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Separability of Dirac equation in higher dimensional Kerr–NUT–de Sitter spacetime

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Abstract

It is shown that the Dirac equations in general higher dimensional Kerr–NUT–de Sitter spacetimes are separated into ordinary differential equations.

Recently, the separability of Klein–Gordon equations in higher dimensional Kerr–NUT–de Sitter spacetimes [1] was shown by Frolov et al. [2]. This separation is deeply related to that of geodesic Hamilton–Jacobi equations. Indeed, a geometrical object called conformal Killing–Yano tensor plays an important role in the separability theory [2–9]. However, at present, a similar separation of the variables of Dirac equations is lacking, although the separability in the four-dimensional Kerr geometry was given by Chandrasekhar [10]. In this Letter we shall show that Dirac equations can also be separated in general Kerr–NUT–de Sitter spacetimes.

The D-dimensional Kerr–NUT–de Sitter metrics are written as follows [1]:

(a) $D = 2n$

$$g^{(2n)} = \sum_{\mu=1}^{n} \frac{dx_\mu^2}{Q_\mu(x)} + \sum_{\mu=1}^{n} Q_\mu(x) \left(\sum_{k=0}^{n-1} A_\mu^{(k)} \, d\psi_k \right)^2, \tag{1}$$

(b) $D = 2n + 1$

$$g^{(2n+1)} = \sum_{\mu=1}^{n} \frac{dx_\mu^2}{Q_\mu(x)} + \sum_{\mu=1}^{n} Q_\mu(x) \left(\sum_{k=0}^{n-1} A_\mu^{(k)} \, d\psi_k \right)^2 + \frac{c}{A^{(n)}} \left(\sum_{k=0}^{n} A^{(k)} \, d\psi_k \right)^2. \tag{2}$$

The functions Q_μ ($\mu = 1, 2, \dots, n$) are given by

$$Q_\mu(x) = \frac{X_\mu}{U_\mu}, \quad U_\mu = \prod_{\substack{\nu=1 \\ (\nu\neq\mu)}}^{n} \left(x_\mu^2 - x_\nu^2\right), \tag{3}$$

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where X_μ is a function depending only on the coordinate x_μ, and $A^{(k)}$ and $A_\mu^{(k)}$ are the elementary symmetric functions of $\{x_\nu^2\}$ and $\{x_\nu^2\}_{\nu\neq\mu}$ respectively:

$$\prod_{\nu=1}^{n}\left(t-x_\nu^2\right)=A^{(0)}t^n-A^{(1)}t^{n-1}+\cdots+(-1)^nA^{(n)}, \tag{4}$$

$$\prod_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\left(t-x_\nu^2\right)=A_\mu^{(0)}t^{n-1}-A_\mu^{(1)}t^{n-1}+\cdots+(-1)^{n-1}A_\mu^{(n-1)}. \tag{5}$$

The metrics are Einstein if X_μ takes the form [1,11]
(a) $D=2n$

$$X_\mu=\sum_{k=0}^{n}c_{2k}x_\mu^{2k}+b_\mu x_\mu, \tag{6}$$

(b) $D=2n+1$

$$X_\mu=\sum_{k=0}^{n}c_{2k}x_\mu^{2k}+b_\mu+\frac{(-1)^nc}{x_\mu^2}, \tag{7}$$

where c, c_{2k} and b_μ are free parameters.

1. $D=2n$

For the metric (1) we introduce the following orthonormal basis $\{e^a\}=\{e^\mu,e^{n+\mu}\}$ $(\mu=1,2,\dots,n)$:

$$e^\mu=\frac{dx_\mu}{\sqrt{Q_\mu}},\qquad e^{n+\mu}=\sqrt{Q_\mu}\sum_{k=0}^{n-1}A_\mu^{(k)}\,d\psi_k. \tag{8}$$

The dual vector fields are given by

$$e_\mu=\sqrt{Q_\mu}\frac{\partial}{\partial x_\mu},\qquad e_{n+\mu}=\sum_{k=0}^{n-1}\frac{(-1)^kx_\mu^{2(n-1-k)}}{\sqrt{Q_\mu}U_\mu}\frac{\partial}{\partial\psi_k}. \tag{9}$$

