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# Bandwidth Allocation in Tactical Data Links via Mechanism Design

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# Abstract

Our research focusses on improving the quality and accuracy of the common operating picture of a tactical scenario through the efficient allocation of bandwidth in the tactical data networks among self-interested actors, who may resort to strategic behaviour dictated by self-interest. We propose a two-stage bandwidth allocation mechanism based on modified strictly-proper scoring rules, whereby multiple agents can provide track data estimates of limited precisions and the centre does not have to rely on knowledge of the true state of the world when calculating payments. In particular, our work emphasizes the importance of applying robust optimization techniques to deal with the data uncertainty in the operating environment. We apply our robust optimization – based scoring rules mechanism to an agent-based model framework of the tactical defence scenario, and analyse the results obtained.

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Keywords : Cyber-Physical-Social Systems; Uncertainty; Sensor Networks; Tactical Data Links; Mechanism Design; Robust Optimization;.

# 1. Introduction

The defense sector is undergoing a phase of rapid technological advancement, in the pursuit of its goal of information superiority. This goal depends on a large network of complex interconnected systems – sensors, weapons, soldiers – linked by heterogeneous tactical data networks. Our research focusses on improving the quality and accuracy of the common operating picture through the efficient allocation of bandwidth in the tactical data links. The problem of bandwidth allocation is compounded by the self-interested behavior exhibited by the military commanders of each platform, who are more concerned with the well-being of their own platforms over others.

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Individual platforms benefit from receiving data from other platforms but have no incentive for sharing it. Thus, we can expect a tendency for platforms to under-represent the quality of their data so that the bandwidth is allocated to the transmission of data by others (Rogers et al.<sup>1</sup>, Klein et al.<sup>2</sup>). Against this background, we propose a mechanism that efficiently allocates the flow of data within the tactical data network to ensure that the resulting global performance maximizes information gain of the entire system, despite the self-interested actions of individual actors.

In this paper, we consider a multi-flag, multi-platform military scenario where a number of military platforms have been tasked with the goal of detecting and tracking targets. The platforms must share and exchange tactical data from their onboard sensors in order to establish and maintain a common operating picture (COP) of the tactical situation. The track data exchanged among sensor platforms encapsulates the sensor's own position as well as estimates of the position and dynamics of the observed targets. The exchange of tactical data among the platforms is facilitated over a standardized radio network, known as a TActical Data Information Link (TADIL). The sensors onboard the military platform have a partial and inaccurate view of the COP and need to make use of data transmitted from neighboring sensors over the bandwidth-constrained TADIL to improve the accuracy of their own measurements. The mission outcome can be significantly affected by decisions made in real time about which data to share. Ad-hoc bandwidth allocation can have serious repercussions and can even jeopardize a mission.

Reporting Responsibility ( $R^2$ ) rules is a minimal precedence based mechanism which permits only the unit with the best quality data (position, velocity, etc.) to report a surveillance track on the data link. This strategy prevents multiple track reports on the link for a single object, thus minimizing the data latency. However, it precludes any possibility of collaboration in building the COP by disallowing the redundant reporting of a single object. In our work we consider the  $R^2$  minimalist approach as our point of departure. We start with the premise that additional communication per network cycle can significantly improve the quality of the combined data and by enough to warrant the additional latency that comes from a longer cycle time. Thus, we seek to design a mechanism which can efficiently allocate a finite bandwidth, beyond what is used in the  $R^2$  approach, to enhance the quality of the common operating picture. We encapsulate the desired features of the mechanism in Figure 1.

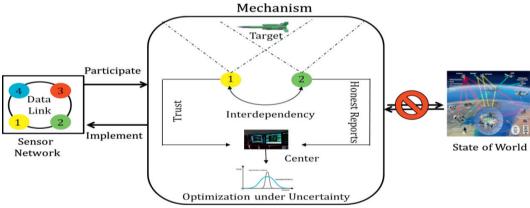


Figure 1 Framework for Bandwidth Allocation in Tactical Data Networks

The heart of the mechanism, which we need to design, resembles a portfolio optimization problem. The portfolio problem assumes that a portfolio needs to be constructed consisting of a set of stocks. Each of the stocks has a return and a risk value associated with it and the objective is to determine the fraction of wealth to be invested in each stock to maximize the portfolio value. In reference to our problem scenario, the stocks represent the observations made by the sensors. The return value of the stock can be regarded as the information content of each observation; the risk indicates the uncertainty in the reported data while the total wealth represents the bandwidth to be allocated on the tactical data link. Thus the objective is to determine which track information to select for transmission, given the fixed bandwidth available to maximize the total information content. The challenges of interdependency of the reported data, selfish behavior, constrained resources and dynamic uncertain environments dictate that our mechanism needs to go beyond a simple portfolio optimization. We highlight these challenges below:

