Nonlinear radiative heat transfer in magnetohydrodynamic (MHD) stagnation point flow of nanofluid past a stretching sheet with convective boundary condition

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Received 21 October 2014; accepted 17 July 2015
Available online 28 November 2015

KEYWORDS
Nonlinear radiative effect; Stretching sheet; Stagnation point flow; Convective boundary condition; Passive control of nanoparticles

Abstract Two-dimensional boundary layer flow of nanofluid fluid past a stretching sheet is examined. The paper reveals the effect of non-linear radiative heat transfer on magnetohydrodynamic (MHD) stagnation point flow past a stretching sheet with convective heating. Condition of zero normal flux of nanoparticles at the wall for the stretched flow is considered. The nanoparticle fractions on the boundary are considered to be passively controlled. The solution for the velocity, temperature and nanoparticle concentration depends on parameters viz. Prandtl number $Pr$, velocity ratio parameter $A$, magnetic parameter $M$, Lewis number $Le$, Brownian motion $Nb$, and the thermophoresis parameter $Nt$. Moreover, the problem is governed by temperature ratio parameter $Nr = \frac{T_1}{T_f}$ and radiation parameter $Rd$. Similarity transformation is used to reduce the governing non-linear boundary-value problems into coupled higher order non-linear ordinary differential equation. These equations were numerically solved using the function bvp4c from the matlab software for different values of governing parameters. Numerical results are obtained for velocity, temperature and concentration, as well as the skin friction coefficient and local Nusselt number. The results indicate that the skin friction coefficient $C_f$ increases as the values of magnetic parameter $M$ increase and decreases as the values of velocity ratio parameter $A$ increase. The local Nusselt number $-\theta'(0)$ decreases as the values of thermophoresis parameter $Nt$ and radiation parameter $Rd$ increase.

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Peer review under responsibility of National Laboratory for Aeronautics and Astronautics, China.
1. Introduction

For many years the study of stagnation point flow of a viscous incompressible fluid past a stretching sheet has got considerable interest due to its vast application in manufacturing industries. Cooling of electronic devices by fan, cooling of nuclear reactors during emergency shutdown, and solar receiver etc. is important applications. Accordingly, Mahapatra and Gupta [1], Ishak et al. [2] and Hayat et al. [3] have examined heat transfer in stagnation point past stretching sheet. The main objective of the synthesis of nanofluid is for proliferation of the thermal conductivity of a convectional heat transfer fluids such as, water, ethylene glycol and engine oil in high technological areas. This synthesized fluid has vast applications in areas such as in nuclear energy, medicine, space exploration, etc. For the past three decades nanofluid has been tremendously studied to mitigate the problem of heat transfer in high technological industries. Many scholars have contributed a lot for the advancement of study of nanofluid in capacitating common fluids by developing models and techniques for the production of nanofluids and producing important and foremost papers in the area. For instance, Choi [4] was a pioneer in injecting the idea of nanofluid for boosting heat transfer capability of convectional fluids. Following him, Buongiorno [5] developed a model which incorporates the effect of thermophoresis and Brownian motion in the convective transport of nanofluids in laminar boundary layer flow analysis.

Furthermore, Kuznetsov and Nield [6] extended the investigation and analyzed the natural convective transport of nanofluid past a vertical surface where the nanoparticle is actively controlled at the boundary. Using the concept of controlled nanoparticle at the surface, Khan and Pop [7] examined the laminar boundary layer flow, heat transfer and nanoparticle fracture over a stretching surface in a nanofluid. The published research papers dealing with laminar boundary layer flow past a surface in nanofluid are numerous. Some of them are available in the references ([8–15]).

