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Solving the flavour problem in supersymmetric Standard Models with three Higgs families

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ABSTRACT

We show how a non-Abelian family symmetry Δ_{27} can be used to solve the flavour problem of supersymmetric Standard Models containing three Higgs families such as the Exceptional Supersymmetric Standard Model (E_6 SSM). The three 27-dimensional families of the E_6 SSM, including the three families of Higgs fields, transform in a triplet representation of the Δ_{27} family symmetry, allowing the family symmetry to commute with a possible high energy E_6 symmetry. The Δ_{27} family symmetry here provides a high energy understanding of the Z_2^H symmetry of the E_6 SSM, which solves the flavour changing neutral current problem of the three families of Higgs fields. The main phenomenological predictions of the model are tri-bi-maximal mixing for leptons, two almost degenerate LSPs and two almost degenerate families of colour triplet D-fermions, providing a clear prediction for the LHC. In addition the model predicts PGBs with masses below the TeV scale, and possibly much lighter, which appears to be a quite general and robust prediction of all models based on the D-term vacuum alignment mechanism.

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1. Introduction

Although TeV scale Supersymmetry (SUSY) is well motivated, the Minimal Supersymmetric Standard Model (MSSM) suffers from the μ problem and the little fine-tuning problem [1]. The simple replacement of the μ -term of the MSSM by a singlet superfield coupling to Higgs superfields leads to a global axial $U(1)$ symmetry whose breaking would lead to an unwanted axion. In the Next-to-Minimal Supersymmetric Standard Model (NMSSM) a cubic singlet coupling is also assumed, which breaks the axial $U(1)$ symmetry down to a discrete Z_3 subgroup. In this way the NMSSM solves the μ problem and the little fine-tuning problem [2]. However when the Z_3 symmetry of the NMSSM is broken it leads to cosmological domain walls. One way to overcome such problems is not to add the cubic singlet term but to gauge the axial $U(1)$ symmetry so that the would-be axion gets eaten by a Higgs mechanism resulting in a massive observable Z' [3]. Of course such models come at a price since the gauged $U(1)$ symmetry must be made anomaly-free, and this generally involves adding additional fermions which are sometimes ignored in simple phenomenological applications such as in the USSM [4].

The E_6 SSM [5] is a non-minimal supersymmetric model based on an underlying high-energy E_6 symmetry where an additional

low-energy gauged $U(1)$ is identified amongst the E_6 generators, and to cancel anomalies, complete 27 dimensional E_6 families are assumed to survive down to the TeV scale. There are many possible choices of gauged $U(1)$ [6], but the E_6 SSM [5] is uniquely defined by a particular choice $U(1)_N$ under which the right-handed neutrinos are neutral, and so may become heavy, allowing naturally small physical neutrino masses. As in the NMSSM, the E_6 SSM superpotential does not have an explicit μ -term for the Higgs doublets but instead contains a trilinear term that represents the interaction between the Higgs doublets and an additional Standard Model singlet field S . When this singlet field obtains a vacuum expectation value (VEV) the trilinear term reduces to an effective μ -term, thus providing a solution to the μ problem of the MSSM. The E_6 SSM also contains an additional low-energy $U(1)$ gauge symmetry which, as in USSM models, can be considered to be a gauged Peccei–Quinn symmetry. This local $U(1)$ symmetry prevents the presence of an unwanted Goldstone boson once the singlet field obtains its VEV, which is eaten by the Higgs mechanism resulting in a massive Z' . For this $U(1)$ group to be anomaly free, the entire matter content of the (three families of) 27 multiplets of E_6 are assumed to survive to low-energies (apart from the right-handed neutrinos). Each 27 multiplet contains one generation of quarks and leptons (including a right-handed neutrino), up and down type Higgs doublet fields (as well as their coloured partners), and a Standard Model singlet. The E_6 SSM thus contains the particle spectrum of the MSSM plus two additional MSSM Higgs families, three families of coloured partners to the three Higgs families, and three singlet fields. To achieve gauge coupling unification at the

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GUT scale, the E_6 SSM also contains two additional electro-weak doublets from a $27'$ and $\overline{27}'$ incomplete multiplets of E_6 which are sterile in the E_6 SSM superpotential. In a minimal version of the E_6 SSM, with string scale unification, these additional states are not present [7].

The E_6 SSM [5], in common with the MSSM and NMSSM, is subject to the usual flavour problems to do with the unexplained spectrum of fermion (including neutrino) masses and mixings of the one hand, and the absence of flavour changing neutral currents (FCNCs) generically expected in SUSY models on the other hand. In fact, the E_6 SSM also faces additional FCNC challenges due to the three Higgs and singlet families. These challenges are resolved in the E_6 SSM by invoking a Z_2^H symmetry which only allows the third family of Higgs and singlet to couple to quarks and leptons, with the first two Higgs and singlet families being inert (having zero VEVs) and fermiophobic (not coupling to quarks and leptons). While this is perfectly acceptable from the phenomenological point of view, from a theoretical standpoint the Z_2^H symmetry looks rather *ad hoc* and adds an additional complication to the flavour problem of the E_6 SSM not present in the MSSM or NMSSM.

The discovery of neutrino mass and approximately tri-bi-maximal lepton mixing [8] suggests some kind of a non-Abelian discrete family symmetry might be at work, at least in the lepton sector, and, assuming a GUT-type of structure relating quarks and leptons at a certain high energy scale, within the quark sector too. The observed neutrino flavour symmetry may arise either directly or indirectly from a range of discrete symmetry groups [9]. Examples of the direct approach, in which one or more generators of the discrete family symmetry appears in the neutrino flavour group, are typically based on S_4 [10] or a related group such as A_4 [11,12] or $PSL(2, 7)$ [13]. Models of the indirect kind, in which the neutrino flavour symmetry arises accidentally, include also A_4 [14] as well as Δ_{27} [15] and the continuous flavour symmetries like, e.g., $SO(3)$ [16] or $SU(3)$ [17] which accommodate the discrete groups above as subgroups [18].