The spin connection is calculated as [11]

$$\begin{aligned}
\omega_{\mu\nu}&=-\frac{x_\nu\sqrt{Q_\nu}}{x_\mu^2-x_\nu^2}e^\mu-\frac{x_\mu\sqrt{Q_\mu}}{x_\mu^2-x_\nu^2}e^\nu\quad(\mu\neq\nu),\\
\omega_{\mu,n+\mu}&=-\left(\partial_\mu\sqrt{Q_\mu}\right)e^{n+\mu}-\sum_{\rho\neq\mu}\frac{x_\mu\sqrt{Q_\rho}}{x_\rho^2-x_\mu^2}e^{n+\rho}\quad(\text{no sum over }\mu),\\
\omega_{\mu,n+\nu}&=\frac{x_\mu\sqrt{Q_\nu}}{x_\mu^2-x_\nu^2}e^{n+\mu}-\frac{x_\mu\sqrt{Q_\mu}}{x_\mu^2-x_\nu^2}e^{n+\nu}\quad(\mu\neq\nu),\\
\omega_{n+\mu,n+\nu}&=-\frac{x_\mu\sqrt{Q_\nu}}{x_\mu^2-x_\nu^2}e^\mu-\frac{x_\nu\sqrt{Q_\mu}}{x_\mu^2-x_\nu^2}e^\nu\quad(\mu\neq\nu).
\end{aligned} \tag{10}$$

Then, the Dirac equation is written in the form

$$\left(\gamma^aD_a+m\right)\Psi=0, \tag{11}$$

where D_a is a covariant differentiation,

$$D_a=e_a+\frac{1}{4}\omega_{bc}(e_a)\gamma^b\gamma^c. \tag{12}$$

From (9), (10) and (12), we obtain the explicit expression for the Dirac equation

$$\sum_{\mu=1}^{n}\gamma^{\mu}\sqrt{Q_\mu}\left(\frac{\partial}{\partial x_\mu}+\frac{1}{2}\frac{X'_\mu}{X_\mu}+\frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{x_\mu}{x_\mu^2-x_\nu^2}\right)\hat{\Psi}$$
$$+\sum_{\mu=1}^{n}\gamma^{n+\mu}\sqrt{Q_\mu}\left(\sum_{k=0}^{n-1}\frac{(-1)^k x_\mu^{2(n-1-k)}}{X_\mu}\frac{\partial}{\partial\psi_k}+\frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{x_\nu}{x_\mu^2-x_\nu^2}\left(\gamma^\nu\gamma^{n+\nu}\right)\right)\hat{\Psi}+m\hat{\Psi}=0. \tag{13}$$

Let us use the following representation of γ-matrices: $\{\gamma^a,\gamma^b\}=2\delta^{ab}$,

$$\gamma^\mu=\underbrace{\sigma_3\otimes\sigma_3\otimes\cdots\otimes\sigma_3}_{\mu-1}\otimes\sigma_1\otimes I\otimes\cdots\otimes I,$$
$$\gamma^{n+\mu}=\underbrace{\sigma_3\otimes\sigma_3\otimes\cdots\otimes\sigma_3}_{\mu-1}\otimes\sigma_2\otimes I\otimes\cdots\otimes I, \tag{14}$$

where I is the 2×2 identity matrix and σ_i are the Pauli matrices. In this representation, we write the 2^n components of the spinor field as $\Psi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(\epsilon_\mu=\pm1)$, and it follows that

$$\left(\gamma^\mu\Psi\right)_{\epsilon_1\epsilon_2\cdots\epsilon_n}=\left(\prod_{\nu=1}^{\mu-1}\epsilon_\nu\right)\Psi_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n},$$
$$\left(\gamma^{n+\mu}\Psi\right)_{\epsilon_1\epsilon_2\cdots\epsilon_n}=-i\epsilon_\mu\left(\prod_{\nu=1}^{\mu-1}\epsilon_\nu\right)\Psi_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n}. \tag{15}$$