The study of stagnation point flow of nanofluid past a stretching sheet was examined by Mustafa et al. [16] and Wubshet et al. [17]. The investigation indicated that when the free stream velocity exceeds the stretching velocity, the velocity boundary layer thickness is increasing. Moreover, the study shows that the skin friction coefficient decreases as the velocity of free stream velocity greater than stretching velocity. By varying the thermal boundary conditions, Wubshet and Shanker [18] and Wubshet and Makinde [19] have examined the effects of convective boundary condition and double stratification on heat transfer of nanofluid past a vertical surface. Moreover, Wubshet and Shanker [20,21] analyzed the magnetohydrodynamic boundary layer flow and heat transfer of a nanofluid over non-isothermal stretching sheet with slip effect.

Their study showed that the surface temperature increases with an increase in Lewis number for prescribed heat flux case. So far the above literature described the examination of nanofluid past a vertical plate or stretching surface when the nanoparticle at the surface is actively controlled. However, very recently, Kuznetsov and Nield [22] revisited their previous study model of natural convective boundary layer flow a nanofluid past a vertical plate and they assumed that the nanoparticle fraction at the boundary is passively controlled rather than actively and the nanoparticle flux at the wall is zero. In such condition the previous model is more physically realistic one. Furthermore, Khan et al. [23] applied the passive controlled model and examined triple diffusive free convection along a horizontal plate.

All the above studies discussed the boundary layer flow towards stretching sheet when nanoparticles flux at a surface is non-zero. None of them discussed the boundary condition on MHD boundary layer flow and heat transfer of nanofluid over a stretching sheet with zero nanoparticle flux at the wall. Therefore, the aim of this study is to fill this felt out knowledge gap. The nanofluid flow and heat transfer analysis have much practical applications in nuclear reactors, food technology, transportation and in electronics as well as in biomedicine fields. Specifically, this study has a good application area in MHD flow and heat transfer process which occurs in many industrial cooling applications, such as the cooling of nuclear reactor, the geothermal system, aerodynamic process, next-generation solar film collector and heat exchange technology design, etc.

2. Mathematical formulation

In this analysis the stagnation point flow of a steady two-dimensional viscous flow of a nanofluid past a stretching sheet with convective boundary condition and thermal radiation has been considered. At its lower surface, the sheet heated convectively with temperature $T_f$ and a heat transfer coefficient $h_f$. The uniform ambient temperature and concentration respectively, are $T_{\infty}$ and $C_{\infty}$. It is assumed that there is no nanoparticle flux at the surface and the effect of thermophoresis is taken into account in the boundary condition. The velocity of the stretching sheet at the surface is $u_w(x) = ax$. Where $a$ is a
The flow is subjected to a constant transverse magnetic field of strength $B = B_0$ which is assumed to be applied in the positive $y$-direction, normal to the surface. The induced magnetic field is assumed to be small compared to the applied magnetic field and is neglected. It is further assumed that the base fluid and the suspended nanoparticles are in thermal equilibrium. It has been chosen that the coordinate system $x$-axis is along the stretching sheet and $y$-axis is normal to the sheet.

Under the above assumptions and boundary layer approximations, the governing equation of the conservation of mass, momentum, energy and nanoparticles fraction in the presence of magnetic field past a stretching sheet as given by Wubshet et al. [17]:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$

$$
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \nu \left( \frac{\partial^2 u}{\partial y^2} \right) + U_\infty \frac{\partial U_\infty}{\partial x} + \frac{\sigma B^2}{\rho_f} (U_\infty - u) \tag{2}
$$

$$
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial y^2} \right) + \Gamma \left[ \frac{\partial C}{\partial y} + \frac{D_f}{\rho_f} \left( \frac{\partial^2 C}{\partial y^2} \right) \right] + \frac{1}{\rho_f} \frac{\partial q_r}{\partial y} \tag{3}
$$

$$
u \frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_B \left( \frac{\partial^2 C}{\partial y^2} \right) + \frac{D_f}{\rho_f} \left( \frac{\partial^2 C}{\partial y^2} \right) \tag{4}
$$