• Voluntary Participation – Since sensor platforms are individually owned by different stakeholders, the mechanism must ensure that platforms participate voluntarily in lieu of some incentive of participation.

- Honest Reports (Incentive Compatibility) The sensor platforms may resort to self-interested behavior
  and optimize their own gain from the network at a cost to the overall network performance. Hence the
  mechanism has to incentivize the platforms to truthfully reveal their private information (track data).
- Interdependency In tactical sensor networks, since the observations made by the sensors are polluted by uncertainty and noise, the information content of a sensor's observation will be affected by the observations made by other sensor platforms. The mechanism must account for this information interdependency in the reported observations.
- Lack of access to the state of world The mechanism should work even when the center has no access to the true state of world. The dynamic and uncertain nature of the operating environment means that the track data evolves between the time the information is reported and the time when it can be observed. Thus the center needs to evaluate the received reports without any knowledge of the true outcome.
- Optimization under uncertainty The mechanism needs to account for the possibility that given the dynamic operating environment, there might be some uncertainty in the reported data.
- Implementation The mechanism has to ensure that once the sensor platforms have been allocated to their respective targets, the selected platforms invest all their resources to track their assigned targets.

#### 2. Literature Review

In order to address the requirements of private information and selfish behavior, mechanism design has been used in literature for achieving globally optimal behavior. The field of mechanism design lies at the intersection of economics and game theory and is concerned with designing protocols, and institutions that are mathematically proven to satisfy certain system-wide objectives under the assumption that individuals interacting through such institutions act in a self-interested manner and may hold private information that is relevant to a required decision. Mechanism design finds application in problems involving the allocation of scarce resources where both human and computational entities are inclined to resort to strategic behavior dictated by guile and self-interest; examples of this includes allocation of network bandwidth, storage capacity and power. Rogers et al.<sup>1</sup> have studied the use of tools and techniques from computational mechanism design for information fusion within Sensor Networks. Klein et al.<sup>2</sup> proposed an interdependent value mechanism design for bandwidth allocation in Tactical Data Networks. Both these bodies of literature use a modified version of the Vickrey-Clarke-Groves (VCG) mechanism to achieve efficient bandwidth allocation and ensure truthful reporting by conditioning payments on the realized value for data shared between agents. The VCG mechanism is a sealed-bid second-price auction in which all bidders submit sealed bids individually but the winner pays the second-highest bid rather than their highest winning bid. The VCG mechanism suffers from well-documented vulnerabilities of bidder collusion and spiteful bidding and doesn't address our requirements of optimization under uncertainty, lack of access to the state of world and implementation. Given the shortcomings of auction-based mechanism we shift our focus from the realms of auction based models, to another promising alternative approach within the Mechanism Design research domain, in the form of *scoring rules*.

#### 3. Scoring Rules

Scoring Rules have been proposed as a methodology to address the shortcomings of auction-based mechanism design for expected value maximization. Scoring Rules are used to assess the accuracy of probabilistic forecasts, by awarding a score based on the forecast and the event that materializes. Scoring rules provide a framework wherein the agents are incentivized to invest their resources in making accurate, high-quality assessments and reporting them truthfully. In our work, we are interested in *strictly proper* scoring rules. A *strictly proper* scoring rule is the one in which an actor can maximize his score by reporting exactly his or her true beliefs about the event. We shall restrict our discussion to the four most popular strictly proper scoring rules – quadratic, spherical, logarithmic and parametric – as we can analytically derive and express their expected values in closed forms.