By the isometry the spinor field takes the form

$$\Psi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x,\psi)=\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)\exp\left(i\sum_{k=0}^{n-1}N_k\psi_k\right) \tag{16}$$

with arbitrary constants N_k. Substituting (15) into (13), we obtain

$$\sum_{\mu=1}^{n}\sqrt{Q_\mu}\left(\prod_{\rho=1}^{\mu-1}\epsilon_\rho\right)\left(\frac{\partial}{\partial x_\mu}+\frac{1}{2}\frac{X'_\mu}{X_\mu}+\frac{1}{2}\frac{\epsilon_\mu Y_\mu}{X_\mu}+\frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{1}{x_\mu-\epsilon_\mu\epsilon_\nu x_\nu}\right)\Psi_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n}+m\Psi_{\epsilon_1\epsilon_2\cdots\epsilon_n}=0, \tag{17}$$

where we have introduced the function

$$Y_\mu=\sum_{k=0}^{n-1}(-1)^k x_\mu^{2(n-1-k)}N_k, \tag{18}$$

which depends only on x_μ.

Consider now the region $x_\mu-x_\nu>0$ for $\mu<\nu$ and $x_\mu+x_\nu>0$. Let us define

$$\Phi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)=\prod_{1\leqslant\mu<\nu\leqslant n}\frac{1}{\sqrt{x_\mu+\epsilon_\mu\epsilon_\nu x_\nu}}. \tag{19}$$

Then, one can obtain an equality

$$\frac{\Phi_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n}(x)}{\Phi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)}=(-\epsilon_\mu)^{\mu-1}\left(\prod_{\rho=1}^{\mu-1}\epsilon_\rho\right)\frac{\sqrt{(-1)^{\mu-1}U_\mu}}{\prod_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}(x_\mu-\epsilon_\mu\epsilon_\nu x_\nu)}. \tag{20}$$

Now we show that the Dirac equation allows a separation of variables by setting

$$\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)=\Phi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)\prod_{\mu=1}^{n}\chi_{\epsilon_\mu}^{(\mu)}(x_\mu). \tag{21}$$

It should be noticed that

$$\frac{\partial}{\partial x_\mu}\log\hat{\Psi}_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n}=\frac{d}{dx_\mu}\log\chi_{-\epsilon_\mu}^{(\mu)}-\frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{1}{x_\mu-\epsilon_\mu\epsilon_\nu x_\nu}. \tag{22}$$

By using (20) and (22), the substitution of (21) into (17) leads to

$$\sum_{\mu=1}^{n} \frac{P^{(\mu)}_{\epsilon_\mu}(x_\mu)}{\prod_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}(\epsilon_\mu x_\mu - \epsilon_\nu x_\nu)} + m = 0, \tag{23}$$

where $P^{(\mu)}_{\epsilon_\mu}$ is a function of the coordinate x_μ only,

$$P^{(\mu)}_{\epsilon_\mu} = (-1)^{\mu-1}(\epsilon_\mu)^{n-\mu}\sqrt{(-1)^{\mu-1}X_\mu}\,\frac{1}{\chi^{(\mu)}_{\epsilon_\mu}}\left(\frac{d}{dx_\mu} + \frac{1}{2}\frac{X'_\mu}{X_\mu} + \frac{\epsilon_\mu Y_\mu}{X_\mu}\right)\chi^{(\mu)}_{-\epsilon_\mu}. \tag{24}$$

Putting

$$Q(y) = -my^{n-1} + \sum_{j=0}^{n-2} q_j y^j \tag{25}$$

with arbitrary constants q_j, we find

$$P^{(\mu)}_{\epsilon_\mu}(x_\mu) = Q(\epsilon_\mu x_\mu). \tag{26}$$

Thus, the functions $\chi^{(\mu)}_{\epsilon_\mu}$ satisfy the ordinary differential equations

$$\left(\frac{d}{dx_\mu} + \frac{1}{2}\frac{X'_\mu}{X_\mu} + \frac{\epsilon_\mu Y_\mu}{X_\mu}\right)\chi^{(\mu)}_{-\epsilon_\mu} - \frac{(-1)^{\mu-1}(\epsilon_\mu)^{n-\mu}Q(\epsilon_\mu x_\mu)}{\sqrt{(-1)^{\mu-1}X_\mu}}\chi^{(\mu)}_{\epsilon_\mu} = 0. \tag{27}$$

