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H_∞ Optimal Control of Aero engine Distributed Control System with Packet Dropout compensator

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Abstract

This article proposes a compensator to overcome the instability of the aero engine distributed control system which has network time delay and packet dropout. The method models the closed-loop in the distributed control system as an asynchronous dynamical-switched system and deduces its robust asymptotic stability criterion and the sufficient solution which satisfy certain H_∞ performance index. Then the corresponding design method of compensator and H_∞ optimal controller is presented. The effectiveness of the proposed compensator and controller is demonstrated by final numerical example.

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Keywords: aero engine; distributed control system; packet dropout; compensator; H_∞ robust control;

1. Main text

Distributed control system(DCS) is a centralized control system which is consisted of several distributed intelligent cells based on microprocessor and communicate network[1]. Comparing with conventional centralized control system, aero engine DCS is higher in system performance and system reliability[2]. But some uncertainty such as time delay and data packet dropout also appeared along with the transmission network. It's equal to introduce the uncertain time delay into the control loop when there

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isn't any compensation for data packet dropout and this will lead to poor performance and instability[3]. Therefore, this paper proposes a compensator to reduce influence of packet dropout on system performance and designs a H_∞ optimal control method with uncertain time delay $\tau_k < T$ and bounded packet dropout for a class of turbo fan DCS.

2. Model of closed-loop DCS with compensator

A class of aero-engine DCS with packet dropout compensator as depicted in Fig.1 is consisted of four components: a controller, an aero-engine plant to be controlled, a CAN bus network to connect intelligent sensors and actuator with controller and a compensator. x_k , $\overline{x}_k$, $\hat{x}_k$, u_k , τ_k , r_k , z_k are respectively the state of plant, the input of controller, the state of compensator, the output of controller, the network induced time delay, an additive disturbance vector and the controlled output of plant.

When there isn't packet dropout in the network at a certain time, the switch in Fig.1 will be located at S_1, and the input of controller is the output of plant. On the other hand, when there is packet dropout in the network, the switch is located at S_2 and the input of controller is the output of compensator. The compensator always calculate its own state $\hat{x}_k$ every sampling period based on current controller's output u_k .

The little departure state space model of a turbo fan at a stable point can be described as follows

$$\begin{cases} \dot{x}(t) = A_p x(t) + B_p v(t) + H_0 r(t) \\ z(t) = C_p x(t) + H_1 r(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$, $v(t) \in R^l$, $z(t) \in R^m$ are respectively the state variable, the input and the controlled output. A_p , B_p , C_p , H_0 and H_1 are known real constant matrices with appropriate dimensions.

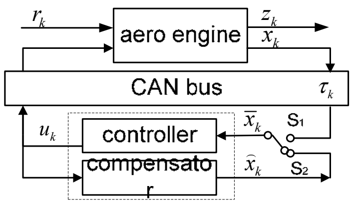

Fig.1. Closed-loop DCS of aero engine with compensator

The compensator is described by

$$\hat{x}(k+1) = A_c \hat{x}(k) + B_c u(k) \tag{2}$$

where $\hat{x} \in R^n$ is the state of compensator; A_c, B_c are unknown matrices which need to be designed.

The state feedback controller law is

$$u(k) = K\overline{x}(k) \tag{3}$$

We assume that the time delay $\tau_k < \overline{\tau} < T$, If controller doesn't receive new sensor's packet until $t = kT + \overline{\tau}$ in the time period $[kT, (k+1)T]$, it will use compensator's state instead of actual aero engine's state to calculate u_k .

Consider the time delay, the input of plant is

$$v(t) = \begin{cases} u(k-1), kT \le t < kT + \tau_k \\ u(k), kT + \tau_k \le t < (k+1)T \end{cases} \tag{4}$$

Where τ_k is the update time for the control vector.

If not considering the disturbance, the discrete-time model of generalized controlled plant[4] is

$$x(k+1) = A^{'} x(k) + B_0^{'} u(k) + B_1^{'} u(k-1) \tag{5}$$

where $A^{'} = e^{AT}$, $B_0^{'} = \int_o^{T-\tau_k} e^{As} Bds$, $B_1^{'} = \int_{T-\tau_k}^{T} e^{As} Bds$.

Because Equation (5) includes the information of $u(k)$ and $u(k-1)$, according to whether packet dropout happen at time point k and $k-1$, the system can be described as an asynchronous dynamical-switched system among the following four event.

Event 1: packet doesn't dropout at time point k and $k-1$.

Event 2: packet doesn't dropout at time point k, but it dropout at time point $k-1$.

Event 3: packet dropout at time point k, but not at time point $k-1$.

Event 4: packet dropout at time point k and $k-1$.