In this Letter we show how a Δ_{27} family symmetry can resolve all the flavour problems of supersymmetric Standard Models containing three Higgs families such as the E_6 SSM [5], including the fermion mass and mixing puzzle present in the SM, the FCNC problems introduced by the MSSM and the additional puzzle of the origin of the Z_2^H symmetry peculiar to models such as the E_6 SSM. The Δ_{27} family symmetry is chosen rather than, for example, A_4 or S_4 , since it allows complex representations whereas A_4 and S_4 only contain real representations. Complex representations are required in family symmetry models in which the left-handed matter fields F and right-handed matter fields F^c both transform in triplet representations. This is to avoid the trivial combination $FF^c h$ where h is the Higgs field. To be concrete we focus on the E_6 SSM where, under the discrete non-Abelian Δ_{27} family symmetry, we shall assume that the quarks, leptons and Higgs fields of the E_6 SSM all transform in triplet representations so that the family symmetry commutes with a possible high energy E_6 symmetry. It is known that such a family symmetry can account for various Yukawa couplings responsible for the masses of quarks and leptons, and serves to predict tri-bi-maximal mixing for leptons [15]. It is also known that such family symmetries when applied to supersymmetry can provide a solution to the SUSY flavour and CP problems [19]. The qualitatively new feature here is that such a family symmetry can solve the flavour changing neutral current problems introduced by extended Higgs sectors by controlling the Higgs couplings, similar to the Z_2^H symmetry of the E_6 SSM. As a consequence, we shall find predictions for the mass structure of the three families of Higgs and Higgsino fields and coloured D-fermions of the E_6 SSM. We remark that in a recent paper [20] we also considered a non-Abelian Δ_{27} family symmetry in E_6 SSM

models [5,7], however the Higgs fields (and D-fermions) were taken to be singlets of the family symmetry, rather than triplets, and the Z_2^H symmetry was simply assumed.

The outline of this Letter is as follows. In the next section we describe how the Δ_{27} family symmetry is applied to the E_6 SSM. This section is split up into subsections which look at how each term in the E_6 SSM superpotential is generated from higher-order operators once we apply the Δ_{27} symmetry. In Section 2.1 we introduce the E_6 SSM renormalizable superpotential, in the absence of any family symmetry. In Section 2.2 we introduce the vacuum alignment required for the various Δ_{27} flavon fields. Then in Section 2.3 we describe the non-renormalizable operators allowed by Δ_{27} and other symmetries that lead to the quark and lepton Yukawa interactions with the Higgs fields. We also describe the types of messenger fields that are integrated out to generate the higher-order operators, and illustrate how the Z_2^H symmetry of the E_6 SSM effectively emerges from the high-energy theory. In Section 2.4 we discuss how tri-bi-maximal mixing is generated in our model from the Δ_{27} family symmetry and sequential dominance and then, in Section 2.5, we describe how the effective μ -term of the MSSM is generated and discuss the mass structure of the LSPs that are formed from the inert Higgsinos and singlinos. In Sections 2.6 and 2.7 the mass structure of the D-fermions is explained and their decay channels are discussed. Finally, in Section 3, we summarize our findings.

2. Δ_{27} family symmetry in the E_6 SSM

2.1. The E_6 SSM superpotential without family symmetry

The quark and lepton Yukawa couplings in the E_6 SSM are derived from the E_6 tensor product $\lambda^{ijk} 27_i 27_j 27_k$ where i, j, k label the three generations. To better understand where the Yukawa couplings come from it is useful to re-write the superpotential in terms of the Pati–Salam subgroup of E_6 under which each 27 multiplet decomposes to the following representations: $27 \subset F + F^c + h + D + S$ where F contains the left-handed quark and lepton fields, F^c contains the charge conjugated quark and lepton fields, h contains up and down Higgs-like fields, D are the colour triplet partners of the Higgs fields, and S are Standard Model singlets though they are charged under the gauged $U(1)_N$ symmetry which is broken at low energies [5]. In this Pati–Salam notation the E_6 tensor product $27.27.27$ from which the E_6 superpotential is formed, decomposes in the following way, dropping all couplings and indices for clarity:

$$27.27.27 \rightarrow FF^c h + Shh + SDD + FFD + F^c F^c D. \quad (1)$$

The part of the E_6 SSM superpotential that contains the Yukawa interactions for the quarks and leptons is $\lambda^{ijk} F_i F_j^c h_k$ and is the subject of Section 2.3. In Section 2.5 we discuss the superpotential term $\lambda_{ijk} S^i h^j h^k$ from which the MSSM effective μ -term is generated. Section 2.6 describes the term $\lambda^{ijk} S_i D_j D_k$ from which the exotic D-fermion states get mass, and Section 2.7 looks at the operators $\lambda^{ijk} F_i F_j D_k + \lambda^{ijk} F_i^c F_j^c D_k$ which provide decay channels for the exotic D-fermions.

Although the above operators are written in a Pati–Salam notation we only do this for ease of notation. The model presented in this Letter is in fact based on the Standard Model gauge symmetry rather than a Pati–Salam symmetry. For the rest of this Letter we use a Pati–Salam notation unless stated otherwise.

2.2. Vacuum alignment

In this Letter we add a Δ_{27} family symmetry which is broken via the VEVs of the flavon triplets ϕ introduced in Table 1. A num-

Table 1

This table illustrates how all the flavon fields and Pati–Salam states of the three copies of a 27 E_6 multiplet transform under the Δ_{27} family symmetry and the additional constraining $U(1) \times Z_2 \times Z_2^h \times Z_2^S$ symmetry. An R-symmetry is also applied to the model which breaks to an R-parity once S_3 obtains a vacuum expectation value.

	Δ_{27}	$U(1)$	Z_2	Z_2^h	Z_2^S	$U(1)_R$
F	3	0	+	+	+	1
F^c	3	0	+	+	+	1
h	3	0	+	–	+	0
D	3	0	+	–	+	0
S	3	0	+	+	–	2
ϕ_3	$\bar{3}$	0	–	+	+	0
ϕ_{23}	$\bar{3}$	–1	–	+	+	0
ϕ_{123}	$\bar{3}$	1	–	+	+	0
$\bar{\phi}_1$	3	–4	–	+	+	0
$\bar{\phi}_3$	3	3	+	+	+	0
$\bar{\phi}_{123}$	3	–1	+	+	+	0
ϕ_3^h	$\bar{3}$	0	+	–	+	0
ϕ_3^S	$\bar{3}$	0	+	+	–	0
$\bar{\phi}_3^h$	3	0	+	+	+	0
$\bar{\phi}_3^S$	3	0	+	–	–	0
H_{45}	1	2	+	+	+	0

ber of different triplet flavon fields with different expectation value directions in Δ_{27} space are required to produce the desired mass structure for the E_6 SSM particles. Four different types of directions for the flavon VEVs are used in this Letter (dropping the VEV brackets):

$$\begin{aligned} \phi_{123} &\propto (1\ 1\ 1), & \phi_3 &\propto (0\ 0\ 1), \\ \phi_1 &\propto (1\ 0\ 0), & \phi_{23} &\propto (0\ 1\ -1). \end{aligned} \quad (2)$$

