2} - \frac{4\nu(2\mu-\nu)}{R}\right] + \frac{3\Delta^2}{\pi^2}\mu^2 + \frac{g\nu}{8\pi^2}\left[\nu^2(4\mu-3\nu) - \frac{2\mu-\nu}{R^2}\right] - B. \tag{10}$$

For the quark number densities, they are

$$n_{u,d} \approx \frac{2\Delta^2}{\pi^2}\mu - \frac{g\nu}{4\pi^2 R^2} + \frac{g\nu^3}{6\pi^2} \tag{11}$$

and

$$n_s \approx \frac{2\Delta^2}{\pi^2}\mu - \frac{g\nu}{4\pi^2 R^2} + \frac{g\nu^3}{6\pi^2} - \frac{3g m_s}{16\pi^2}\left[\frac{4\nu}{R} - \frac{\pi}{R^2}\right]. \tag{12}$$

It is obvious from Eqs. (11) and (12) that $n_s$ is smaller than $n_{u,d}$ because $n_s$ has an extra negative term. The corresponding strangelet, CFL slet-1, is thus positively charged.

Secondly, assume $\nu$ is modest, i.e., it is smaller than $m_s$ but larger than $m_{u,d}$. In this case, one can still take the limit of $m_{u,d} \to 0$ for $u/d$ quarks. But for $s$ quarks, expressions should be expanded according to $\nu$, rather than $m_s$. Accordingly, Eq. (9) becomes

$$P = \frac{3\Delta^2}{\pi^2}\mu^2 - \frac{g m_s \nu}{6\pi^2 R^2} - B. \tag{13}$$

The $u/d$ quark number density is still the same as Eq. (11). For $s$ quarks, however, one now has

$$n_s \approx \frac{2\Delta^2}{\pi^2}\mu + \frac{3g\nu}{16\pi R^2}\left(\frac{8}{3} - \frac{\pi\nu}{m_s}\right) - \frac{3g\nu^2}{16\pi R}. \tag{14}$$
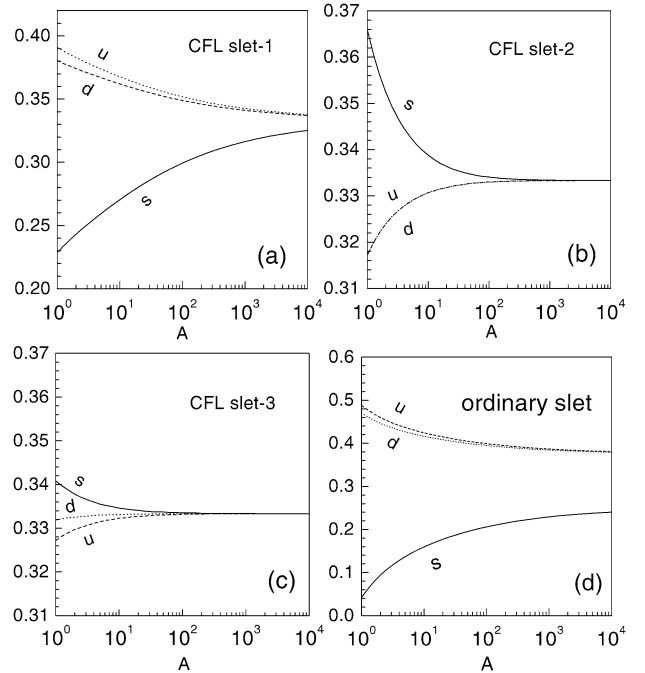


Fig. 3. Quark fractions of different strangelets. Figures (a)–(c) are for the three kinds of CFL strangelets. Figure (d) is for the ordinary strangelets. The vertical axis for each figure is the quark number density in unit of the total quark number density, or the ratio of the corresponding quark number to the total quark number.

Please note the curvature (the $R^{-2}$ term) contribution. It is negative for $u/d$ quarks in Eq. (11). However, it is positive for $s$ quarks in Eq. (14). This makes $n_s$ bigger than $n_{u,d}$. Consequently, the corresponding strangelet, CFL slet-2, is negatively charged.

