

On ionosphere-delay processing methods for single-frequency precise-point positioning

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Abstract: In single-frequency precise-point positioning of a satellite, ionosphere delay is one of the most important factors impacting the accuracy. Because of the instability of the ionosphere and uncertainty of its physical properties, the positioning accuracy is seriously limited when using a precision-limited model for correction. In order to reduce the error, we propose to introduce some ionosphere parameter for real-time ionosphere-delay estimation by applying various mapping functions. Through calculation with data from the IGS (International GPS Service) tracking station and comparison among results of using several different models and mapping functions, the feasibility and effectiveness of the new method are verified.

Key words: single-frequency precise-point positioning; ionosphere delay; model correction; mapping function; parameter estimation

1 Introduction

In single-frequency Precise-Point Positioning (PPP) of a satellite, ionosphere delay is one of the most important factors that influence the positioning accuracy. Thus to assess the ionosphere delay accurately is key to improving the precision of single-frequency PPP. At present, the methods used for correcting the ionosphere-caused errors include half-closed model, single-layer model, Klobuchar model, and grid model^[1-4]. Since there are many factors that influence the ionosphere and each factor has its randomness, the conventional models cannot describe the mutual relations, variability, and internal mechanisms of these factors comprehensively. As a result, by using these models one cannot achieve very high precision in the estimation of ionosphere delay^[5,6]. In the present study, we analyzed

the characteristics of conventional ionosphere-mapping functions and delay functions, and propose to reduce ionosphere-caused errors in real-time estimation by using different mapping functions in which the ionosphere delay is solved as some undetermined parameter in the observed equation. We tested the new scheme on data from WUHN (Wuhan station) and BJFS (Beijing Fangshan station) and compared the results of correction with the traditional single-layer, Klobuchar, and grid models.

2 The ionosphere-parameter estimation model

2.1 The mathematical model

Half-closed models^[7] are generally used in single-frequency PPP. After eliminating satellite-orbit error, satellite-clock error, hardware delay, troposphere delay, and relativistic-effect error, the observation is represented by the equation

$$C_1 = \rho_r^s + c \cdot dt_r + d_{\text{ion}} + \varepsilon(C_1)$$

$$\frac{C_1 + \Phi_1}{2} = \rho_r^s + cdt_r + \lambda_1 N_1 / 2 +$$

Received:2010-09-25; Accepted:2010-12-20

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This work was supported by the National Natural Science Foundation of China(40902081,40774001,40841021)

$$\frac{\varepsilon(\Phi_1) + \varepsilon(C_1)}{2} \quad (1)$$

where, C_1 is the C/A code pseudo-range observation, Φ_1 is the L1 phase observation, ρ_r^s is the geometric distance between station and satellite, C is the speed of light, dt_r is receiver clock, d_{ion} is the ionosphere error, λ is L1 carrier wavelength, N_1 is an ambiguity parameter, and $\varepsilon(C_1)$, $\varepsilon(\Phi_1)$ are observation noises.

In Equation (1), the ionosphere error is conventionally corrected with an ionosphere model^[1-3]. However, these models cannot describe the inter-factor relationship, variability, and internal mechanisms of the affecting factors in the ionosphere comprehensively, and thus considerable residual remains after the correction. The correction effectiveness of single-layer model is about 50%. With the Klobuchar model, it is 50% - 60% in mid-latitude range, but significant poorer in low- and high-latitude regions due to changes of ionosphere activity. Even with the grid model, which is the best of the current correction models, it can only reach 70% - 80%^[8-10]. The limited accuracy of ionosphere-correction models has seriously affected the accuracy of single-frequency PPP. Here, we propose to construct ionosphere parameters by using different mapping functions and to make real-time estimation of the ionosphere-delay error, in order to minimize its effect on positioning accuracy.

2.2 Ionosphere-mapping functions

Usually the ionosphere delay is mapped to the zenith direction, using an ionosphere-mapping function related to the puncture point, and the estimated TEC (Total Electronic Content) of the GPS propagation paths is naturalized to geocentric zenith direction by using mapping function, and then, ionosphere delays are calculated according to the VTEC. The commonly used mapping functions are as follows:

1) Trigonometric-type Single-Layer Model (SLM) mapping function

Based on the single-layer model, the most convenient mapping function is the trigonometric function $F(Z)$ ^[7]:

$$\sin Z' = \frac{R}{R+H} \sin Z \quad (2)$$

$$F(Z) = \frac{1}{\cos Z'} = \frac{1}{\sqrt{1 - \sin^2 Z'}} \quad (3)$$

where R is the earth's radius, H is the single-layer height, Z is the station-satellite zenith, and Z' is the satellite zenith of the puncture point.