The boundary conditions are

$$
u = u_\infty = ax, \quad v = 0, \quad -k \frac{\partial T}{\partial y} = h_f (T_f - T), \quad \frac{D_B}{T_\infty} \frac{\partial C}{\partial y} + \frac{D_f}{T_\infty} \left[ \frac{\partial T}{\partial y} \right] = 0 \quad \text{at} \quad y = 0
$$

$$u \to U_\infty = bx, \quad v = 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty \tag{5}
$$

$x$ and $y$ represent coordinate axes along the continuous surface in the direction of motion and normal to it, respectively. The velocity components along $x$ and $y$-axis are $u$ and $v$ respectively, $\nu$ is the kinematics viscosity, $T$ is the temperature inside the boundary layer, $\Gamma$ parameter defined by $\Gamma = \frac{(\rho_c)p(\rho_c)_f}{(\rho_c)_c}$ effective heat capacity of a nanoparticle, $(\rho_c)_f$ heat capacity of the base fluid, $\rho$ is the density, $T_\infty$ is ambient the temperature far away from the sheet.

By introducing the following similarity transforms and dimensionless quantities as:

$$\eta = \sqrt{\frac{u}{\nu}}, \quad \psi = \sqrt{aw_\infty}(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_\infty} \tag{6}
$$

where $\psi(x,y)$ represent the stream function and is defined as,

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \tag{7}$$
so that Eq. (1) is satisfied automatically.

Using Rosseland approximation of radiation

$$q_r = -\frac{4\sigma^* \alpha T^4}{3k^*} \frac{\partial T}{\partial y} = -\frac{16\sigma^* T^4}{3k^*} \frac{\partial T}{\partial y}$$

(8)

where $k^*$ is the mean absorption coefficient and $\sigma^*$ is the Stefan Boltzmann constant. Eq. (8) results in a highly nonlinear energy equation in $T$. The Rosseland approximation can be linearized about ambient temperature $T_\infty$. This means simply replace $T^3$ in Eq. (8) by $T_\infty^3$.

Then Eq. (3) becomes:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \alpha + \frac{16\sigma^* T^3}{3(\rho c)k^*} \right) \frac{\partial T}{\partial y} \right]$$

$$+ \frac{T_\infty}{T} \left\{ D_{r B} \left( \frac{\partial C}{\partial y} \right) + D_{r T} \left( \frac{\partial T}{\partial y} \right)^2 \right\}$$

(9)

Define the non-dimensional temperature $\theta(\eta) = \frac{T - T_\infty}{T_\infty - T_\infty}$ with $T = T + (1 + (Nr - 1) \theta) \alpha$ and $Nr = \frac{T}{T_\infty}$, where $Nr$ is the temperature ratio parameter. The first term on the right hand side of Eq. (9) can be written as

$$\alpha \left( \frac{\partial^2}{\partial y^2} \right) \left( 1 + Rd(1 + (Nr - 1) \theta)^3 \right)$$

where $Rd = \frac{16\sigma^* T^3}{3k^*}$ denotes the radiation parameter, and $Rd = 0$ indicates no thermal radiation effect.

The governing Eqs. (2)–(4) and Eq. (9) are reduced by using Eq. (6) and Eq. (8) as follows:

$$f'' + f'f - f^2 + A^2 + M(A - f') = 0 \quad (10)$$

$$[1 + Rd(1 + (Nr - 1) \theta)^3] \theta' + Pr(f' \theta' + Nb f \theta' + Nt \theta'^2) = 0 \quad (11)$$

$$\phi' + Le Pr f \phi' + \frac{Nt}{Nb} \theta'' = 0 \quad (12)$$

With boundary conditions

$$f(0) = 0, \quad f'(0) = 1, \quad \theta'(0) = Bi(\theta(0) - 1),$$

$$Nb f'(0) + Nt \theta'(0) = 0, \quad \text{at} \ \eta = 0,$$

$$f'(\infty) \to 0, \quad \theta(\infty) \to 0, \quad (\phi(\infty) \to 0, \quad \text{as} \ \eta \to \infty) \quad (13)$$

