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UNCERTAINTY PROPAGATION ANALYSIS FOR YONGGWANG NUCLEAR UNIT 4 BY MCCARD/MASTER CORE ANALYSIS SYSTEM

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This paper concerns estimating uncertainties of the core neutronics design parameters of power reactors by direct sampling method (DSM) calculations based on the two-step McCARD/MASTER design system in which McCARD is used to generate the fuel assembly (FA) homogenized few group constants (FGCs) while MASTER is used to conduct the core neutronics design computation. It presents an extended application of the uncertainty propagation analysis method originally designed for uncertainty quantification of the FA FGCs as a way to produce the covariances between the FGCs of any pair of FAs comprising the core, or the covariance matrix of the FA FGCs required for random sampling of the FA FGCs input sets into direct sampling core calculations by MASTER. For illustrative purposes, the uncertainties of core design parameters such as the effective multiplication factor (k_{eff}), normalized FA power densities, power peaking factors, etc. for the beginning of life (BOL) core of Yonggwang nuclear unit 4 (YGN4) at the hot zero power and all rods out are estimated by the McCARD/MASTER-based DSM computations. The results are compared with those from the uncertainty propagation analysis method based on the McCARD-predicted sensitivity coefficients of nuclear design parameters and the cross section covariance data.

KEYWORDS : McCARD, Uncertainty Propagation, Direct Sampling Method, Few Group Constant, Covariance Matrix, Yonggwang Nuclear Unit 4

1. INTRODUCTION

In reference 1, we presented the B_1 theory-augmented Monte Carlo (MC) method as a new way to generate fuel or fuel assembly (FA) homogenized few group diffusion theory constants (few group constants or FGCs hereafter) and demonstrated that FGCs from it are well qualified for high-accuracy, two-step core neutronics design calculations. Because the FGCs from the new MC method are bound to have uncertainties due to nuclear cross section and nuclide number density input data uncertainties of the constituent nuclides as well as statistical uncertainties inherent in the MC method, we subsequently presented in reference 2 an uncertainty propagation analysis method designed to estimate the uncertainties of FGCs which arise solely from MC input data uncertainties. We first used it to quantify uncertainties of the burnup-dependent homogenized two group constants of a low-enriched UO_2 fuel pin cell and a FA of a pressurized water reactor (PWR) caused by nuclear cross section data uncertainties of U-235 and U-238 as reflected in the covariance files of the JENDL 3.3 library, and then to examine the effects of the resulting uncertainties of the two group constants on those of the effective multiplication factors (k_{eff}) of the UO_2 pin cell and the PWR FA. As a follow-up study result of the above-mentioned references, this paper presents an extended application of the uncertainty propagation analysis method [2] as a way to quantify the uncertainties of the core design parameters of commercial power reactors caused by those of the FGCs from the B_1 theory-augmented MC method.

Quantification of reactor core design parameter uncertainties has been of great interest to nuclear designers for various reasons. First of all, it enables them to evaluate the safety of nuclear system designs. Furthermore, it provides them with useful information not only to assess the feasibility of new nuclear systems but also to improve the evaluated nuclear cross section data including the covariance data. The common approaches available for this subject are the direct sampling method (DSM), often called the brute force method [3, 4, 5], and the perturbation-theory based method [2, 6]. Because both approaches require the covariance matrix of the uncertain FGCs data set inputted into a nuclear design computation, it is prerequisite to work out how to estimate the covariance of each pair of the uncertain FGCs inputted as part of the input data of the nuclear design calculation. The objectives of this paper are to present an extended application of the uncertainty propagation analysis method [2] as a practical way to estimate the necessary covariance matrix of the FGCs generated from the B_1 theory-augmented MC method [1] and to describe how to utilize them to determine uncertainties of core design parameters by the DSM.

To do so, we employed the McCARD/MASTER-based two-step neutronics design system in which McCARD [7] is used as the MC code for generation of FA FGCs while MASTER [8] is used as the modern nodal code for deterministic core design calculations. We adopted the DSM to determine the uncertainties of core design parameters of the YGN4. This method involves a random sampling of a finite number of the FGCs sets – two group constants sets in Yonggwang Nuclear Unit 4 (YGN4) PWR core [9] – each set consisting of FGCs of all the FAs comprising YGN4 core and conducting MASTER design computations for each set of FGCs. The random variate sampling scheme was used first to generate 10,000 two group constants input data sets for each FA type comprising the YGN4 core from the B_1 heory-augmented MC method and covariance matrix estimated from the uncertainty propagation method. Then, 10,000 independent core neutronics MASTER computations for the initial YGN4 core at the hot zero power (HZP) state and all rods out (ARO) were conducted to determine the uncertainties of neutronics design parameters such as k_{eff} and the power peaking factor of the core.