If not considering the disturbance, the discrete-time state space models of the four events are

Event 1: $x(k+1) = (A^{'} + B_0^{'}K)x(k) + B_1^{'}Kx(k-1)$ *(6)*

Event 2: $x(k+1) = (A^{'} + B_0^{'}K)x(k) + B_1^{'}K\hat{x}(k-1)$ *(7)*

Event 3: $x(k+1) = A^{'}x(k) + \tilde{B}_0^{'}K\hat{x}(k) + \tilde{B}_1^{'}Kx(k-1)$ *(8)*

Event 4: $x(k+1) = A^{'}x(k) + \tilde{B}_0^{'}K\hat{x}(k) + \tilde{B}_1^{'}K\hat{x}(k-1)$ *(9)*

where $\tilde{B}_0^{'} = \int_o^{T-\bar{\tau}} e^{A_p s} B_p ds$, $\tilde{B}_1^{'} = \int_{T-\bar{\tau}}^{T} e^{A_p s} B_p ds$.

According to matrix theory, we can obtain

$$B_0^{'} = B_0 + DF(\tau_k)E \quad (10)$$

$$B_1^{'} = B_1 - DF(\tau_k)E \quad (11)$$

$$F^T(\tau_k)F(\tau_k) \le I \quad (12)$$

where B_0, B_1, D, E are constant matrices.

As for Event 1 and Event 2, $u(k) = Kx(k)$, the state equation is

$$\hat{x}(k+1) = A_f\hat{x}(k) + B_f Kx(k) \quad (13)$$

As for Event 3 and Event 4, $u(k) = K\hat{x}(k)$, the state equation is

$$\hat{x}(k+1) = (A_f + B_f K)\hat{x}(k) \quad (14)$$

We defined $w(k) = \begin{bmatrix} x^T(k) & \hat{x}^T(k) \end{bmatrix}^T$

Combining (6)~(14), the model of closed-loop DCS with compensator becomes

$$w(k+1) = \Phi_{1i}w(k) + \Phi_{2i}w(k-1), \quad i = 1,2\cdots4 \quad (15)$$

where

$$\Phi_{11} = \Phi_{12} = \begin{bmatrix} A^{'} + B_0^{'}K & 0 \\ B_c K & A_c \end{bmatrix}, \Phi_{13} = \Phi_{14} = \begin{bmatrix} A^{'} & \tilde{B}_0^{'}K \\ 0 & A_c + B_c K \end{bmatrix}, \Phi_{21} = \begin{bmatrix} B_1^{'}K & 0 \\ 0 & 0 \end{bmatrix}, \Phi_{22} = \begin{bmatrix} 0 & B_1^{'}K \\ 0 & 0 \end{bmatrix},$$

$\Phi_{23} = \begin{bmatrix} \tilde{B}_1^{'}K & 0 \\ 0 & 0 \end{bmatrix}, \Phi_{22} = \begin{bmatrix} 0 & \tilde{B}_1^{'}K \\ 0 & 0 \end{bmatrix}$, $B_0^{'}$ and $B_1^{'}$ are variable matrix, the others are constant matrix.

3. The robustly asymptotic stability and H_∞ performance analysis

Lemma 1. For any real matrix W , M , N , $F(k)$; and W is symmetric, $F(k)$ satisfies $F^T(k)F(k) \le I$, if there exists a scalar quantity $\varepsilon > 0$ such that

$$W + \varepsilon^{-1}N^T N + \varepsilon MM^T < 0 \quad (16)$$

then we have

$$W + N^T F^T(k)M^T + MF(k)N < 0 \quad (17)$$

Define 1. There is a positive constant γ. If: a. closed-loop system is asymptotic stable, b. satisfy restriction $\|z(k)\|_2 \le \gamma\|r(k)\|_2$ on zero initial condition, then the closed-loop system satisfy H_∞ performance index γ.

Theorem 2. For given $\gamma > 0$ and controller gain K , if there are positive symmetry matrices P_1, P_2, Q_1, Q_2 and scalar quantity $\varepsilon_1, \varepsilon_2$, such that

$$\begin{bmatrix} \bar{\Lambda} & \Psi_{2i} \\ * & \Psi_{3i} \end{bmatrix} < 0, i = 1 \sim 4 \tag{18}$$

where

$$\bar{\Lambda} = \begin{bmatrix} \Lambda_1 & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix}, \quad \bar{\Lambda}_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 \end{bmatrix}^T, \quad \bar{\Lambda}_{22} = \begin{bmatrix} -\gamma^2 I & H_1^T \\ * & -I \end{bmatrix}, \ \Lambda_1 = diag\{Q_1 - P_1, Q_2 - P_2, -Q_1, -Q_2\}$$