where the governing parameters are defined by:

$$Bi = \frac{h_f}{k} \sqrt{\frac{\nu}{\alpha}}, \quad Pr = \frac{\nu}{\alpha}, \quad A = \frac{a}{D_b}, \quad Le = \frac{\alpha}{D_b}, \quad M = \frac{\sigma B_0^2}{\rho_f a}$$

$$Nb = \frac{(\rho c)_f D_b C_\infty}{(\rho c)_f \nu}, \quad Rd = \frac{16\sigma^* T^3}{3k^*,}$$

$$Nt = \frac{(\rho c)_f D_T(T_f - T_\infty)}{(\rho c)_f \nu T_\infty}, \quad Nr = \frac{T_f}{T_\infty} \quad (14)$$

### Table 1: Comparison of skin friction coefficient $-f''(0)$ for different values of velocity ratio parameter $A$ when $M = Nr = 0$.

<table>
<thead>
<tr>
<th>$A$</th>
<th>Wubshet et al. [17]</th>
<th>Ishak et al. [2]</th>
<th>Present result</th>
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<tr>
<td>0.1</td>
<td>0.9694</td>
<td>0.9694</td>
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<td>0.2</td>
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<td>0.3</td>
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<tr>
<td>0.4</td>
<td>–</td>
<td>–</td>
<td>0.7653</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6673</td>
<td>0.6673</td>
<td>0.6673</td>
</tr>
<tr>
<td>0.8</td>
<td>–</td>
<td>–</td>
<td>0.2994</td>
</tr>
<tr>
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<td>–</td>
<td>–</td>
<td>0.0000</td>
</tr>
<tr>
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<td>2.0175</td>
<td>2.0175</td>
<td>2.0175</td>
</tr>
<tr>
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<td>4.7293</td>
<td>4.7293</td>
<td>4.7293</td>
</tr>
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<td>–</td>
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</tr>
<tr>
<td>10.0</td>
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<td>–</td>
<td>36.2574</td>
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</tbody>
</table>

### Table 2: Comparison of local Nusselt number $-\theta'(0)$ at $Nr = 0$, $Nb -> 0$, $Rd = Nr = 0$ for different values of $Pr$ with previously published data.

<table>
<thead>
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</tr>
<tr>
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<td>1.0116</td>
<td>–</td>
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<td>–</td>
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### Table 3: Computed values of local Nusselt number $-\theta'(0)$ when $Nb = A = 0.5$, $Rd = Pr = M = 1$, for different values of $Le$ $Nr$, $Nt$ and $Bi$.

<table>
<thead>
<tr>
<th>$Le$</th>
<th>$Nr$</th>
<th>$Nt$</th>
<th>$Bi$</th>
<th>$-\theta'(0)$</th>
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<tr>
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</tr>
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<td>2.0</td>
<td>0.1574</td>
</tr>
<tr>
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<td>3.0</td>
<td>0.5</td>
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</tr>
<tr>
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<td>0.5</td>
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<td>0.3331</td>
</tr>
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</tr>
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</tr>
</tbody>
</table>
\( f' \), \( \theta \) and \( \phi \) are the dimensionless velocity, temperature and particle concentration respectively. \( \eta \) is the similarity variable, the prime denotes differentiation with respect to \( \eta \). \( Pr, M, Nb, Nt, Le \) denote Prandtl number, a magnetic parameter, a Brownian motion parameter, a thermophoresis parameter, and a Lewis number, respectively.