2. UNCERTAINTY PROPAGATION ANALYSIS OF TWO-STEP CALCULATION

2.1 Uncertainty Quantification of Core Design Parameters by Direct Sampling Method

Let $\mathbf{X}$ designate the FGCs set inputted into a MASTER-based core neutronics computation and $\Sigma^p_{\alpha,G}$ be the few group cross section of reaction type α (= t, a, f, tr) and group G (=1,2) of FA type p (= A0, B0, …). In terms of the FGCs of FAs comprising the core, $\Sigma^p_{\alpha,G}$, $\mathbf{X}$ may be written as:

$$\mathbf{X} = (\Sigma^p_{\alpha,G}) . \tag{1}$$

From the input and output relation of the MASTER computations, one may represent the reactor core nuclear design parameter Q, one of the outputs of the MASTER computations, as a function of $\mathbf{X}$:

$$Q \equiv Q(\mathbf{X}) . \tag{2}$$

Needless to mention, Q may be k_{eff}, normalized FA power density, the power peaking factor, etc. Because of uncertainties of $\Sigma^p_{\alpha,G}$, there may be an infinite number of different FGCs input sets, $\mathbf{X}^\kappa$(κ = 1, 2,⋯, ∞), which may be sampled from the covariance matrix of $\mathbf{X}$, and so many different outputs from MASTER core calculations may be expressed by:

$$Q^\kappa \equiv Q(\mathbf{X}^\kappa);\ \kappa = 1, 2, \cdots, \infty . \tag{3}$$

κ is the index for different input data sets and outputs from them. From Eq. (3), one can formally define the mean value, $\bar{Q}$, and the variance, $\sigma^2[Q]$:

$$\bar{Q} = \lim_{K\to\infty} \frac{1}{K} \sum_{\kappa=1}^{K} Q^\kappa , \tag{4}$$

and:

$$\sigma^2[Q] = \lim_{K\to\infty} \frac{1}{K-1} \sum_{\kappa=1}^{K} (Q^\kappa - \bar{Q})^2 . \tag{5}$$

DSM computes $\bar{Q}$ and $\sigma^2[Q]$ based on Eqs. (3)-(5). In actual use, however, one makes use of a finite sampling instead of the infinite sampling of the FGCs set, say $\mathbf{X}^\kappa$(κ = 1, 2,⋯, K). One may then approximate the infinite sum over κ for $\bar{Q}$ in Eq. (4) and Eq. (5) for $\sigma^2[Q]$ by a finite sum from κ=1 to κ=K. Here K can be any finite integer of one's choice. In other words, one calculates $\bar{Q}$ and $\sigma^2[Q]$ by:

$$\bar{Q} \cong \frac{1}{K} \sum_{\kappa=1}^{K} Q^\kappa \tag{6}$$

and :

$$\sigma^2[Q] \cong \frac{1}{K-1} \sum_{\kappa=1}^{K} (Q^\kappa - \bar{Q})^2 . \tag{7}$$

Thus, all one has to do to estimate the uncertainty of nuclear design parameter Q, $\sigma^2[Q]$, by the direct sampling scheme is to randomly sample the FGCs input set $\mathbf{X}^\kappa$ and perform the MASTER computation for Q^κ.

In this conjunction, there is a standard random sampling scheme [10] for multiple correlated random variables like the FGCs of which the covariance matrix is known. Suppose that $\mathbf{C}_\Sigma$ is the known covariance matrix of the FGCs, and also that a lower triangular matrix $\mathbf{B}$ is known through the Cholesky decomposition of $\mathbf{C}_\Sigma$ as:

$$\mathbf{C}_\Sigma = \mathbf{B} \cdot \mathbf{B}^T \tag{8}$$

where $\mathbf{B}^T$ is the transpose matrix of $\mathbf{B}$. Then one can sample the FGCs set $\mathbf{X}^\kappa$ by:

$$\mathbf{X}^\kappa = \bar{\mathbf{X}} + \mathbf{B} \cdot \mathbf{Z} . \tag{9}$$

$\bar{\mathbf{X}}$ is the mean vector of $\mathbf{X}$ defined by the mean value of the FGCs, $\overline{\Sigma^p_{\alpha,g}}$, namely $\bar{\mathbf{X}}$ =($\overline{\Sigma^p_{\alpha,g}}$). $\mathbf{Z}$ is a random normal vector which can be constructed directly from random sampling of the standard normal distribution function for each element of $\mathbf{X}$, $\Sigma^p_{\alpha,g}$.