2. $D = 2n+1$

For the metric (2) we introduce the orthonormal basis $\{\hat{e}^a\} = \{\hat{e}^\mu, \hat{e}^{n+\mu}, \hat{e}^{2n+1}\}$ ($\mu = 1, 2, \ldots, n$):

$$\hat{e}^\mu = e^\mu, \qquad \hat{e}^{n+\mu} = e^{n+\mu}, \qquad \hat{e}^{2n+1} = \sqrt{S}\sum_{k=0}^{n} A^{(k)}\,d\psi_k \tag{28}$$

with $S = c/A^{(n)}$. The 1-forms e^μ and $e^{n+\mu}$ are defined by (8). The dual vector fields are given by

$$\hat{e}_\mu = e_\mu, \qquad \hat{e}_{n+\mu} = e_{n+\mu} + \frac{(-1)^n}{x_\mu^2\sqrt{Q_\mu}U_\mu}\frac{\partial}{\partial\psi_n}, \qquad \hat{e}_{2n+1} = \frac{1}{\sqrt{S}A^{(n)}}\frac{\partial}{\partial\psi_n} \tag{29}$$

with (9). The spin connection is calculated as [11]

$$\begin{aligned}
&\hat{\omega}_{\mu\nu} = \omega_{\mu\nu}, \qquad \hat{\omega}_{\mu,n+\nu} = \omega_{\mu,n+\nu} + \delta_{\mu\nu}\frac{\sqrt{S}}{x_\mu}\hat{e}^{2n+1}, \qquad \hat{\omega}_{n+\mu,n+\nu} = \omega_{n+\mu,n+\nu},\\
&\hat{\omega}_{\mu,2n+1} = \frac{\sqrt{S}}{x_\mu}\hat{e}^{n+\mu} - \frac{\sqrt{Q_\mu}}{x_\mu}\hat{e}^{2n+1}, \qquad \hat{\omega}_{n+\mu,2n+1} = -\frac{\sqrt{S}}{x_\mu}\hat{e}^\mu.
\end{aligned} \tag{30}$$

A similar calculation to the even-dimensional case yields the following Dirac equation,

$$\begin{aligned}
&\sum_{\mu=1}^{n}\gamma^\mu\sqrt{Q_\mu}\left(\frac{\partial}{\partial x_\mu} + \frac{1}{2}\frac{X'_\mu}{X_\mu} + \frac{1}{2x_\mu} + \frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{x_\mu}{x_\mu^2 - x_\nu^2}\right)\Psi\\
&\quad + \sum_{\mu=1}^{n}\gamma^{n+\mu}\sqrt{Q_\mu}\left(\sum_{k=0}^{n-1}\frac{(-1)^k x_\mu^{2(n-1-k)}}{X_\mu}\frac{\partial}{\partial\psi_k} + \frac{(-1)^n}{x_\mu^2 X_\mu}\frac{\partial}{\partial\psi_n} + \frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{x_\nu}{x_\mu^2 - x_\nu^2}\left(\gamma^\nu\gamma^{n+\nu}\right)\right)\Psi\\
&\quad + \gamma^{2n+1}\sqrt{S}\left(-\sum_{\mu=1}^{n}\frac{1}{2x_\mu}\left(\gamma^\mu\gamma^{n+\mu}\right) + \frac{1}{c}\frac{\partial}{\partial\psi_n}\right)\Psi + m\Psi = 0.
\end{aligned} \tag{31}$$

We use the representation of γ-matrices given by (14) together with

$$\gamma^{2n+1} = \sigma_3 \otimes \sigma_3 \otimes \cdots \otimes \sigma_3. \tag{32}$$