2) Broadcast satellite-orbit mapping function

Klobuchar proposed the following mapping function used for GPS-broadcast ionosphere model^[7]

$$F(E) = 1.0 + 16.0 \times (0.53 - E)^3 \quad (4)$$

$$F(h) = 1.0 + 0.516 \times (1.6745 - h)^3$$

where E and h are the satellite's elevation angle in units of π and radian, respectively.

3) Modified mapping function of SLM (MSLM)

In order to reduce the difference in TEC between the above two ionosphere-mapping functions, the Center for Orbit Determination in European (CODE) modified the SLM mapping function of the single-layer model, and the MSLM is^[5]:

$$F(Z) = \frac{1}{\cos Z'} = \frac{1}{\sqrt{1 - \sin^2 Z'}} \quad (5)$$

where $\sin Z' = \frac{R}{R+H} \sin(\alpha Z)$, $R = 6371$ km, $H = 506.7$ km, Z is the observed satellite zenith distance, and $\alpha = 0.9782$.

4) Ionosphere "slab" mapping function

JPL (Jet Propulsion Laboratory) used a "slab" model to establish the following "slab" mapping function^[3]:

$$F(z, r) = \frac{Y_{\text{slab}}(z, r) + Y_{\text{upper}}(z, r) Y_{\text{lower}}(z, r)}{Y_{\text{slab}}(0, \infty) + Y_{\text{upper}}(0, \infty) + Y_{\text{lower}}(0, \infty)} \quad (6)$$

where $Y_{\text{slab}}(z, r)$, $Y_{\text{upper}}(z, r)$, $Y_{\text{lower}}(z, r)$ represent, respectively, the degrees of contribution to the whole ionosphere of the "slab" layer and of the upper and lower parts of the ionosphere, z is satellite-zenith distance, and r is the station-to-satellite distance.

In addition, Clynych^[10] used the least-squares fitting to solve the Q-factor mapping function, Ou Jikun^[11] proposed an ionosphere mapping function which can adapt to changing values of elevation-angle in subsections, and Cohen^[12] put forward a three-parameter estimation of delay functions.

The SLM, broadcast satellite-orbit, and MSLM mapping functions are developed based on the single-layer

ionosphere model. They are simple, but in practice it is difficult to determine the single-layer height properly. The ionosphere "slab" model constructs mapping functions which can convert slant TEC into vertical TEC; these functions are related to satellite elevation. This model is hard to build.

2.3 Ionosphere-delay functions

When using one parameter, the estimated ionosphere delay can be expressed as:

$$I = F d_{\text{ion}}^{\text{zenith}} \quad (7)$$

where F is the ionosphere-mapping function, $d_{\text{ion}}^{\text{zenith}}$ is an unknown parameter.

When using two parameters, the delay function is:

$$I = F A d_{\text{ion}}^A + F E d_{\text{ion}}^E \quad (8)$$

where A is the satellite azimuth, E is the satellite elevation, and d_{ion}^A and d_{ion}^E are unknown ionosphere parameters.

Cohen^[10] used three-parameter estimation for the delay function:

$$I = (1 + \sin \varphi^*) J_1 + \cos \varphi^* \cos \lambda^* C_{11} \varphi^* \sin \lambda^* S_{11} \quad (9)$$

where φ^* , λ^* , respectively, represent the longitude and latitude of the foot Galileo point in the sun conjugate coordinates, and J_1 , C_{11} , S_{11} are unknown parameters introduced to estimate the ionosphere delays.

Different ionosphere-delay parameters indicate different decompositions of ionosphere mapping. By using one parameter, we can only map ionosphere delay to the zenith direction; by using two parameters, we can map it, to satellite elevation and azimuth direction; and by using three parameters, we can map it to the feet Galileo point in three orthogonal directions.