In the case of direct models, in which one or more generators of the discrete family symmetry appears in the neutrino flavour group, the usual mechanism of vacuum alignment of flavon fields is based on F-term alignment which exploits driving fields in the superpotential as discussed in [12]. This mechanism is also available to indirect models, in which the neutrino flavour symmetry arises accidentally, as discussed in [18]. However, in the case of indirect models, an additional and elegant possibility for vacuum alignment becomes available that is not possible for direct models, namely D-term vacuum alignment introduced in [14,15] as discussed below. One advantage of the D-term method is that the terms required to achieve vacuum alignment originate from the Kähler potential and so are not restricted by the symmetries which are introduced to control the terms in the holomorphic superpotential. Thus the required terms may always be present independent of the symmetries of the model. In the case of the present model we shall use the D-term vacuum alignment method to generate the above flavon VEVs, following largely the discussion in [15,14].

An elegant way to obtain the alignments of Eq. (2) is to start with a flavon scalar potential of the form [14,15]:

$$V = m^2 \sum_i \phi^{i\dagger} \phi^i + \lambda \left(\sum_i \phi^{i\dagger} \phi^i \right)^2 + \Delta V + \dots, \quad (3)$$

where the index i labels the components of a particular flavon triplet ϕ and:

$$\Delta V = \kappa \sum_i \phi^{i\dagger} \phi^i \phi^{i\dagger} \phi^i. \quad (4)$$

In a supersymmetric theory the quartic terms may arise from D-terms, after which this vacuum alignment mechanism is named [14,15], which take the form:

$$\frac{[\hat{\chi}^\dagger \hat{\chi} (\hat{\phi}^\dagger \hat{\phi} \hat{\phi}^\dagger \hat{\phi})]_D}{M_P^4} \rightarrow \frac{F_\chi^2}{M_P^4} (\phi^\dagger \phi \phi^\dagger \phi) \sim \frac{m_{3/2}^2}{M_P^2} (\phi^\dagger \phi \phi^\dagger \phi)$$

where the F-component of the Δ_{27} singlet χ acquires a SUSY breaking VEV F_χ , leading to a gravitino mass $m_{3/2}^2 \sim F_\chi^2/M_P^2$. Hence supersymmetry is broken and the scalar potential V gets a contribution of the type $(\lambda, \kappa) \phi^\dagger \phi \phi^\dagger \phi$ with small $(\lambda, \kappa) \sim m_{3/2}^2/M_P^2$. The quadratic mass term originates also from a soft supersymmetry breaking mass term, and this term is thus expected to have a soft mass squared $m^2 \sim m_{3/2}^2 \sim (\text{TeV})^2$.

We suppose that the flavon mass squared m^2 is positive at the (reduced) Planck scale M_P which prevents symmetry breaking at M_P (if $m^2 < 0$ at M_P then we would expect a VEV given by $\sqrt{-m^2/\lambda} \sim M_P$). Then we assume that the soft mass squared m^2 of a given flavon is driven negative by radiative corrections at some scale $\Lambda < M_P$, which triggers a VEV for that flavon set by the scale Λ , a mechanism which has been widely used in different contexts (see, for example, [21]). To see this explicitly here, we may approximate the potential, including radiatively corrected logarithmically running masses, to be of the form¹ $V \approx m^2 \phi^\dagger \phi \ln(\phi^\dagger \phi / \Lambda^2)$, dropping the small quartic terms. The first derivative of the potential with respect to ϕ^\dagger is $V' \approx m^2 \phi [\ln(\phi^\dagger \phi / \Lambda^2) + 1]$ which is zero for $\langle \phi^\dagger \phi \rangle \approx \Lambda^2/e$. As the different flavons have different superpotential couplings to heavy states, and since the soft masses run logarithmically with energy scale, the Λ scales defined above may differ greatly for the different flavons. Thus a hierarchy between the VEVs of various flavon fields is possible, and also stable, in the framework of the radiative breaking mechanism [21,15,14].

Only the term ΔV in Eqs. (3), (4) determines the alignment, where $\kappa \sim \kappa_0 m_{3/2}^2/M_P^2$, and the sign of $\kappa_0 \sim O(1)$ is undetermined. For $\kappa > 0$ we obtain the alignment along the direction of the VEV of ϕ_{123} , while $\kappa < 0$ can give rise to that of ϕ_3 . The configuration $\phi_{23} \propto (0, -1, 1)^T$ can then be generated using a leading higher order term that requires that the VEV is orthogonal to both $(1, 0, 0)^T$ and $(1, 1, 1)^T$. All these operators can be used to generate the VEV configurations of the flavons in Eq. (2). A more detailed discussion on this subject can be found in [15], where the vacuum alignment operators required for the flavon fields that are used in this Letter is provided except for the flavon fields ϕ_3^h , $\bar{\phi}_3^h$, ϕ_3^S and $\bar{\phi}_3^S$. We assume the same vacuum alignment operators used in [15] which align all the flavon fields except for ϕ_3^h , $\bar{\phi}_3^h$, ϕ_3^S and $\bar{\phi}_3^S$. For these additional flavon fields to get the required direction of vacuum expectation values, we use the following D-terms (omitting $m_{3/2}^2/M_P^2$ factors): $(\phi_3^h)^i \bar{\phi}_{3i}^h \bar{\phi}_{3i}^{\dagger h} (\phi_3^{\dagger h})^i$ and $\phi_3^i \bar{\phi}_{3i}^h \bar{\phi}_{3i}^{\dagger h} \phi_3^{\dagger i}$ both with negative coefficients, and similarly for the ϕ_3^S and $\bar{\phi}_3^S$ flavons. These terms cause $\bar{\phi}_3^h$ and ϕ_3^h to get VEVs in the same direction as the pre-aligned fields $\bar{\phi}_{3i}$ and ϕ_3^i respectively.