$$\Psi_{21} = \begin{bmatrix} (A' + B_0K)^T & (B_cK)^T & (A' + B_0K)^T & (EK)^T \\ 0 & A_c^T & 0 & 0 \\ (B_1K)^T & 0 & (B_1K)^T & -(EK)^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & H_0^T & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Psi_{22} = \begin{bmatrix} (A' + B_0K)^T & (B_cK)^T & (A' + B_0K)^T & (EK)^T \\ 0 & A_c^T & 0 & 0 \\ 0 & 0 & 0 & 0 \\ (B_1K)^T & 0 & (B_1K)^T & -(EK)^T \\ 0 & 0 & H_0^T & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Psi_{23} = \begin{bmatrix} A'^T & 0 & A'^T \\ (\tilde{B}_0K)^T & (A_c + B_cK)^T & (\tilde{B}_0K)^T \\ (\tilde{B}_1'K)^T & 0 & (\tilde{B}_1'K)^T \\ 0 & 0 & 0 \\ 0 & 0 & H_0^T \\ 0 & 0 & 0 \end{bmatrix}, \Psi_{24} = \begin{bmatrix} A'^T & 0 & A'^T \\ (\tilde{B}_0K)^T & (A_c + B_cK)^T & (\tilde{B}_0K)^T \\ 0 & 0 & 0 \\ (\tilde{B}_1'K)^T & 0 & (\tilde{B}_1'K)^T \\ 0 & 0 & H_0^T \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Psi_{3i} = \begin{bmatrix} \varepsilon_i DD^T - P_1^{-1} & 0 & \varepsilon_i DD^T & 0 \\ & -P_2^{-1} & 0 & 0 \\ & & \varepsilon_i DD^T - P_1^{-1} & 0 \\ * & & & -\varepsilon_i I \end{bmatrix}, i = 1,2 \ ; \Psi_{33} = \Psi_{34} = diag\{-P_1^{-1}, -P_2^{-1}, -P_1^{-1}\}$$

then the closed-loop system （15） satisfy H_∞ performance index γ.

Simple proof process: Define Lyapunov function as

$$V(w(k)) = w^T(k)Pw(k) + w^T(k-1)Qw(k-1) \tag{19}$$

By schur complement lemma, we will have the solution of $\Delta V(k) < 0$. Then define

$$J_z = \sum_{k=0}^{\infty} \left[z^T(k)z(k) - \gamma^2 r^T(k)r(k) \right] \tag{20}$$

By Define 1, if $J_z \le 0$ then $\|z(k)\|_2 \le \gamma \|r(k)\|_2$. For system (15), On zero initial condition, $\forall r(k) \in L_2[0,\infty]$, we have

$$J_z \le \sum_{k=0}^{\infty} \left[z^T(k)z(k) - \gamma^2 r^T(k)r(k) + \Delta V(k) \right] \tag{21}$$

Because $z^T(k)z(k) - \gamma^2 r^T(k)r(k) + \Delta V(k) = \left[w^T(k) \ w^T(k-1) \ r^T(k) \right] \Omega_1 \left[w^T(k) \ w^T(k-1) \ r^T(k) \right]^T$, $J_z \le 0$ is equivalent to $\Omega_1 < 0$, by schur complement lemma, and combining (10), (11) to $\Omega_1 < 0$, then applying Lemma 1 and schur complement lemma, if there exists $\varepsilon_i > 0$ such that (18), then system(15) is robustly asymptotic stable and satisfy H_∞ performance index γ.

4. Corresponding design method of Compensator and H_∞ optimal controller

By Theorem 2, when controller gain is known, compensator's parameters which satisfy certain H_∞ performance index can be calculated using cone complementary linearization iterative algorithm

(CCLIA). Similarly, controller gain which satisfies certain H_∞ performance index can be solved as well when compensator's parameters are known. In this section, we will develop how to design both the compensator and controller to optimize H_∞ index of system (15). Combining $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $Y = B_c K$ and (18), then we have the following deduction.

Deduction 1. For aero engine closed-loop control system represented by Equation (15) and given $\gamma > 0$, if there are positive symmetric matrices $P_1, P_2, Q_1, Q_2, X_1, X_2$, and matrix Y, and scalar quantity $\varepsilon_1 > 0, \varepsilon_2 > 0$, such that

$$\begin{cases} B_c = Y / K \\ \min\limits_{P_1,P_2,Q_1,Q_2,X_1,X_2,Y,\varepsilon_1,\varepsilon_2,A_c,K} tr([P_1X_1 + P_2X_2]) \end{cases} \tag{22}$$

$$s.t. \begin{bmatrix} \bar{\Lambda} & \Psi_{2i} \\ * & \Psi_{3i} \end{bmatrix} < 0, i = 1 \sim 4 \tag{23}$$

$$\begin{bmatrix} P_1 & 0 \\ 0 & X_1 \end{bmatrix} \geq 0, \begin{bmatrix} P_2 & 0 \\ 0 & X_2 \end{bmatrix} \geq 0 \tag{24}$$