Thirdly, assume $\nu$ is extremely small so that expansion can be done with respect to $\nu$ for all the three flavors. In this case, the pressure gives

$$P = \frac{3\Delta^2}{\pi^2}\mu^2 + \frac{g}{2\pi^2}(\mu - \bar{m})\frac{\nu}{R^2} - B, \tag{15}$$

where $\bar{m} \equiv (m_u + m_d + m_s)/3$, while the quark number densities are

$$n_u \approx n_d \approx n_s \approx \frac{2\Delta^2}{\pi^2}\mu + \frac{g\nu}{2\pi^2 R^2}. \tag{16}$$

Namely, the three flavors of quarks are nearly equal in this case. The corresponding strangelet, CFL slet-3, is almost neutral.

Naturally, the above expanded expressions are merely approximate. Real calculations have been performed by directly solving the original system equations. Fig. 3 shows the quark fraction in different phases. They are qualitatively consistent with the above analysis.

Now we discuss the determination of parameters. For the $u/d$ quark mass, we take $m_u = 5$ MeV and $m_d = 10$ MeV, which are closer to the accepted current mass of light quarks [22]. Decreasing the $u/d$ quark mass has little effects on CFL slet-1 and slet-2, while the charge of CFL slet-3 becomes smaller and smaller until it is charge-neutral. The strange quark mass is expected to be density-dependent, lying between the

current mass $\sim$100 MeV and the vacuum constituent quark mass $\sim$500 MeV. The $\Delta$ value varies from several tens to several hundreds of MeV in literature. For example, it can range from 20 MeV to 90 MeV [23], or from 50 MeV to more than 100 MeV [24]. Sophisticated treatments of the instanton interaction, including form factors from suitable Fourier transformation of instanton profiles, give larger values for $\Delta$, as large as more than 200 MeV [25]. Therefore, we treat $\Delta$ as a free parameter in the present investigation. For the above calculations in Figs. 2 and 3, we have taken $\Delta = 150$ MeV, $B^{1/4} = 155$ MeV (this $B$ value was also used in Ref. [21]), and $m_s = 150$ MeV. How these parameters influence the stability of CFL strangelets will be discussed a little later.

Although the 'common Fermi momentum' $\nu$ in CFL slet-2 and slet-3 is small, the chemical potential $\mu$ is still large. To get an approximate expression for $\mu$ from the equality $P = 0$, one can take $\nu = 0$ in Eq. (13) or (15), resulting

$$\mu = \frac{\pi}{\sqrt{3}} \frac{\sqrt{B}}{\Delta}. \qquad (17)$$

For the parameters chosen for Figs. 2 and 3, Eq. (17) gives $\mu \approx$ 290 MeV, very close to the actual value from the numerical calculation, which gives the ratio $m_s^2/\mu$ to be about 77 MeV. On the other hand, $\mu$ varies in the range of 240–263 MeV for CFL slet-1.

The radius of CFL slet-2 and slet-3 can be approximately expressed as

$$R_{\text{slet-2,3}} \approx \left( \frac{3\sqrt{3}A}{8\Delta\sqrt{B}} \right)^{1/3}. \qquad (18)$$

This equation means $R \propto A^{1/3}$, which is a known fact in nuclear physics. One may perhaps imagine $\Lambda$, composed of $(uds)$, as the simplest CFL slet-3. The H particle [26], composed of $(uuddss)$, is probably the next simplest CFL slet-3.

For information on the stability of CFL strangelets, we should investigate the energy per baryon $E/n_{\text{b}}$. It is generally a function of $A, m_s, \Delta$, and $B$, i.e., $E/n_{\text{b}} = f(A, m_s, \Delta, B)$. If $E/n_{\text{b}}$ is less than 930 MeV (the mass of $^{56}$Fe divided by 56), the strangelets are absolutely more stable than normal nuclear matter. Otherwise, they are meta-stable or unstable. The full line in Fig. 4(a) gives $\Delta$ as a function of $B$ at $A = 20$, $m_s = 150$ MeV, $E/n_{\text{b}} = 930$ MeV. In fact, this line does not depend strongly on the concrete values of $m_s$ and $A$. Because $E = \Omega + \sum_i \mu_i n_i = \Omega + 3\mu n_{\text{b}}$ and $\Omega \approx -P = 0$, one has $E/n_{\text{b}} \approx 3\mu$. With a view to Eq. (17), we immediately have $E/n_{\text{b}} \approx \sqrt{3}\pi\sqrt{B}/\Delta$. Therefore, if