Thus, the spinor field $\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}$ defined by

$$\Psi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x,\psi)=\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)\exp\left(i\sum_{k=0}^{n}N_k\psi_k\right) \tag{33}$$

satisfies the equation

$$\sum_{\mu=1}^{n}\sqrt{Q_\mu}\left(\prod_{\rho=1}^{\mu-1}\epsilon_\rho\right)\left(\frac{\partial}{\partial x_\mu}+\frac{1}{2}\frac{X'_\mu}{X_\mu}+\frac{1}{2}\frac{\epsilon_\mu\hat{Y}_\mu}{X_\mu}+\frac{1}{2x_\mu}+\frac{1}{2}\sum_{\substack{\nu=1\\(\nu\neq\mu)}}^{n}\frac{1}{x_\mu-\epsilon_\mu\epsilon_\nu x_\nu}\right)\hat{\Psi}_{\epsilon_1\cdots\epsilon_{\mu-1}(-\epsilon_\mu)\epsilon_{\mu+1}\cdots\epsilon_n}$$
$$+\left(i\sqrt{S}\left(\prod_{\rho=1}^{n}\epsilon_\rho\right)\left(-\sum_{\mu=1}^{n}\frac{\epsilon_\mu}{2x_\mu}+\frac{N_n}{c}\right)+m\right)\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}=0, \tag{34}$$

where

$$\hat{Y}_\mu=\sum_{k=0}^{n}(-1)^k x_\mu^{2(n-1-k)}N_k. \tag{35}$$

We find that the Dirac equation above allows a separation of variables

$$\hat{\Psi}_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)=\Phi_{\epsilon_1\epsilon_2\cdots\epsilon_n}(x)\prod_{\mu=1}^{n}\left(\frac{\chi^{(\mu)}_{\epsilon_\mu}(x_\mu)}{\sqrt{x_\mu}}\right) \tag{36}$$

with $\Phi_{\epsilon_1\epsilon_2\cdots\epsilon_n}$ defined by (19). Indeed, (34) becomes

$$\sum_{\mu=1}^{n}\frac{P^{(\mu)}_{\epsilon_\mu}(x_\mu)}{\prod^{n}_{\substack{\nu=1\\(\nu\neq\mu)}}(\epsilon_\mu x_\mu-\epsilon_\nu x_\nu)}+\frac{i\sqrt{c}}{\prod_{\rho=1}^{n}(\epsilon_\rho x_\rho)}\left(-\sum_{\mu=1}^{n}\frac{\epsilon_\mu}{2x_\mu}+\frac{N_n}{c}\right)+m=0 \tag{37}$$

with the help of (24). Let us introduce the function

$$\hat{Q}(y)=\sum_{j=-2}^{n-1}q_j y^j \tag{38}$$

where

$$q_{n-1}=-m,\qquad q_{-1}=\frac{i}{2}(-1)^{n-1}\sqrt{c},\qquad q_{-2}=\frac{i}{\sqrt{c}}(-1)^n N_n. \tag{39}$$

Using the identities

$$\sum_{\mu=1}^{n}\frac{1}{y_\mu^2\prod_{\substack{\nu\\(\nu\neq\mu)}}(y_\mu-y_\nu)}=\frac{(-1)^{n-1}}{\prod_{\mu=1}^{n}y_\mu}\sum_{\nu=1}^{n}\frac{1}{y_\nu}, \tag{40}$$

$$\sum_{\mu=1}^{n}\frac{1}{y_\mu\prod_{\substack{\nu\\(\nu\neq\mu)}}(y_\mu-y_\nu)}=\frac{(-1)^{n-1}}{\prod_{\mu=1}^{n}y_\mu} \tag{41}$$

we can confirm that the functions $\chi^{(\mu)}_{\epsilon_\mu}$ satisfy the ordinary differential equations (27) by the replacements $Y_\mu\to\hat{Y}_\mu$ and $Q(\epsilon_\mu x_\mu)\to\hat{Q}(\epsilon_\mu x_\mu)$.

We have shown the separation of variables of Dirac equations in general Kerr–NUT–de Sitter spacetimes. An interesting problem is to investigate the origin of separability. In the case of geodesic Hamilton–Jacobi equations and Klein–Gordon equations we know that the existence of separable coordinates comes from that of a rank-2 closed conformal Killing–Yano tensor. We can also construct the first order differential operators from the closed conformal Killing–Yano tensor which commute with Dirac operators [12–14]. However, we have no clear answer of the separability of Dirac equations. As another problem we can study eigenvalues of Dirac operators on Sasaki–Einstein manifolds. Indeed, as shown in [1,15–17], the BPS limit of odd-dimensional Kerr–NUT–de Sitter metrics leads to Sasaki–Einstein metrics. Especially, the five-dimensional metrics are important from the point of view of AdS/CFT correspondence.

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