Where

$$\bar{\Lambda} = \begin{bmatrix} \Lambda_1 & \bar{\Lambda}_{12} \\ * & \bar{\Lambda}_{22} \end{bmatrix}, \Lambda_1 = diag\{Q_1 - P_1, Q_2 - P_2, -Q_1, -Q_2\}, \bar{\Lambda}_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ C & 0 & 0 & 0 \end{bmatrix}^T, \bar{\Lambda}_{22} = \begin{bmatrix} -\gamma^2 I & H_1^T \\ * & -I \end{bmatrix},$$

then system is robustly asymptotic stable and satisfy H_∞ performance index γ. K and A_c are the feasible solution of (23), B_c is the solution of (22).

We can obtain optimal H_∞ controller with binary search. Firstly, make certain bound $\gamma \in [\gamma_1, \gamma_2]$, which γ_1 is little enough to make (22) haven't feasible solutions, and γ_2 is big enough to make (22) have feasible solutions. Then seeking the minimum γ by binary search.

Strictly speaking, the solution above algorithm searched isn't the global optimal solution, but is the suboptimal solution.

5. Example

The comparatively state equation of the aero engine in certain status can be written as

$$\Delta\dot{\bar{x}}(t) = \begin{bmatrix} -3.2501 & 0.067731 \\ 2.0448 & -3.6263 \end{bmatrix} \Delta\bar{x}(t) + \begin{bmatrix} 0.60994 & 0.48945 \\ 0.63024 & 0.3949 \end{bmatrix} \Delta\bar{u}(t) \tag{25}$$

where $x = [n_H, n_L]^T$, $u = [mf, A_8]^T$, $\Delta x = (x - x_0)/x_0$, $\Delta u = (u - u_0)/u_0$, and n_H is rotate speed of compressor, n_L is rotate speed of fan, mf is main fuel supply quantity, A_8 is acreage of nozzle's laryngeal. Set the time delay $\bar{\tau}_k = 10ms$, sample period $T = 20ms$, packet dropout rate $r = 0.01$, scalar quantity $\varepsilon_1 = \varepsilon_2 = 0.015$. Then the coefficient matrices in discrete-time state equation (15) are

$$A' = \begin{bmatrix} 0.9371 & 0.0013 \\ 0.0382 & 0.9301 \end{bmatrix}, B_0 = \begin{bmatrix} 0.1927 & 0.1985 \\ 0.2437 & 0.2208 \end{bmatrix}, B_1 = \begin{bmatrix} -0.1809 & -0.1863 \\ -0.2340 & -0.2130 \end{bmatrix}, D = \begin{bmatrix} -0.0858 & 0.0257 \\ -0.2899 & -0.2296 \end{bmatrix}$$

$$E = \begin{bmatrix} 1.6997 & 1.7245 \\ -1.1475 & -1.2666 \end{bmatrix}, H_0 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \tilde{B}_0' = \begin{bmatrix} 0.0060 & 0.0062 \\ 0.0049 & 0.0039 \end{bmatrix}, \tilde{B}_1' = \begin{bmatrix} 0.0058 & 0.0060 \\ 0.0048 & 0.0039 \end{bmatrix}$$

Controlled output equation is

$$\Delta z(k+1) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Delta x(k) + \begin{bmatrix} 0.2 & 0 \\ 0.1 & 0.1 \end{bmatrix} \Delta r(t) \tag{26}$$

Firstly, we get the bound of γ, $[\gamma_1, \gamma_2] = [0.01, 24.26]$; and then seek the optimal H_∞ performance index, the last feasible solutions are

$$A_c = \begin{bmatrix} 0.4691 & 0.0022 \\ 0.0022 & 0.4667 \end{bmatrix},\ K = \begin{bmatrix} 0.0980 & -0.2882 \\ -0.2882 & 0.2135 \end{bmatrix},\ Y = \begin{bmatrix} 0.0052 & -0.0450 \\ -0.0376 & 0.0028 \end{bmatrix},\ B_c = \begin{bmatrix} 0.2912 & 0.1821 \\ 0.1771 & 0.2520 \end{bmatrix}$$

And K , A_c and B_c are the optimal H_∞ controller gain and compensator coefficient matrices.

6. Conclusions

Network induce time delay, packet dropout and uncertain disturbance are some main factors which influence the closed-loop system in aero engine. In order to reach its optimal H_∞ control, this article set up the mathematic model of a class of aero engine DCS with packet dropout compensator. The corresponding design algorithm of compensator and H_∞ optimal controller has been proposed by Lyapunov 2^{nd} and a set of iterative LMIs method restricted by matrices inversion. The numerical example proves the effectiveness of the proposed method. How to design the compensator and controller over the whole flight envelop is the direction for future research.

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