2.4 The mathematical model for ionosphere-parameter estimation

To use parameter estimation for ionosphere delay is to introduce the ionosphere parameters and position parameters together in the positioning calculation. At a station, the ionosphere delay can be expressed as an unknown parameter in a mathematical model as follows:

$$C_1 = \rho_r^s + cdt_r + I + \varepsilon(C_1)$$

$$\frac{C_1 + \Phi_1}{2} = \rho_r^s + cdt_r + \lambda_1 N_1 / 2 + \frac{\varepsilon(\Phi_1) + \varepsilon(C_1)}{2} \quad (10)$$

where I is the ionosphere delay, with specific forms given in equation (7), (8) and (9).

3 Examples of analysis

By using data from IGS continuous operation stations WUHN and BJFS on January 1, 2005 (WUHN0010.05 o and BJFS0010.05 o) together with the single-frequency PPP software, developed by ourselves, and the station coordinate published by IGS website as the true value, we carried out a precision analysis. In this experiment, the ionosphere parameters are estimated once per epoch, its variance is 0.001, the positioning-parameter variance is 0, the ambiguity-parameter variance is 0, and the receiver-clock-parameter variance is 30.

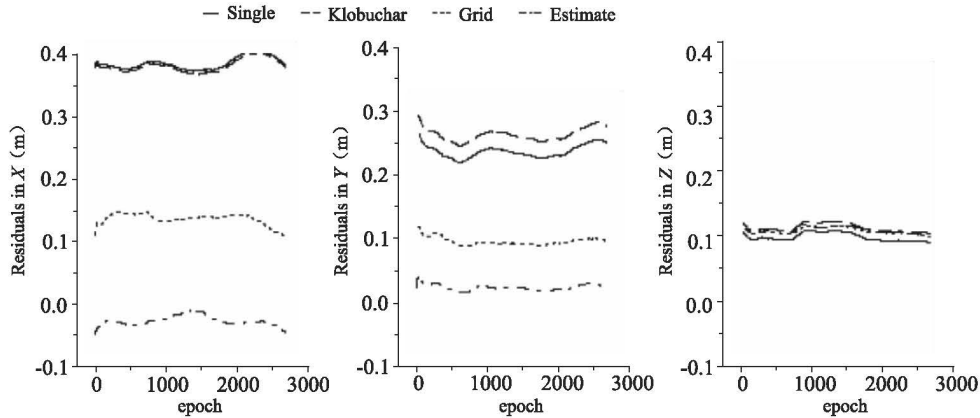
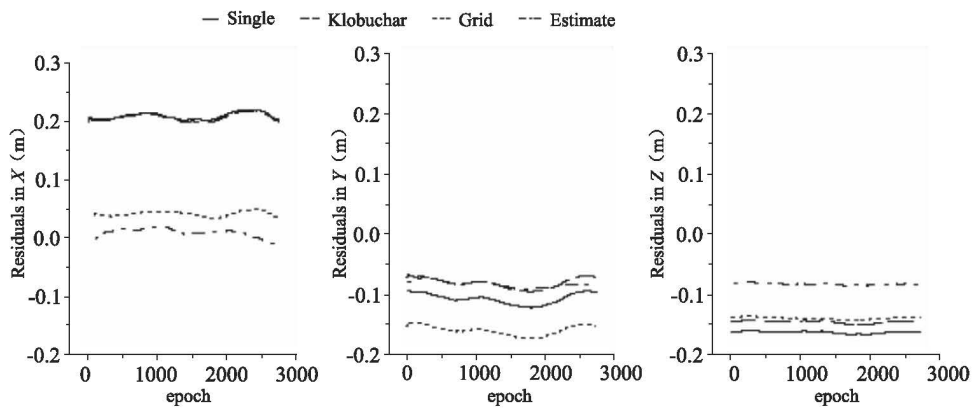
3.1 Comparison of model corrections and parameter estimates

In order to compare the results of parameter estimation and direct model correction, we made correction calculations with data from the Wuhan and Beijing stations, by using: 1) the single-layer model, 2) the Klobuchar model, 3) the grid model; and 4) the three-parameter model, as well as the mapping function given in equation (9).

We calculated coordinates for each epoch after convergence, and compared them with the true values. The residual errors of WUHN and BJFS in X , Y , Z direction are shown in Figures 1 and 2, and the precision statistics are given in Table 1.

