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DISCRETE MATHEMATICS

Note

Minus domination in regular graphs

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Abstract

A three-valued function f defined on the vertices of a graph $G=(V,E)$, $f: V \to \{-1,0,1\}$, is a minus dominating function if the sum of its function values over any closed neighborhood is at least one. That is, for every $v \in V$, $f(N[v]) \geqslant 1$, where $N[v]$ consists of v and every vertex adjacent to v. The weight of a minus dominating function is $f(V)=\sum f(v)$, over all vertices $v \in V$. The minus domination number of a graph G, denoted $\gamma^-(G)$, equals the minimum weight of a minus dominating function of G. In this note, we establish a sharp lower bound on $\gamma^-(G)$ for regular graphs G.

1. Introduction

Let $G=(V,E)$ be a graph with vertex set V and edge set E, and let v be a vertex in V. If $v \in V$, the degree of v in G is written as $\deg v$. The graph G is r-regular if $\deg v = r$ for all $v \in V$. In particular, if $r=3$, then we call G a cubic graph. The *open neighborhood* of v is defined as the set of vertices adjacent to v, i.e., $N(v)=\{u \mid uv \in E\}$. The *closed neighborhood* of v is $N[v]=N(v)\bigcup\{v\}$.

For any real valued function $g: V \to R$ and $S \subseteq V$, let $g(S)=\sum g(u)$ over all $u \in S$. A *minus dominating function* is defined in [2] as a function $g: V \to \{-1,0,1\}$ such that $g(N[v]) \geqslant 1$ for all $v \in V$. The *minus domination number* for a graph G is $\gamma^-(G)=\min\{g(V) \mid g$ is a minus dominating function on $G\}$. The concept of minus domination in graphs is studied in [1–3].

Zelinka [4] established the following lower bound on $\gamma^-(G)$ for a cubic graph G.

Theorem A. *For every cubic graph G of order n, $\gamma^-(G) \geqslant n/4$.*

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In this note we generalize the result of Theorem A to r-regular graphs.

Theorem 1. *For every r-regular graph $G = (V,E)$ of order n,*

$$\gamma^{-}(G) \geqslant \frac{n}{r+1}$$

and this bound is sharp.

Proof. Let f be a minus dominating function on G satisfying $f(V) = \gamma^{-}(G)$. We consider the sum $N = \sum\sum f(u)$, where the outer sum is over all $v \in V$ and the inner sum is over all $u \in N[v]$. This sum counts the value $f(u)$ exactly $deg\, u + 1$ times for each $u \in V$, so

$$N = \sum_{u \in V}(deg\, u + 1)f(u) = (r+1)\sum_{u \in V} f(u) = (r+1)f(V).$$

On the other hand,

$$N = \sum_{v \in V}\sum_{u \in N[v]} f(u) = \sum_{v \in V} f(N[v]) \geqslant \sum_{v \in V} 1 = n.$$

Consequently, $\gamma^{-}(G) = f(V) \geqslant n/(r+1)$. That the lower bound is sharp is easily seen by considering a complete graph on $r+1$ vertices and assigning the value 1 to one vertex and the value 0 to the remaining r vertices to produce a minus dominating function of weight $n/(r+1) = 1$. □

References

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