One of the drawbacks of the auction-based Mechanism Design was that it did not account for agents not investing all their available resources in generating the observations. Miller et al.<sup>3</sup> combat this issue through the introduction of scaling parameters. They show that the affine transformation of the scoring rules, does not affect the inherent

properties of the scoring rules, like, incentive compatibility. We model an agent's noisy private measurement, x, as Gaussian random variable,  $x \sim N(x_0, \theta^{-1})$  where,  $x_0$  is the true state of the observable and  $\theta$  is the information content of the observation. If we denote the scoring rule by the function  $S(x_0; x, \theta)$  and the expected score as  $\overline{S}(\theta)$  then we can formulate the expected payment and utility as

$$\bar{P}(\theta) = \alpha \bar{S}(\theta) + \beta \qquad \bar{U}(\theta) = \alpha \bar{S}(\theta) + \beta - c(\theta) \tag{1}$$

where  $\alpha$  and  $\beta$  are the scaling parameters and  $c(\theta)$  is the cost of generating an observation with precision  $\theta$ .

We can now compare the four different scoring rules – quadratic, spherical, logarithmic and parametric – for Gaussian probability density function  $N(x_0; x, \theta^{-1})$ . An important property of the strictly proper scoring rules is the concavity of the expected scoring rules to incentivize an agent to produce truthful observations. Hence the parameter k for the parametric scoring rule family is restricted to the space (1, 3) to ensure concavity. We also calculate the expected values along with the parameter expressions for the strictly proper scoring rules in Table 1.

Table 1 Scoring Rules for Gaussian distributions				
	Quadratic	Spherical	Logarithmic	Parametric
$S(x_0; x, \theta)$	$2N - \left(\frac{\theta}{4\pi}\right)^{1/2}$	$\left(\frac{4\pi}{\theta}\right)^{1/4}N$	log N	$kN^{(k-1)} - \frac{k-1}{\sqrt{k}} \left(\frac{2\pi}{\theta}\right)^{(1-k)/2}$
$\bar{S}(\theta)$	$\left(\frac{\theta}{4\pi}\right)^{1/2}$	$\left(\frac{\theta}{4\pi}\right)^{1/4}$	$\frac{1}{2}\log\left(\frac{\theta}{2\pi}\right) - \frac{1}{2}$	$\frac{1}{\sqrt{k}} \left(\frac{2\pi}{\theta}\right)^{(1-k)/2}$
α	$4c'( heta_0)\sqrt{ heta\pi}$	$4c'(\theta_0)(4\pi\theta^3)^{1/4}$	$2c'(\theta_0)\theta_0$	$\frac{2c'(\theta_0)\theta_0\sqrt{k}}{k-1} \left(\frac{\theta_0}{2\pi}\right)^{(1-k)/2}$
β	$c( heta_0)$ $- 2 heta_0 c'( heta_0)$	$c( heta_0) \ - 4 heta_0 c'( heta_0)$	$c(\theta_0) - 2\theta_0 c'(\theta_0)$ $\left(\frac{1}{2}\log\left(\frac{\theta_0}{2\pi}\right) - \frac{1}{2}\right)$	$c(\theta_0) - \frac{2\theta_0 c'(\theta_0)}{k-1}$

Papakonstantinou et al.<sup>4</sup> extended the concept of *modified* scaled strictly proper scoring rules to handle the lack of knowledge of the outcome while preserving the property of incentive compatibility. In modified strictly proper scoring rules, the trusted center fuses the observations from all the other agents and excludes the agent whose reported observation is being evaluated. In the absence of access to the true outcome, the center uses this fused set of observations to evaluate the agent's reported observations. Thus, based on the modified strictly proper scoring rule, an agent can maximize its expected score and by extension, their expected payments by truthfully reporting its observations, assuming that other agents in the system also honestly report their observations. This makes truthful revelation a Nash equilibrium and the optimal strategy for all agents in the system.