Note that the leading order potential, including ΔV , is invariant under a product of global $U(1)$ symmetries, one for each flavon component. However this symmetry is broken explicitly by superpotential terms including the Yukawa superpotential discussed in the following subsection. It is also broken by higher order terms in the scalar potential involving purely flavon superfields as indicated by the dots in Eq. (3). An example of such a term in the potential which is invariant under all the symmetries in Table 1 but which would violate the additional global $U(1)$ symmetries is $(m_{3/2}^2/M_P^2) (\phi_{123} \bar{\phi}_1 \bar{\phi}_3)^2 (\phi_{23} \phi_{123})^\dagger$. Inserting flavon VEVs of order Λ , such a term would lead to pseudo-Goldstone-boson (PGB) square masses of order $m_{\text{PGB}}^2 \sim m_{3/2}^2 \Lambda^6/M_P^6$, where generally we

¹ I am indebted to G. Ross (private communication) for this argument.

expect $\Lambda < M_P$ leading to PGB masses below the TeV scale (and perhaps much lighter) with possible interesting phenomenological and astrophysical implications beyond the scope of this Letter. Similarly, inserting the flavon VEVs of order Λ , one sees that this additional eighth order term gives a contribution to the potential suppressed relative to the previous quartic terms by a factor of Λ^4/M_P^4 so that, assuming $\Lambda < M_P$, it will not perturb the previous vacuum alignment arguments appreciably.

It may seem surprising that both the symmetry breaking and vacuum alignment are governed by quartic terms with very small coefficients ($\lambda, \kappa \sim m_{3/2}^2/M_P^2$). In particular one may worry that other quartic operators with larger coefficients are present in the theory and that they could destabilize the symmetry breaking and alignment scheme described above. However, given the field content and symmetries assumed for the model, in particular the gauged $U(1)_N$ and the $U(1)_R$ symmetries, it is not possible to write down any operators which would lead to terms in the flavon potential with competing or larger coefficients than those of the quartic terms above and which would destabilize the vacuum, and so the quartic terms considered above are the dominant ones. On the other hand, we have seen that such symmetries imply approximate global $U(1)$ symmetries which are only broken by higher order operators, leading to PGB masses below the TeV scale and possibly much lighter. The presence of such PGBs seems to be generic to the type of radiative symmetry breaking associated with D-term vacuum alignment, and this observation appears to be robust although it has not been remarked upon before.

2.3. The effective Yukawa operators

Under the Δ_{27} family symmetry the F , F^c and h transform as triplets so that the superpotential $\lambda^{ijk} F_i F_j^c h_k$ becomes $\epsilon^{ijk} F_i F_j^c h_k$ where i, j, k are now Δ_{27} indices. Table 1 describes how all the Pati–Salam states from a 27 representation and the flavons transform under the family symmetry and the additional symmetries that constrain the model such as Z_2^h symmetry which distinguishes the Higgs and D fields (but unlike Z_2^h treats all three Higgs families identically). The lowest order Yukawa superpotential consistent with the symmetries of Table 1 is:

$$\begin{aligned} W_{Yuk} \sim & \frac{1}{M^3} F_i F_j^c h_k \phi_3^i \phi_3^j (\phi_3^h)^k \\ & + \frac{1}{M^4} F_i F_j^c h_k \phi_{23}^i \phi_{23}^j (\phi_3^h)^k H_{45} \\ & + \frac{1}{M^3} F_i F_j^c h_k \phi_{123}^i \phi_{23}^j (\phi_3^h)^k + \frac{1}{M^3} F_i F_j^c h_k \phi_{123}^j \phi_{23}^i (\phi_3^h)^k \\ & + \frac{1}{M^6} F_i F_j^c h_k \phi_{123}^i \phi_3^j (\phi_{123}^m \bar{\phi}_{1m}) (\phi_3^h)^k H_{45} \\ & + \frac{1}{M^6} F_i F_j^c h_k \phi_{123}^j \phi_3^i (\phi_{123}^m \bar{\phi}_{1m}) (\phi_3^h)^k H_{45} \\ & + \frac{1}{M^7} F_i F_j^c h_k \phi_{123}^i \phi_{123}^j (\phi_3^l \bar{\phi}_{123l}) (\phi_3^m \bar{\phi}_{123m}) (\phi_3^h)^k \end{aligned} \quad (5)$$

where all order 1 coupling constants are suppressed. ϕ_{23} , ϕ_3^h , ϕ_{123} , ϕ_3 are all anti-triplets of the Δ_{27} family symmetry, $\bar{\phi}_1$ is a triplet of Δ_{27} , H_{45} is a singlet of Δ_{27} but a 45 of the $SU(5)$ subgroup of E_6 , and M is some messenger scale that is discussed further in Section 2.3.2. The ϕ_3^h field is taken to transform under the Z_2^h symmetry so that all of the above operators respect this symmetry. We assume that this flavon field and ϕ_3 get a VEV in the third component of Δ_{27} , ϕ_{23} gets an equal but opposite VEV in the second and third components of Δ_{27} , and ϕ_{123} gets an equal VEV in the first, second and third components of Δ_{27} . The vacuum alignment of these fields was discussed in Section 2.2. For ease of notation

we denote ϕ as a field that transforms as $\bar{3}$ under Δ_{27} and $\bar{\phi}$ as a field that transforms as a 3.

In addition to the Δ_{27} family symmetry, Table 1 also contains the vertical symmetries $U(1) \times Z_2 \times Z_2^h \times Z_2^S \times U(1)_R$. These symmetries prevent interactions that would otherwise be allowed by the Δ_{27} symmetry and would introduce certain phenomenological issues to the model. In particular, The Z_2^h symmetry is used to differentiate certain superfields including the Higgs from others, and is instrumental in preventing flavour changing neutral currents due to the extended Higgs sector of the model, as discussed further in the following section. The Z_2^S is introduced in Section 2.5 and is primarily used to lead to a non-renormalizable operator that reduces to an effective μ -term at the soft SUSY scale, while forbidding other operators. Note that Z_2^S also forbids the renormalizable operator $\epsilon_{ijk} S^i h_j^j h_k^k$ but this is not a crucial requirement and similar terms are generated at higher order. The $U(1) \times Z_2$ symmetries are invoked to allow only certain combinations of flavon fields, as in [15]. For example, the $U(1)$ symmetry prevents the effective Yukawa operator $\frac{1}{M^2} F_i F_j^c \phi_{123}^i \phi_3^j$ from appearing in Eq. (5). Finally, the $U(1)_R$ symmetry is an R-symmetry that is assumed to reduce to R-parity allowing the LSP of the model to be stable.

2.3.1. Preventing flavour changing neutral currents

As discussed in the Introduction, the E_6 SSM [5] is subject to the “usual” FCNC and CP problems generically expected in SUSY models, due to SUSY loop diagrams with off-diagonal soft mass insertions, as well as “additional” FCNC challenges arising from tree-level Higgs exchange due to the three Higgs and singlet families. The introduction of a family symmetry, together with the idea of spontaneous CP violation by the flavons, leads to a natural suppression of the “usual” FCNC and CP violating operators and provides a solution to the SUSY flavour and CP problems [19].