In Figures 1 and 2 and Table 1, we may see:

1) When using the models to correct ionosphere delay, the accuracy of single-layer and Klobuchar models are comparable, and the grid model is better. Compared with single-layer and Klobuchar models, the grid model improves the accuracy by 64% and 61% in X and Y directions, respectively, with no change in Z direction at WUHN; at BJFS, the accuracy is 79% and 14% better in X and Z directions, respectively, but is

Figure 1 Residuals in X, Y, Z directions with 4 schemes at WUHN stationFigure 2 Residuals in X, Y, Z directions with 4 schemes at BJFS station**Table 1** Mean and RMS of experimental results compared with true values under 4 schemes (unit:m)

name	plan	Mean(X)	RMS(X)	Mean(Y)	RMS(Y)	Mean(Z)	RMS(Z)
WUHN	1	0.381	0.385	0.234	0.236	0.097	0.098
	2	0.380	0.380	0.262	0.262	0.112	0.112
	3	0.136	0.136	0.095	0.096	0.107	0.107
	4	-0.027	0.028	0.023	0.024	0.109	0.109
BJFS	1	0.210	0.210	-0.106	0.107	-0.162	0.162
	2	0.207	0.208	-0.080	0.081	-0.145	0.145
	3	0.043	0.043	-0.159	0.160	-0.139	0.139
	4	0.011	0.013	-0.082	0.082	-0.082	0.083

slightly lower in Y direction.

2) When parameter estimation is used, the accuracy is better than model correction. At WUHN, the improvements are 92% and 91% in X and Y directions, respectively, but there is no significant improvement in Z direction. At BJFS, the improvements are 93% and 50%, respectively, in X and Z directions, but in Y direction the improvement is the same as Klobuchar model, but much better than the single-layer and grid models.

3) While using parameter estimation, the accuracy

in X direction is improved the fastest. At WUHN, the improvement is 92%, and the RMS value is changed from 0.385 m to 0.028 m. At BJFS, the increase is 93%, and the RMS value is changed from 0.210 m to 0.013 m. In Y direction, there is an increase of 0.2 m at WUHN, and 0.08 m at BJFS. In Z direction, there is no increase at WUHN but an increase to 0.08 m at BJFS.

Overall, the parameter-estimation method is better than the grid-model correction, which in turn is better than the corrections by single-layer and Klobuchar

models.

3.2 Comparison of different ionosphere-delay parameters

To compare the influence on single-frequency PPP by using different ionosphere-delay parameters, we used the following three sets of ionosphere delay and mapping function for analysis:

- 1) 1 parameter (Eq. 7), MSLM (Eq. 5);
- 2) 2 parameters (Eq. 8), MSLM (Eq. 5);
- 3) 3 parameters (Eq. 9), (Eq. 9);

We calculated coordinates for each epoch after convergence, and compared them with the true values, and acquired the residual errors of the data from WUHN and BJFS in X , Y , and Z directions (Figs. 3 and 4); the precision statistics are given in Table 2.

From Figures 3 and 4 and Table 2 we see:

- 1) By choosing different parameters to estimated ionosphere delay, the improvements are different; using three parameters is better than using two parameters, which in turn is slightly better than using one parameter;

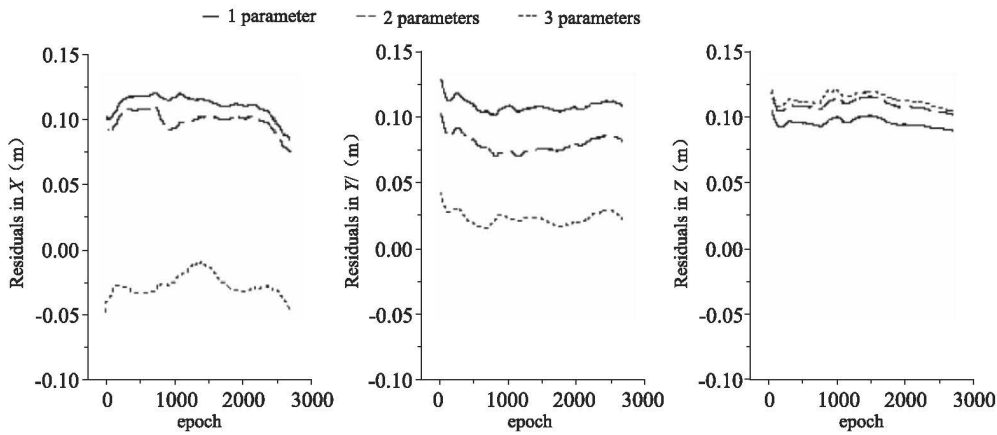


Figure 3 Residuals in X, Y, Z directions with 3 parameter estimation schemes at WUHN station