#### 4. Interdependent Valuation

In tactical sensor networks, individual sensors have a limited and partial view of the common operating picture and produce uncertain and noisy observations. The value of one sensor platform for an allocation of bandwidth depends on private information held by other platforms, namely on the quality of their observed track data. The resulting information structure is one of interdependent valuations. A naïve extension of the VCG mechanism is known not to work in the case of interdependent valuations. Jehiel & Moldovanu<sup>5</sup> have showed that, in an interdependent valuation setting, no standard one-stage mechanism can achieve both efficiency and incentive compatibility for procurement of estimates from multiple sources. Mezzetti<sup>6</sup> addressed this challenge to a certain extent and showed that an efficient allocation with multidimensional types is possible, if two-stage mechanisms can be adopted in which the payments are made contingent on realized values reported in a second stage. Mezzetti designed a two-stage mechanism: in the first stage the agents would submit their reports to the center, which would in turn, determine the allocation of the items among the bidding agents. In the second stage, the agents report their observations and receive the final payments from the center. Accordingly, we design a two-stage mechanism based on modified strictly proper scoring rules.

#### 5. Robust Optimization

A two-stage mechanism can be constructed based on modified scaled strictly proper scoring rules, which selects a set of sensor agents to provide observations for a target. Since there are numerous targets in the system, we end up with different sets of sensor – target pairs. However we can only allocate a limited bandwidth for transmitting information over the tactical data network. Hence, we need a methodology to decide which sensor-target pairs to select for transmission to ensure the recovery of the highest gain in information for a given quantum of bandwidth. The problem is compounded by the inherent uncertainty in the information content of the observations. Deterministic optimization techniques that rely on nominal data, no longer work in these settings. Robust techniques provide an attractive choice in addressing the feasibility and optimality of the optimization solution, given the uncertainty in the data. We formulate our problem as a robust portfolio optimization problem.

$$\begin{array}{ll} maximize & \sum_{\substack{k \in K, j \in L_k \\ subject \ to \end{array}}} \theta_{jk} z_{jk} \\ \sum_{\substack{k \in K, j \in L_k \\ z_{jk} \in \{0,1\}}} z_{jk} = N_{auc} \end{array}$$

$$(2)$$

where,

*K* : Set of targets in the system

 $L_k$ : Set of agents selected through the proper-scoring rules algorithm for target k

 $\theta_{ik}$ : Quantification of covariance (information content) of the reported observation made by agent j of target k

 $N_{auc}$ : The total number of agent-target pairs that can selected for transmission

 $z_{ik}$ : Binary decision variable corresponding to which sensor-target pair is selected

The information content which is a calculated using the covariance of the reported observation is assumed to be uncertain. In other words, we model the information content as a random variable  $\tilde{\theta}_{jk}$  that has a symmetric distribution in the interval  $[\theta_{jk} - \hat{\theta}_{jk}]$ ,  $\theta_{jk} + \hat{\theta}_{jk}]$ .  $\theta_{jk}$  is the expected information gain, while  $\hat{\theta}_{jk}$  is the measure of the uncertainty of the information content. We adopt the robust linear framework proposed by Bertsimas & Sim<sup>7</sup> to solve the portfolio problem. The Bertsimas-Sim framework is based on the premise that given a set of uncertain dara elements, only a small subset of the elements takes their worst-case values at the same time. The formulation provides a protection-level  $\Gamma$  to control the degree of robustness of the solution. The parameter  $\Gamma$  guarantees a feasible solution for instances in which fewer than  $\Gamma$  parameters take their worst-case values. The approach even provides a probabilistic guarantee, that if more than  $\Gamma$  parameters change, the robust solution will still be feasible to a high degree of probability. The linear nature of the problem makes it extensible to discrete optimization problems.

$$\begin{array}{ll} maximize & \sum_{k \in K, j \in L_{k}} \bar{\theta}_{jk} \, z_{jk} - \beta(z_{jk}, \, \Gamma) \\ subject to & \sum_{k \in K, j \in L_{k}} z_{jk} = N_{auc} \\ \beta(z_{jk}, \, \Gamma) = & \max_{\{S \cup \{t\} | S \subseteq J, \, |S| = |\Gamma|, \, t \in J \setminus S\}} \left\{ \sum_{jk \in S} \hat{\theta}_{jk} \, z_{jk} + (\Gamma - |\Gamma|) \hat{\theta}_{jk_{t}} z_{jk_{t}} \right\} \end{array}$$
(3)

## 6. Algorithm

We design a two-stage mechanism based on the modified strictly proper scoring rules and robust optimization. In the first stage, the center preselects M of the N available agents based on the reported cost functions through a

single  $(M + 1)^{th}$  sub-auction. In the second stage, the center announces the modified scaled strictly proper scoring rules and asks the *M* preselected agents to produce their observations. Each of the preselected agents produce and report their observations to the center, which in turn, calculates their payments based on the announced scoring rule. The center then selects the final sensor-target pairs based on the robust portfolio optimization and the selected sensor platforms report the observations on their allocated targets on the data link.