The important physical quantities of interest in this problem are the skin friction coefficient \( C_f \) and local Nusselt number \( N_u \) are defined as:

\[
C_f = \frac{\tau_w}{\rho u_w^2}, \quad N_u = \frac{\theta_q}{k(T_f - T_\infty)}
\]

where the wall shear stress \( \tau_w \) and wall heat flux \( q_w \) are given by

\[
\tau_w = \mu \left( \frac{\partial u}{\partial y} \right)_{y=0},
\]

\[
q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} + (q_f)_w
\]

\[
= -k(T_f - T_\infty) \left[ \frac{\partial}{\partial y} \left( 1 + Rd\theta_\infty^3 \right) \right] \theta'(0),
\]

By using the above equations, we get

\[
C_f \sqrt{Re_s} = -f''(0),
\]

\[
\frac{N_u}{\sqrt{Re_s}} = -(1 + Rd\theta_\infty^3 N_r) \theta'(0)
\]

where \( Re_s = \frac{u^2}{v} \) and \( N_u \) are local Reynolds number and local Nusselt number, respectively.

3. Numerical solution and accuracy

The dimensionless ordinary differential equations Eqs. (10)–(12) subjected to the boundary conditions Eq. (13) have been solved by the function bvp4c from matlab for different values of governing parameters. The step size and the convergence criteria were taken as \( \Delta \eta = 0.01 \) and \( 10^{-8} \), respectively. The asymptotic boundary conditions were given by Eq. (13) were replaced by a value of \( \eta_{\text{max}} = 10 \).

![Figure 1](image1.png) Velocity profile for different values of velocity ratio \( A \) when \( Nb = Nt = 0.5, Pr = 1, Le = Bi = 5, M = 1 \).

![Figure 2](image2.png) Velocity profile for different values of magnetic parameter \( M \) when \( A = Nb = Nt = 0.5, Pr = 1, Le = Bi = 5 \).

![Figure 3](image3.png) Temperature profile for different values of temperature ratio parameter \( N_r \) when \( A = Nt = Nb = 0.5, Bi = Le = 5, Rd = M = 1 \).

![Figure 4](image4.png) Temperature profile for different values of radiation parameter \( Rd \) when \( A = Nt = Nb = 0.5, Bi = Le = 5, M = 1, N_r = 1.1 \).
The choice of \( \eta_{\text{max}} = 10 \) ensure that all numerical solutions approached the asymptotic value correctly.

Furthermore, in order to evaluate the precision of the method used, comparison with previously reported data available in the literature has been made. From Table 1 it can be seen that the numerical values of the skin friction coefficient \( -f''(0) \) in this paper for different values of velocity ratio \( A \) when \( M = 0 \) are in an excellent agreement with the results published in Ishak et al. [2] and Wubshet et al. [17]. To further validate the numerical method used in the paper, comparison of local Nusselt number \( -\theta'(0) \) for different values of Prandtl number \( Pr \) by ignoring the effects of \( Nt \) and \( Nb \) parameters has been shown in Table 2, which is also in excellent agreement with Refs. [1,3] and [18]. Comparison of the results of this study with literature values have shown in Tables 1 and 2 indicate an excellent agreement and this gives us a confidence to use the present code.

Furthermore, Table 3 shows the computed values of the local Nusselt number \( -\theta'(0) \) for different values of the governing parameters such as \( Nr, Le, Nt \) and \( Bi \). It is found
that the local Nusselt number \( \theta_0(0) \) is a decreasing function of the parameter \( Le, Nr \) and \( Nt \) and an increasing function of Biot number \( Bi \).

4. Result and discussion

The set of ordinary differential equations which are obtained from momentum, energy and concentration Eqs. (10)–(12) subjected to the boundary conditions Eq. (13) were numerically solved using bvp4c from a matlab. Velocity, temperature and concentration graphs for different values of governing parameters have been obtained. The results are displayed through figures and tables.