The impact of the higher order Kähler operators has been systematically explored for $SU(3)$ in [19] and the results there are directly applicable to the present model based on Δ_{27} . In the exact Δ_{27} family symmetry limit the soft masses are universal:

$$\hat{m}_Q^2 \propto \hat{m}_{u^c}^2 \propto \hat{m}_{d^c}^2 \propto \hat{m}_l^2 \propto \hat{m}_{e^c}^2 \propto \hat{m}_{\nu^c}^2 \propto \mathbf{1}. \quad (6)$$

However, in this limit, the Yukawa couplings and trilinear terms vanish. In reality the Δ_{27} family symmetry has to be broken, leading to violations of universality. For example, the soft masses allowed by the Δ_{27} family symmetry can be written as:

$$\hat{m}_{F, F^c}^2 = m_0^2 \left(b_0^{F, F^c} \mathbf{1} + \sum_A^{F, F^c} \frac{\langle \phi_A \phi_A^* \rangle}{M_{F, F^c}^m} + \dots \right)$$

where the generic subscript A runs over all the relevant flavon species. The above off-diagonal soft mass terms are suppressed by ϵ factors from the ratios of the flavon VEVs and the messenger masses. In [19] it was rigorously shown that off-diagonal terms from flavons of the type in Table 1 lead to suppressed FCNCs consistent with present experimental limits. The family symmetry thus naturally suppresses the “usual” type of induced FCNCs due to off-diagonal soft masses and can therefore provide a resolution to the generic SUSY FCNC problem.

In addition to the FCNCs associated with the soft SUSY potential, FCNCs can also originate from models with the extended Higgs sectors, leading to “additional” FCNC problems arising from tree-level Higgs exchange. However, as we shall now show, the Z_2^h symmetry, in combination with the Δ_{27} family symmetry, suppresses FCNCs due to tree-level Higgs exchange in the present model. This can be understood by noting that, since ϕ_3^h transforms under Z_2^h , it will couple to the Higgs fields but not to the quarks and leptons. This can be understood by considering the messenger diagrams of

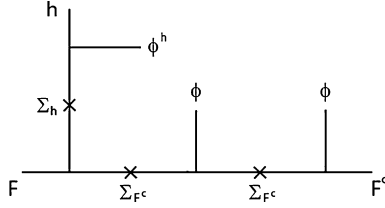


Fig. 1. This figure illustrates the type of messenger diagram that provides the dominant contribution to the Yukawa operators in Eq. (5).

the above higher-order operators where ϕ_3^h will only be allowed to attach itself to the Higgs fields (and the Higgs-like messenger fields) if all the messenger fields are even under Z_2^h . This is illustrated by Fig. 1. Once ϕ_3^h gets a VEV, only the third generation of the up and down Higgs fields h_3 are allowed to couple to the quarks and leptons. It is these up and down Higgs fields which we therefore take to obtain electro-weak scale VEVs, and thus act like the up and down Higgs fields of the MSSM.

The Z_2^h and Δ_{27} symmetries prevent the first and second generation of Higgs fields from interacting with the quarks and leptons at tree-level and so there can be no tree-level FCNC processes involving the neutral scalar components of these fields. In the E_6 SSM a Z_2 symmetry called Z_2^H is applied to all the 27 fields except for the third generation of Higgs fields and singlet fields to prevent the first and second generation of Higgs fields from interacting with the quarks and leptons at tree-level. The Z_2^h in this Letter is therefore acting as the Z_2^H symmetry of the E_6 SSM even though it does not distinguish between the different Higgs fields. The Z_2^H symmetry in the E_6 SSM is broken by an additional discrete Z_2 symmetry that forbids the colour triplets of the Higgs fields causing rapid proton decay [5]. This additional discrete symmetry will not break the Z_2^h symmetry here, however any misalignment of ϕ_3^h will play the role of Z_2^H breaking, as discussed later.

2.3.2. The messenger fields

The messenger fields Σ that are responsible for the suppression factors in Eq. (5) include fields that transform in the same way as quarks and leptons under the Standard Model gauge group and as singlets, triplets and anti-triplets of Δ_{27} . We refer to these type of messenger field as quark and lepton-like messengers Σ_{F, F^c} . In addition there are also messengers that are singlets of Δ_{27} and transform in the same way as Higgs fields under the Standard Model gauge group. We refer to these messenger fields as Higgs-like messengers Σ_h . All these messenger fields are taken to carry positive Z_2^h parity, and we assume that the Higgs-like messengers Σ_h are heavier than the quark and lepton-like messengers Σ_{F, F^c} so that the latter dominate the messenger diagrams. We further assume that the right-handed quark and lepton messengers Σ_{F^c} are heavier than their left-handed counterparts Σ_F so that the former dominate over the latter. The messenger diagrams are illustrated by Fig. 1.

To create a smaller hierarchy in the down quark sector compared to the up quark sector, we take the mass of the 3 and $\bar{3}$ up and down Higgs messengers M_3^h to be equal, but the up right-handed quark messengers Σ_{u^c} that are 3 and $\bar{3}$ and singlets of $SU(3)$ to have a mass M^u that is greater than the right-handed down quark messengers Σ_{d^c} by approximately a factor of three. For the top Yukawa coupling constant to be greater than the bottom Yukawa coupling constant we take the ϕ_3 flavon to transform as a $3 + 1$ of the $SU(2)_R$ subgroup of E_6 and choose its VEV so that $\langle \phi_3 \rangle / M_d = \langle \phi_3 \rangle / M_u$ as in [15]. In terms of these messenger

masses, the VEV scales for the various flavon fields are then taken to be the following:

$$\frac{\langle \phi_3^h \rangle}{M_3^h} \approx \frac{\langle \phi_3 \rangle}{M^u} \approx 0.8, \quad \frac{\langle \phi_{23} \rangle}{M^u} \approx \epsilon_u, \quad \frac{\langle \phi_{123} \rangle}{M^u} \approx \epsilon_u^2 \quad (7)$$

where $\epsilon_u \approx 0.05$. At the GUT scale the Yukawa coupling for the top and bottom quark is expected to be about 0.5 in third family Yukawa unification models based on the MSSM with large $\tan \beta$ [22]. We therefore assume that $\langle \phi_3^h \rangle / M_3^h \approx \langle \phi_3 \rangle / M^u \approx 0.8$. By comparison, in Δ_{27} models in which the Higgs is a singlet and there is a messenger-scale suppression factor to the second power, $\langle \phi_3 \rangle / M^u$ is assumed to be about 0.7 [15]. Note that if $\langle \phi_3 \rangle / M^u$ was taken to be exactly or very close to 1 then higher-order operators would be no-more suppressed than lower-order operators, so such large expansion parameters are of some concern, and in this respect the model is on a similar footing to that [15].