# 1. First Stage

- 1.1. The trusted centre asks  $N \ge 2$  sensor agents to report their cost functions  $\hat{c}_i(\theta)$  and their maximum information content  $\hat{\theta}_i^c$ ,  $\forall i \in \{1, 2, ..., N\}$
- 1.2. The centre selects M ( $1 \le M < N$ ) sensor agents with the lowest costs, associates them with the  $(M + 1)^{th}$  cost and discards the rest of the sensor agents.

# 2. Second Stage

- 2.1. The centre asks sensor agent *j*, selected in Step 1.2, to generate the observations and presents it with a modified strictly proper scoring rule with parameters  $\alpha_i$  and  $\beta_i$
- 2.2. Each of these sensor agents will produce an estimate  $x_j$  with information content  $\theta_j$  and report  $(\hat{x}_i, \hat{\theta}_i)$  to the centre which, in turn, issues the payments to all the sensor agents.
- 2.3. The center solves the robust portfolio optimization to select target-sensor pairs for transmission. The selected sensors are asked to broadcast the observations on their allocated targets.

# 7. Results

In order to study the application of mechanism design in a practical context, we need a surrogate model for the real-world operation which exhibits the necessary fidelity and complexity. To this end, we leverage the Discrete – Agent Framework (DAF) developed at Purdue University to design an Agent-Based Model (ABM). We conduct multiple runs of the ABM by changing the starting positions and dynamics of the targets in the scenario and analyse the results of applying our modified scaled strictly proper scoring rules based mechanism to the simulation model.

# 7.1 Maximum Number of Preselected Sensor Platforms (M)

In the first stage of the proposed mechanism, the trusted centre preselects M sensor platforms from the N available sensor platforms with the lowest cost functions through one single reverse  $(M + 1)^{th}$  auction. In our simulation scenario, since we consider four sensor platforms and the R<sup>2</sup> tracks have already been pre-assigned, there are only N = 3 sensor platforms available for selection for transmitting non  $- R^2$  track data. Thus  $M \in [1,3]$  dictates the maximum number of sensor platforms that can track any one target. For example, M = 2 indicates that a maximum of 3 platforms can be assigned to one target; one for R<sup>2</sup>-track data and two for non-R<sup>2</sup> track information.

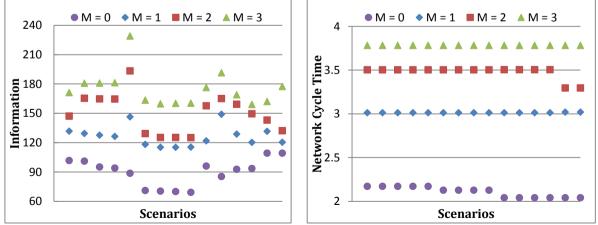


Figure 2(a) Information for different values of M, and (b) Network Cycle Time for different values of M

Figure 2(a) and (b) represents the variation of the transmitted information and the net cycle time as the maximum number of pre-selected platforms (M) is varied from 1 to 3. The x-axis represents the diverse simulation scenarios (runs) with different starting positions and dynamics of the targets. The baseline case of R<sup>2</sup> reporting is represented as M = 0 and corresponds to the lowest information flow with the minimum Network Cycle Time. As the value of M increases from 1 to 3, more platforms are selected to transmit non – R<sup>2</sup> track data and the information flow in the network increases. However this increased situational awareness comes at the cost of information latency, as the Network Cycle Time (NCT) increases with M. This represents an intuitive result of the tradeoff between information content and information latency. Increasing the value of M allows additional track data to be transmitted over the tactical data network, though it also results in increased latency between successive track updates.