Figures 1 and 2 depict the graphs of the non-dimensional velocity profile \( f'(\eta) \) for different values of velocity ratio parameter \( A \) and magnetic parameter \( M \). Figure 1 indicates that the hydrodynamic boundary layer thickness increases with increasing values of \( A (A > 1) \) and it decreases with decreasing values of \( A (A < 1) \). Physically, when the free stream velocity greater than stretching velocity, the ratio of free stream velocity to stretching velocity is greater than 1, as a result, a retarding force is diminished and the flow velocity increased. Figure 2 reveals the effect of Hartmann number called magnetic parameter \( M \) on velocity profile. The existence of magnetic field sets in a resistive force called Lorentz force, which is a retarding force on the velocity field, as a result a flow velocity is reduced.

The effects of temperature ratio parameter \( Nr \), radiation parameter \( Rd \), thermophoresis parameter \( Nt \), convective parameter called Biot number \( Bi \) and velocity ratio parameter \( A \) on temperature profiles are given in Figures 3–6. Figure 3 shows the influences of temperature ratio parameter \( Nr \) on temperature profile. It is observed that the fluid temperature increases as \( Nr \) increases due to the fact that the conduction effect of the nanofluid increases with increasing temperature ratio parameter. The inclusion of temperature ratio parameter induces higher surface heat flux as a result the temperature increases with in the boundary layer region.

Figure 4 examines the behavior of radiation parameter \( Rd \) on temperature graph. Increasing thermal radiation effect
permits the thermal effect to penetrate deeper into the quiescent fluid and thus both temperature and thermal boundary layer thickness increase with an increase in \(R_d\). Figure 5 depicts the influence of thermophoresis parameter \(N_t\) on temperature profile. As the thermophoretic effect increases, the migration of nanoparticles from the hot surface to cold ambient fluid is occurred, as results the temperature increases in the boundary layer, this results in the growth of thermal boundary layer thickness. Figure 6 shows the impact of convective heating called Biot number \(Bi\) on temperature profile. Physically Biot number is the ratio of convection at the surface to conduction within the surface of a body. As the Biot number effect (convection at the surface) increases, temperature at the surface increases this result in an increase in the thermal boundary layer thickness. The influence of velocity ratio parameter \(A\) on temperature profile is given by Figure 7. As the values of velocity ratio \(A\) increase, temperature at the surface a sheet decreases; moreover, thermal boundary layer thickness diminishes.

The effects of governing parameters such as Brownian motion parameter \(Nb\), thermophoresis parameter \(N_t\), Prandtl number \(Pr\), Lewis number \(Le\), Biot number \(Bi\) and velocity ratio parameter \(A\) on concentration profiles are displayed in
Figures 8–13. Figure 8 illustrates the influences of Brownian motion parameter Nb on concentration profile \( \phi(\eta) \). As Brownian motion effect increase, the concentration gradient increases as a result the Brownian force increases which boost the nanoparticles concentration at the surface. Hence the concentration profile \( \phi(\eta) \) increases at the surface. Figure 9 shows the impact of thermophoresis on concentration profile \( \phi(\eta) \). Since the impact of Brownian force is to counter balance the influences of thermophoretic force, as the influences of thermophoretic force increases the concentration gradient at the surface decreases, as result the concentration profile at a surface decreases, which is opposite to the case for Brownian motion effect.

The variation of Prandtl number on concentration profile is shown in Figure 10. It can be seen from a graph that as the influence of \( Pr \) increases, the nanoparticles diffuses out towards outside, as a result the nanoparticles concentration at the surface decreases.

The influences of Lewis number \( Le \), Biot number \( Bi \) and velocity ratio \( A \) on concentration profile in this study are similar as shown in the Figures 11–13. The Lewis number effect on nanoparticle on concentration graph is described in Figure 11. Increasing the Lewis number corresponds to a poor Brownian diffusion coefficient which leads to short penetration depth for concentration profile \( \phi(\eta) \). As a result the concentration at the surface decreases with increasing the influences of Lewis number. Similar effects are observed when the values of Biot number \( Bi \), velocity ratio parameter \( A \) (Figure 12) and radiation parameter \( Nr \) (Figure 13) increase.