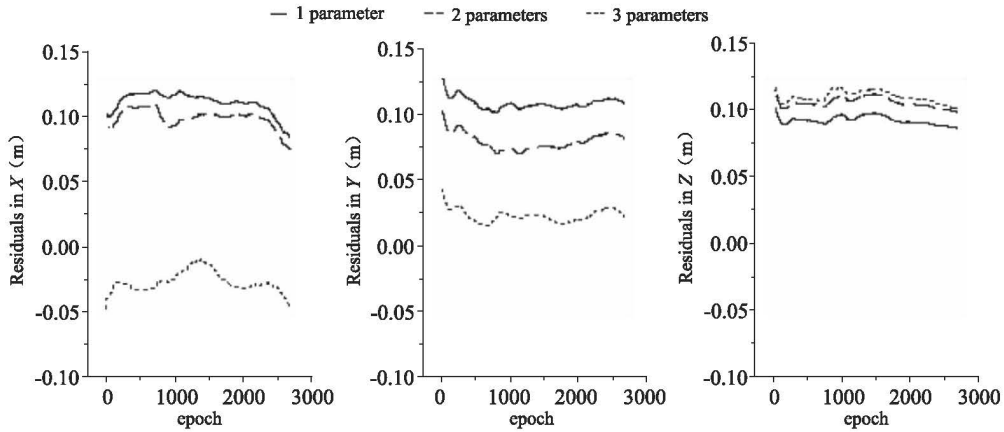


Figure 4 Residuals in X, Y, Z directions with 3 parameter estimation schemes at BJFS station

Table 2 Mean and RMS of experimental results compared with true values under 3 parameter estimation schemes (unit: m)

name	Plan	Mean(X)	RMS(X)	Mean(Y)	RMS(Y)	Mean(Z)	RMS(Z)
WUHN	1	0.111	0.111	0.109	0.109	0.092	0.092
	2	0.098	0.099	0.079	0.080	0.105	0.105
	3	-0.027	0.027	0.022	0.023	0.109	0.109
BJFS	1	0.017	0.018	-0.104	0.105	-0.099	0.100
	2	0.049	0.050	-0.084	0.084	-0.097	0.097
	3	0.011	0.013	-0.082	0.082	-0.082	0.083

2) By using three parameters, the improvements are 7 cm and 6 cm in X and Y directions, respectively, with no change in Z direction, at WUHN. The improvements are 1 – 2 cm in all three directions at BJFS;

3) With the same mapping function, the positioning results are comparable when using one parameter and two parameters. This result shows that using a mapping function which accurately portrays the physical characteristics of the ionosphere is basic to accuracy improvement in ionosphere-parameter estimation.

4 Conclusions and suggestions

1) The complex physical characteristics of the ionosphere have limited the accuracy of model correction. In traditional model corrections, the grid model has the highest precision, and the single-layer and Klobuchar models have comparable precisions. In the single-frequency PPP experiment, the improvements are 64% and 61% in X and Y directions, respectively, with no improvement in Z direction, at WUHN. The improvements are 79% and 14% in X and Z directions, respectively, but with lower accuracy in Y direction, at BJFS.

2) When parameter estimation is used for ionosphere delay, the accuracy of single-frequency PPP is better than model correction by 92% and 91%, respectively, in X and Y directions with no difference in Z direction, in our example at WUHN. At BJFS, the accuracy is 93% and 50% better in X and Z directions, respectively; while in the Y direction, the accuracy of parameter estimation is the same as Klobuchar-model correction, but much better than single-layer and grid models. Also, the parameter-estimation method can make real-time estimate for ionosphere delay.

3) When using parameter estimation to correct ionosphere delay, the accuracy is also influenced by choice of different parameters. Our results show that it is best to use three parameters for estimation, and that the ac-

curacies are comparable with one parameter and two parameters. The basic reason for this result is that when choosing different parameters the ionosphere-mapping functions are different. Thus, using a precise mapping function is basic to correctly estimating ionosphere delays.

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