## 7.2 Scoring Rules

In order to facilitate the discussion on the comparison of the four strictly proper scoring rules – Quadratic, Spherical, Logarithmic and Parametric - we generate the plots of the total expected payment and the minimum payment made by the center for the parameter space of k = (1,3). We present these results in Figure 3

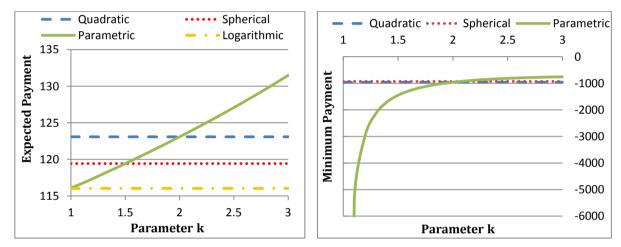


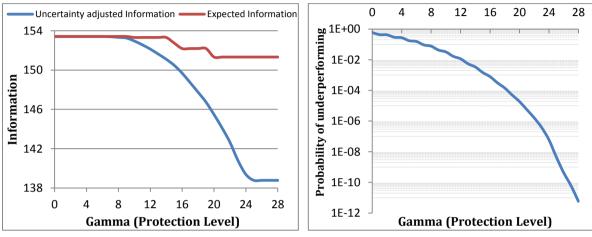
Figure 3(a) Expected Payment for different values scoring rules, and (b) Minimum Payment for different values scoring rules

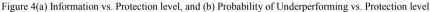
Figure 3(a) illustrates that the payment scheme based on the logarithmic scoring rule and the parametric scoring rule for  $k \rightarrow 1$  results in the center making the lowest expected payments to the sensor platforms. Another distinctive trait is that the expected payment resulting from the logarithmic, spherical and quadratic scoring rules is the same as those based on the parametric scoring rule, for values of the parameter  $k \rightarrow 1$ , k = 1.5 and k = 2 respectively. This result serves as a validation for the analytical derivation where the parametric scoring rule takes the same expression for the expected payments as the logarithmic, spherical and quadratic scoring rules, for values of the parameter  $k \rightarrow 1$ , k = 1.5 and k = 2 respectively. Figure 3(b) plots the lower bounds of the payment of the parameter scoring rule for the parameter space k = [1.1, 3) against the spherical and quadratic scoring rule. The logarithmic scoring rule and the limiting case of the parameter scoring rule family ( $k \rightarrow 1$ ) results in large negative payments when the sensor platforms produce imprecise observations and hence are omitted from the figure. The effect of the platform's imprecise estimate can be minimized for the parametric family by choosing the parameter carefully. From Figure 3 it appears that a value of  $k \in [1.1, 1.5]$  is a judicious compromise between the different factors. This set of parameter values produces low expected payments close to the ones obtained from the logarithmic scoring rule, and at the same time, imposes a finite lower bound on the minimum payments.

# 7.3 Protection Level ( $\Gamma$ )

Next, we solve the robust portfolio optimization problem in the second stage by using the Bertsimas - Sim formulation for different values of the protection level ( $\Gamma$ ). Figure 4(a) shows the decrease in the expected information flow and the uncertainty-adjusted information flow in the mechanism as the value of  $\Gamma$  increases. The uncertainty-adjusted information value represents the difference between the expected information flow and the risk

function (uncertainty) when at most  $\Gamma$  variables are allowed to take their worst values. The figure illustrates the phase transitions that occur as the value of  $\Gamma$  increase and the transition points for the expected information flow coincides with the protection levels where the composition of the portfolio changes. The Bertsimas-Sim framework provides probabilistic bounds of constraint violation, i.e. a theoretical bound on the fraction of portfolios with information values which fall below the threshold value of the uncertainty adjusted information. We plot this probability of underperforming as a function of the protection level  $\Gamma$  in Figure 4(b). For low protection levels, the probability of the portfolio solution falling below the optimal solution is quite high. As the protection levels increase, probability of underperforming decreases by several orders of magnitude.





#### 8. Conclusion and Future Work

In conclusion, we have successfully applied our proposed robust-optimization based scoring rules algorithm to the MAS simulation model. The algorithm provides a unique insight into the role of computational mechanism design, especially strictly proper scoring rules, in decision making. The applicability of the proposed mechanism goes beyond tactical data links and is amenable to any settings which involve exchange of information or services between buyers and sellers. Ensuring trust in the auctioneer, handling the correlation in data uncertainty and preventing bidder collusion represent some of the potential avenues for extending the scope of this research work.

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