In order to avoid problems with unification, we assume that the mass of the above messenger fields have masses close to the conventional GUT scale, which suggests that the Δ_{27} family symmetry is also broken close to GUT scale. In principle this could lead to GUT scale threshold effects, but we do not consider this further here. Below the GUT scale the particle content of the model below the GUT scale is then the equivalent to that of the E_6 SSM, with the possible inclusion of additional SM singlet flavon fields close to the GUT scale that will not affect the running of the gauge coupling constants. As in the E_6 SSM, below the GUT scale there are three copies of complete 27 representations of E_6 plus two additional electroweak doublets with masses of order the TeV scale, with the gauge coupling constants predicted to unify at the conventional GUT scale but with a larger unified gauge coupling constant than in the MSSM [5].

In Section 2.5 we also discuss the messenger fields which are responsible for the higher-order operators that generate the effective μ -term. These particular messengers include Higgs-like messengers and messengers that transform as singlets of the Standard Model gauge group.

2.3.3. The effective Yukawa matrices

Inputting the above flavon VEVs into the superpotential given by Eq. (5) generates the following effective Yukawa matrices for the quarks and leptons:

$$\lambda_u^{ij} = \lambda_t \begin{pmatrix} 0 & \epsilon_u^3 & \mathcal{O}(\epsilon_u^3) \\ \epsilon_u^3 & \epsilon_u^2 & \mathcal{O}(\epsilon_u^2) \\ \mathcal{O}(\epsilon_u^3) & \mathcal{O}(\epsilon_u^2) & 1 \end{pmatrix},$$

$$\lambda_d^{ij} = \lambda_t \begin{pmatrix} 0 & 1.5\epsilon_d^3 & 0.4\epsilon_d^3 \\ 1.5\epsilon_d^3 & \epsilon_d^2 & 1.3\epsilon_d^2 \\ 0.4\epsilon_d^3 & 1.3\epsilon_d^2 & 1 \end{pmatrix},$$

where $\epsilon_d = 3 \times \epsilon_u \approx 0.15$, and $\lambda_t \approx 0.5$ at the GUT scale [22].

The above form of Yukawa matrices have been shown to produce a realistic CKM matrix and realistic mass hierarchies for the up and down quarks [22]. The H_{45} Higgs field in the superpotential equation (5) is used to generate the Georgi–Jarlskog relations in the down quark and charged lepton matrices so that the correct hierarchy in the charged leptons is generated at the electroweak scale [23].

2.4. Tri-bi-maximal mixing

To generate tri-bi-maximal mixing we use constrained sequential dominance [24] in which the right-handed neutrinos have a

hierarchy in mass. This hierarchy is generated by the following operators:

$$\begin{aligned}
W_{Maj} \sim & \frac{1}{M} F_i^c F_j^c \theta^i \theta^j \\
& + \frac{1}{M^5} F_i^c F_j^c \phi_{23}^i \phi_{23}^j (\theta^k \bar{\phi}_{123k}) (\theta^l \bar{\phi}_{3l}) \\
& + \frac{1}{M^5} F_i^c F_j^c \phi_{123}^i \phi_{123}^j (\theta^k \bar{\phi}_{123k}) (\theta^l \bar{\phi}_{123l}) \quad (8)
\end{aligned}$$

where $\bar{\phi}_{123}$ and $\bar{\phi}_3$ are triplets of Δ_{27} , and θ is from a $\bar{27}$ of E_6 and a $\bar{3}$ of Δ_{27} .

The above operators together with the Dirac operator involving the left-handed and right-handed neutrinos from the superpotential $F_i F_j^c h_3$ create a conventional type I see-saw mechanism which generates an effective Majorana mass matrix for the left-handed neutrinos. Due to form of the right-handed Majorana mass matrix, and the Dirac mass matrix from Eq. (5), the effective Majorana matrix for the left-handed neutrinos is of a form that is diagonalized by a tri-bi-maximal matrix (see [15] for more details).

2.5. The effective μ -term and inert Higgsino/singlino masses

The $U(1)_N$ symmetry of the E_6 SSM forbids any bilinear superpotential terms for the different Higgs fields. Instead effective bilinear terms come from the Pati–Salam superpotential term $\lambda^{ijk} S_i h_j h_k$ from Eq. (1), where $i, j, k = 1 \dots 3$, once S_3 gets a VEV. In terms of the Standard Model gauge group, which is the symmetry of the model discussed in this Letter, this superpotential term reduces to $\lambda^{ijk} S_i h_{uj} h_{dk}$ where h_i and h_d denote up and down Higgs fields. In this Letter we take the singlet fields S_i , like the Higgs fields, to transform as a triplet of the Δ_{27} symmetry. We also take them to be odd under a Z_2^S symmetry and even under the Z_2^h symmetry. The operator $\lambda^{ijk} S_i h_{uj} h_{dk}$ is thus forbidden and is instead generated by the following higher-order operators:

$$\begin{aligned}
W_\mu \sim & \frac{1}{M^3} S_i h_{uj} h_{dk} (\phi_3^S)^i (\phi_3^h)^j (\phi_3^h)^k \\
& + \frac{1}{M^2} \epsilon^{jkl} S_i h_{uj} h_{dk} (\phi_3^S)^i (\bar{\phi}_3^h)_l \\
& + \frac{1}{M^2} \epsilon^{ijl} S_i h_{uj} h_{dk} (\bar{\phi}_3^S)_l (\phi_3^h)^k \\
& + \frac{1}{M^2} \epsilon^{ijl} S_i h_{uk} h_{dj} (\bar{\phi}_3^S)_l (\phi_3^h)^k \quad (9)
\end{aligned}$$

where the flavons have the charges shown in Table 1 and we take the scale of these flavon VEVs to be such that $\langle \phi_3^S \rangle / M_S = \epsilon_S$, $\langle \bar{\phi}_3^S \rangle / M_h = \bar{\epsilon}_S$ and $\langle \bar{\phi}_3^h \rangle / M_S = \bar{\epsilon}_h$ where M_S is the mass scale of the singlet-like messengers and M_h is the mass scale of the Higgs-like messengers. The messenger diagrams responsible for generating the above higher-order operators are represented by Fig. 2.