As the values of \( Nb \) and \( Le \) increase, the dimensionless concentration at a surface increases, but opposite effect is observed as the values of \( Nr \), \( Pr \) and \( Bi \) increases. The graph also reveals that the concentration boundary layer thickness increases as the values of \( Nb \) and \( Le \), but it decreases as the values of \( Nr \), \( Pr \) and \( Bi \) increases. For all the governing parameters considered here, the nanofluid parameters overshooting the dimensionless concentration at the surface and asymptotic to zero far away from the surface.

Figures 14 and 15 show the variation of the coefficient of skin friction with respect to magnetic field parameter \( M \) for different values of velocity ratio \( A \) parameter and vice versa. In both cases it is observed that the magnitude of skin friction coefficient increases as the values of magnetic parameters \( M \) increase and decreases as the values of \( A \) increase.

Figures 16–18 show the variation of local Nusselt number \(-\theta^{(0)}(0)\) with respect to different parameters such as thermophoresis parameter \( Nr \), velocity ratio parameter \( A \), convective parameter \( Bi \) and temperature ratio parameter \( Nr \). The graphs show that as the values of \( Nr \) and \( Nr \) increase, the local Nusselt number \(-\theta^{(0)}(0)\) at the surface decreases, however, it increase as the values of \( A \), \( Pr \) and \( Bi \) increase.

5. Conclusion

In this paper, the effects of nonlinear radiative heat transfer, magnetic field and convective heating on boundary layer flow and heat transfer of nanofluid over a stretching sheet is discussed. The boundary layer equations governing the flow problem are reduced into a coupled high order non-linear ordinary differential equations using the similarity transformation. The differential equations are solved numerically using bvp4c from matlab software. The effects of various physical parameters such as thermal radiation parameter \( Rd \), temperature ratio parameter \( Nr \), magnetic parameter \( M \), Prandtl number \( Pr \), Brownian motion parameter \( Nb \), thermophoresis parameter \( Nr \) and Lewis number \( Le \) on momentum, energy and concentration equation are discussed using figures and tables. Significant effects of governing parameters are plotted and justify our results through tables and graphs. The outcome of this study indicated that the flow velocity and the skin friction coefficient on stretching sheet are strongly influenced by velocity ratio \( A \) and magnetic parameter \( M \). It is observed that the magnetic parameter \( M \) boosts the growth of skin friction coefficient - \( f^{(0)}(0) \), however, velocity ratio parameter \( A \) inhibited the growth of skin friction coefficient. The study also shows that the Biot number \( Bi \), Prandtl number \( Pr \) and velocity ratio \( A \) parameters favor the local Nusselt number \(-\theta^{(0)}(0)\) but thermophoresis parameter \( Nr \) and temperature ratio parameter \( Nr \) act in opposite way. It is also found that the influences of Brownian motion on temperature and nanoparticle volume fraction is minimal.

The following are brief summary conclusions drawn from the analysis:

1. \( A \) and \( M \) have opposite influences on skin friction coefficient.
2. The influences of \( Nb \) and \( Nr \) on concentration profile is opposite.
3. The influence of \( A \) and \( Bi \) on the local Nusselt number is positive.
4. Prandtl number \( Pr \) and convective parameter \( Bi \) have a similar impact on local Nusselt number.
5. The thickness of thermal boundary layer are antagonistic with parameters \( Nr \) and \( Pr \).
6. Nanoparticle volume fraction near the stretching surface is negative and it is zero far away from the surface.
7. Radiation parameter \( Rd \) favors the thermal boundary layer thickness.
8. The temperature ratio parameter \( Nr \) has a diminishing effect on local Nusselt number -\( \theta^{(0)}(0)\).

Acknowledgements

The author wish to express his very sincere thanks to referees for their valuable comments and suggestions.

References

Nonlinear radiative heat transfer in magnetohydrodynamic (MHD) stagnation point


