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Some Elementary Inequalities for Distance-regular Graphs

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In this note we will give some inequalities for distance-regular graphs. In [1, p. 135, remark (ii)] a general inequality is given and the authors ask for useful specializations. The inequalities given below are such specializations; however, the second is an improvement of such a specialization. These specializations are useful since they rule out an intersection array not ruled out until now. For definitions and notations we refer to [1].

THEOREM. *Suppose* $p^k_{ij} \neq 0$. *Then the following inequalities must hold*:
(i) $b_i \leq p^k_{i+1\,j-1} + p^k_{i+1\,j} + p^k_{i+1\,j+1}$;
(ii) $1 + a_i \leq p^k_{i\,j-1} + p^k_{ij} + p^k_{i\,j+1}$;
(iii) $c_i \leq p^k_{i-1\,j-1} + p^k_{i-1\,j} + p^k_{i-1\,j+1}$.

PROOF. Let $\alpha, \beta \in \Gamma$ with $d(\alpha, \beta) = k$. Take $\gamma \in \Gamma_i(\alpha) \cap \Gamma_j(\beta)$. Looking at the neighbours of γ we find the claimed inequalities. For example, (ii) follows from the following inclusion: $\{\gamma\} \cup (\Gamma_i(\alpha) \cap \Gamma(\gamma)) \subset \Gamma_i(\alpha) \cap (\Gamma_{j-1}(\beta) \cup \Gamma_j(\beta) \cup \Gamma_{j+1}(\beta))$. □

COROLLARY. *The array* {39, 32, 20, 2; 1, 4, 16, 30} *cannot be realized as the intersection array of a distance-regular graph.*

PROOF. Such a graph would have $a_4 = 9$, $p^4_{43} = p^4_{45} = 0$ and $p^4_{44} = 4$, which contradicts (ii) with $i = j = k = 4$. □

REFERENCE

1. A. E. Brouwer, A. M. Cohen and A. Neumaier, *Distance-regular Graphs*, Springer-Verlag, Berlin, 1989.

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