$$\frac{\sqrt{B}}{\Delta} < \frac{310\sqrt{3}}{\pi} \approx 170 \text{ MeV}, \qquad (19)$$

then the parameter pair $(B, \Delta)$ is located in the up-left part of Fig. 4(a), and the new strangelets are more stable than iron. For CFL slet-1, a similar solid line is plotted in Fig. 4(b). For different $m_s$ and $A$, this line moves a little up-left (bigger $m_s$, e.g., the dotted line for $m_s = 180$ MeV) or down-right (smaller $m_s$). However, the line in Fig. 4(a) is always located in the region where CFL slet-1 is more stable than $^{56}$Fe for reasonable $m_s$. Therefore, if the condition Eq. (19) is satisfied, all
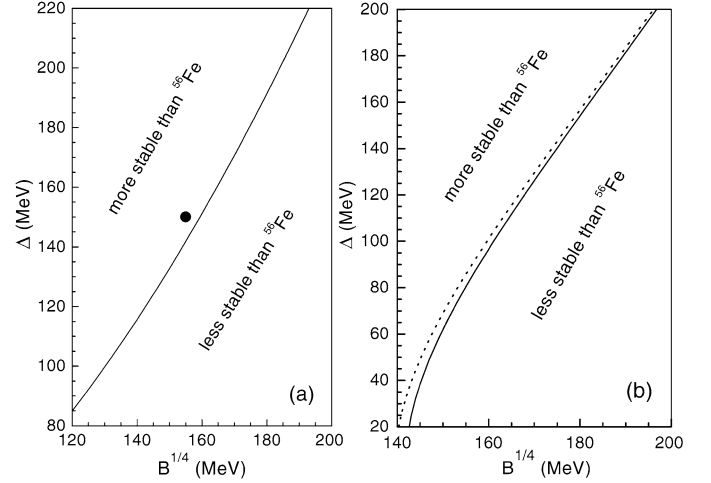


Fig. 4. Parameters for CFL strangelets to be more stable than $^{56}$Fe. (a) is for CFL slet-2 and slet-3 while (b) is for CFL slet-1. The full dot indicates the parameters in this Letter.

the three kinds of CFL strangelets are more stable than $^{56}$Fe, and also more stable than the normal unpaired strangelets. As for the comparative stability between the three kinds of CFL strangelets, it depends on the pairing parameter $\Delta$. If one uses the same $\Delta$ for all the three, then the slet-1 is more stable. In this case, however, the former is denser. Because investigations have shown that $\Delta$ depends on density, most probably increases with increasing densities [25], the comparative stability of the three kinds of CFL strangelets needs to be further studied in the future.

CFL strangelets which are more stable than $^{56}$Fe may have far-reaching consequences. The slet-1 can provide an alternative explanation for cosmic rays beyond the GZK cutoff [16]. The slet-3 is nearly neutral, and so might be a candidate for the miracle dark matter in our universe. The slet-2 and slet-3 are more stable than the normal unpaired strangelets, and so may have chances to be produced in the modern heavy ion collision experiments. However, they are unable to transform our planet into a strange star for the following two reasons. First, the positively charged slet-1 is the energy minimum for the same parameters. And secondly, when the electron's Compton wave length ($\approx$386 fm) is reached, the constraint $n_e = 0$ (or, equivalently, $\mu_u = \mu_d = \mu_s$) is no longer valid, and so the strangelet will be neutralized and ceases to expand its size.

It should be emphasized that the strangelets reported here are different from the previous ones [11,15] in that their electric charge is opposite. The new strangelets are also different from the heavily negatively charged strangelets in Ref. [5]. There the strangelets were not in $\beta$ equilibrium and had no color–flavor locking. It was investigated how the metastable candidates might look like if they are assumed to be stable against strong hadronic decay and subsequently against weak hadronic decay. Here the strangelets are assumed to be in perfect $\beta$ equilibrium and considered as having the possibility of absolute stability. However, the concrete values should not be taken seriously, and further studies are needed.

In summary, there exist new solutions to the system of equations where CFL strangelets are slightly negatively charged or nearly neutral. If the ratio of the squared bag constant to the gap parameter is smaller than 170 MeV, CFL strangelets are more stable than the normal nuclear matter and ordinary unpaired strangelets.

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