The first operator in Eq. (9) is responsible for generating an effective μ -term for the third family of Higgs fields once the flavons fields and the third family singlet field S_3 obtain VEVs. Since we assume that only the third family of Higgs obtains a VEV, this effective μ -term acts like the μ -term of the MSSM Higgs fields. The effective μ -term will have a value $(0.8)^3 \langle S_3 \rangle$, which will be approximately 1 TeV if $\langle S_3 \rangle = 2$ TeV, which is consistent with the experimental bound for the mass of a Z' . If we assume a radiative symmetry breaking explanation for the third family singlet field's VEV then it will be related to the singlet's soft mass term. This then explains the apparent requirement of the μ -term of the MSSM being related to the MSSM soft terms.

The second and third operators in Eq. (9) are responsible for providing mass to the first and second families of Higgsinos and

singlinos once the third family of Higgs fields and singlet field obtain VEVs. This results in a mixing between all of these states which is represented by the following matrix:

$$M^{inert} = \begin{pmatrix} A_{22} & A_{21} \\ A_{21}^T & A_{11} \end{pmatrix}.$$

This matrix is written in the basis $(\tilde{h}_{d2}^0, \tilde{h}_{u2}^0, \tilde{S}_2 | \tilde{h}_{d1}^0, \tilde{h}_{u1}^0, \tilde{S}_1)$ so that $A_{\alpha\beta}$ are 3×3 matrices where $\alpha, \beta = 1, 2$. Because of the anti-symmetric tensor in Eq. (9) we find that $A_{11} = A_{22} = \mathbf{0}$, whereas A_{21} is given by the following:

$$A_{21} = \begin{pmatrix} 0 & \epsilon_S \bar{\epsilon}_h \langle S^3 \rangle & \bar{\epsilon}_S \langle h_u^3 \rangle \\ \epsilon_S \bar{\epsilon}_h \langle S^3 \rangle & 0 & \bar{\epsilon}_S \langle h_d^3 \rangle \\ \bar{\epsilon}_S \langle h_u^3 \rangle & \bar{\epsilon}_S \langle h_d^3 \rangle & 0 \end{pmatrix}, \quad (10)$$

where this matrix couples the states $(\tilde{h}_{d2}^0, \tilde{h}_{u2}^0, \tilde{S}_2)$ to the states $(\tilde{h}_{d1}^0, \tilde{h}_{u1}^0, \tilde{S}_1)$. In the limit of exact Z_2^h and Z_2^S symmetry these Higgsino and singlino states will decouple from the usual inert USSM states such as the third family of Higgsinos, singlinos, wino and hypercharge bino fields. See [25] for a full discussion on the mixing between the usual USSM states and the additional E_6 SSM states where it is also shown that the mixing between the $U(1)_N$ bino and Higgsino and singlino fields is expected to be small.

The above Higgsino and singlino neutral states combine to form two degenerate LSP states, approximately consisting of a Dirac state formed from (dropping the tildes) S_1 and S_2 , together with two generally heavier approximately degenerate Dirac states formed from h_{d1}^0 and h_{u2}^0 on the one hand and h_{d2}^0 and h_{u1}^0 on the other hand. With exact R-parity the Dirac LSP state formed from S_1 and S_2 becomes a dark matter candidate. However the masses of the degenerate LSPs S_1 and S_2 can be split if the first and second generation of Higgs and singlet fields are distinguished from one another. One way of achieving this is to assume that the flavon field ϕ_3^h gets small vacuum expectation values in its first and second components of Δ_{27} such that $\langle \phi_3^h \rangle^T \propto (\delta_1, \delta_2, 1)$ where $\delta_1, \delta_2 \ll 1$. This might be expected to occur from higher-order operators that affect the vacuum alignment of the fields. Two WIMPs that are almost degenerate in mass have been recently used to explain the DAMA data [26].

Note that although the Z_2^h and Z_2^S symmetries of this model have combined to operate in a similar manner to the original Z_2^H symmetry of the E_6 SSM, they allow fewer operators than the latter. The operators allowed by the original Z_2^H symmetry but which are not present in this model are $S_3 h_{u\alpha} h_{d\alpha}$, $S_\alpha h_{u\alpha} h_{d3}$ and $S_\alpha h_{u3} h_{d\alpha}$. Such operators are responsible for the A_{22} and A_{11} matrices being non-zero in the E_6 SSM.

2.6. D-fermion mass terms

For the $U(1)_N$ group of the E_6 SSM to be anomaly free, all the colour partners D_i of the Higgs fields from the three 27 multiplets must have masses lower than the energy scale of $\langle S_3 \rangle$. These masses come from the Pati–Salam superpotential $\lambda^{ijk} S_i D_j D_k$ which is derived from the E_6 superpotential of the E_6 SSM given by Eq. (1). In terms of a Standard Model gauge symmetry we represent this operator as $\lambda^{ijk} S_i D_j \bar{D}_k$ where D is a triplet of the strong force gauge group $SU(3)_c$ but \bar{D} is an anti-triplet.

In this Letter we assume that the exotic particles D_i , like the Higgs fields transform as a triplet of Δ_{27} and have odd Z_2^h parity but even Z_2^S parity. The allowed higher order operator thus mirrors the allowed operators that provide effective μ -terms for the Higgs fields:

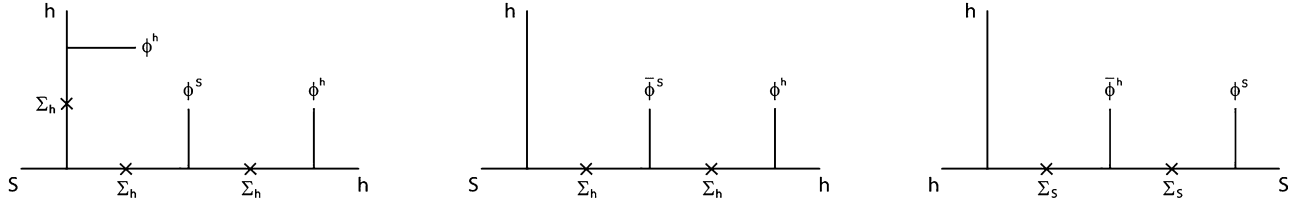


Fig. 2. This figure illustrates the type of messenger diagram that provides the dominant contribution to the effective μ -term operators in Eq. (9).

$$\begin{aligned}
W_D \sim & \frac{1}{M^3} S_i D_j \bar{D}_k (\phi_3^S)^i (\phi_3^h)^j (\phi_3^h)^k \\
& + \frac{1}{M^2} \epsilon^{jkl} S_i D_j \bar{D}_k (\phi_3^S)^i (\bar{\phi}_3^h)_l \\
& + \frac{1}{M^2} \epsilon^{ijl} S_i D_j \bar{D}_k (\bar{\phi}_3^S)_l (\phi_3^h)^k \\
& + \frac{1}{M^2} \epsilon^{ijl} S_i D_k \bar{D}_j (\bar{\phi}_3^S)_l (\phi_3^h)^k. \quad (11)
\end{aligned}$$

The mass scale for the exotic-like messengers $\Sigma_{D,\bar{D}}$ responsible for the operators in Eq. (11) however need not be the same as the Higgs messengers. We define the messenger scales such that $M_D = M_{\bar{D}}$, $\langle \phi_3^h \rangle / M_D \equiv \epsilon_D$, $\langle \bar{\phi}_3^h \rangle / M_S \equiv \bar{\epsilon}_D$ and $\langle \bar{\phi}_3^S \rangle / M_D \equiv \bar{\epsilon}'_S$. We also assume that the exotic-like messengers, like the Higgs-like messengers only have even Z_2^S parity but can carry either even or odd Z_2^S parity. The messenger diagrams that are assumed to be responsible for generating the higher-order operators in Eq. (11) are analogous to those in Fig. 2 but with the Higgs fields and Higgs-like messenger fields replaced with exotic fields and exotic-like messenger fields respectively.

The D-fermions thus obtain mass once the flavons and S_3 obtain an expectation value. We write these masses in matrix form $M^{Dij} D_i \bar{D}_j$ where M^{Dij} is the following:

$$M^{Dij} = \begin{pmatrix} 0 & \epsilon_S \bar{\epsilon}_D & 0 \\ \epsilon_S \bar{\epsilon}_D & 0 & 0 \\ 0 & 0 & \epsilon_S \epsilon_D^2 + \epsilon_D^3 \end{pmatrix} \langle S^3 \rangle.$$

The parameters ϵ_S , ϵ_D and $\bar{\epsilon}_D$ can then be chosen for the exotic masses to be larger than the experimental bound of 300 GeV. Two of the exotic states are predicted to be degenerate in mass with the third also being degenerate in the approximation that $\epsilon_D^2 = \bar{\epsilon}_D$ and $\epsilon_D \ll \epsilon_S$. This mass structure is in stark contrast to the hierarchical structure of the quarks and leptons despite all the states being triplets of the family symmetry.

2.7. D-fermion decay and proton decay suppression

If the exotic D particles are taken to have the same Δ_{27} , Z_2^h and Z_2^S quantum numbers as the Higgs fields h , then they can decay [5] via the following non-renormalizable operators:

$$\begin{aligned}
W_{Exotic} \sim & \frac{1}{M^3} F_i^c F_j^c D_k \phi_3^i \phi_3^j (\phi_3^h)^k \\
& + \frac{1}{M^4} (F_i F_j + F_i^c F_j^c) D_k \phi_{23}^i \phi_{23}^j (\phi_3^h)^k H_{45} \\
& + \frac{1}{M^3} (F_i F_j + F_i^c F_j^c) D_k \phi_{123}^i \phi_{23}^j (\phi_3^h)^k \\
& + \frac{1}{M^3} (F_i F_j + F_i^c F_j^c) D_k (\phi_{123}^i \phi_3^j + \phi_3^i \phi_{123}^j) \\
& \times (\phi_{123}^m \bar{\phi}_{1m}) (\phi_3^h)^k H_{45}. \quad (12)
\end{aligned}$$

However not all these operators can be allowed otherwise this would lead to very rapid proton decay. Thus, we assume either the Z_2^B or Z_2^L discrete symmetries that are used in the E_6 SSM [5],

where these symmetries differentiate between different fermion F, F^c components. Under the Z_2^B symmetry the leptons and D-states are odd whereas, under the Z_2^L symmetry, only the leptons are odd and all other particles are even. These discrete symmetries remove some of the above operators, such that the remaining operators correspond to the D-states coupling as either diquarks or leptquarks. This effectively prevents the proton from decaying due to couplings with the D-states, but still allowing the latter states to decay, thus avoiding any nucleosynthesis difficulties [5,27]. Note that this discrete symmetry breaks the Pati-Salam gauge symmetry but respects the Standard Model gauge symmetry assumed in this Letter.

In the limit that $\langle \phi_3^h \rangle^T \propto (0, 0, 1)$ exactly, the decay channels of the exotic states in the model used in this Letter will be different to those of the E_6 SSM since only the third generation of the exotic states couples directly to quarks and leptons, whereas all three generations of the exotic states in the E_6 SSM interact directly with the quarks and leptons. The difference between the two models occurs because we have taken the exotic states to transform under Z_2^h , which results in an effective Z_2^H symmetry for only the first and second generation of exotic states. In the E_6 SSM however all three generations transform under Z_2^H . This application of the Z_2^h symmetry results in the decay products of the first and second generation of exotic states always involving a singlet field S_i .

If instead $\langle \phi_3^h \rangle^T \propto (\delta_1, \delta_2, 1)$ as discussed in Section 2.5, then all the Δ_{27} components of the exotic states will mix via the mass terms presented in Section 2.6. This results in the same exotic decay channels as used in the E_6 SSM but with some being more suppressed since $\delta_1, \delta_2 \ll 1$.

3. Summary

In this Letter we have shown how FCNC's due to models with three families of Higgs fields may be tamed by the same family symmetry which predicts tri-bi-maximal lepton mixing and provides a solution to the SUSY FCNC and CP problems. To be concrete we have focussed on the E_6 SSM where we have shown how the flavour problem can be solved by using a Δ_{27} family symmetry. The three 27-dimensional families of the E_6 SSM, including the three families of Higgs fields, transform in a triplet representation of the Δ_{27} family symmetry, allowing the family symmetry to commute with a possible high energy E_6 symmetry. The Δ_{27} family symmetry breaking considered here, together with a vertical Z_2^h symmetry which does not distinguish between the three families, gives rise effectively to the Z_2^H symmetry of the E_6 SSM, which solves the flavour changing neutral current problem of the three families of Higgs fields. The main phenomenological predictions of the model are tri-bi-maximal mixing for leptons, two almost degenerate LSPs and two almost degenerate families of colour triplet D-fermions, providing a clear prediction for the LHC. In addition the D-term vacuum alignment mechanism described here leads to PGBs with masses below the TeV scale, and possibly much lighter, which appears to be a quite general and robust prediction of any model based on the D-term vacuum alignment mechanism.

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