2.2 Determination of Covariance Matrix C_Σ

The key step to the DSM as a way to quantify uncertainties of core design parameters involves random sampling of the FGCs set $\mathbf{X}^\kappa$. Obviously, this can be readily implemented, once the covariance matrix $\mathbf{C}_\Sigma$ is known. Therefore, the uncertainty quantification of core design parameters by the DSM boils down to the problem of how to determine the covariance matrix $\mathbf{C}_\Sigma$(=(cov[$\Sigma_{\alpha,G}^{p_1}$, $\Sigma_{\alpha',G'}^{p_2}$])). (cov[$\Sigma_{\alpha,G}^{p_1}$, $\Sigma_{\alpha',G'}^{p_2}$]) is the covariance between the FGCs of type p_1 FA and type p_2 FA with (p_1, p_2) chosen from all the possible parings of FA types comprising the core. Subscripts α and α' denote the reaction type of the FGCs while G and G' designate the few energy group index, respectively. As mentioned in the introduction, an extended application of the uncertainty propagation analysis method [2] leads one to determine covariance matrix $\mathbf{C}_\Sigma$ or cov[$\Sigma_{\alpha,G}^{p_1}$, $\Sigma_{\alpha',G'}^{p_2}$]. This section presents additional mathematical manipulations necessary, but unavailable in reference 2, for implementing it in order to estimate cov[$\Sigma_{\alpha,G}^{p_1}$, $\Sigma_{\alpha',G'}^{p_2}$].

Suppose that we are generating the FGCs of two types of FAs, p_1 and p_2, using the B_1 theory-augmented MC method. As shown in Fig 1, the new MC method generates FGCs of a given FA through a sequence of computational steps: (i) the infinite medium spectrum (φ_g) and fine-group reaction rate ($r_{\alpha,g,j,m}$) calculation for the FA by the MC method, (ii) determination of infinite-medium spectrum weighted fine-group cross sections ($\Sigma_{\alpha,g}$), (iii) the B_1 critical spectrum (ϕ_g^B) and critical buckling (B_c^2) calculation, and (iv) generation of critical spectrum corrected FGCs ($\Sigma_{\alpha,G}$ and D_G). $r_{\alpha,g,j,m}$ is the α-type reaction rate of fine group g for nuclide j in region m and the other notations are standard. If one wants to produce the burnup-dependent FGCs of the FA, the FA depletion calculation step is added as an additional step to determine burnup-dependent nuclide number density ($N_{i',m',n}$) inputs at the given FA burnup state for the first step infinite medium spectrum calculation by the MC method. Because the five step calculations are performed in sequence, uncertainties of nuclear cross section ($\sigma_{\alpha',g,i'}$) and nuclide number density ($N_{i',m',n}$) input data and the statistical uncertainties from the first-step MC calculation propagate to uncertainties of outputs of the ensuing steps and finally to those of the FGCs. The uncertainty propagation analysis method described in detail in reference 2 makes the best of this fact. In brief, it establishes the functional relation of the uncertain input and output (I/O) variables in each step as shown in Table 1, quantifies the fluctuation of output variables about their mean values as a function of the input variables by a first-order Taylor expansion of the functional relation of the I/O variables, and utilizes them step after step to determine first the variances of the output variables of the first step and finally those of the fourth step output, namely FGCs. In reference 2, the method is focused on estimating the variances of the burnup-dependent FGCs as a measure of their uncertainties.

Now, let's apply it to estimate the covariance of the FGCs of a pair of two FAs, say cov[$\Sigma_{\alpha,G}^{p_1}$, $\Sigma_{\alpha',G'}^{p_2}$] at the beginning of the initial core. Note that in this case the nuclear cross sections become a sole uncertain input data set into the first step MC calculation for the FGCs of two FA types p_1 and p_2, because the nuclide number densities of the two FAs are assumed to be known exactly.

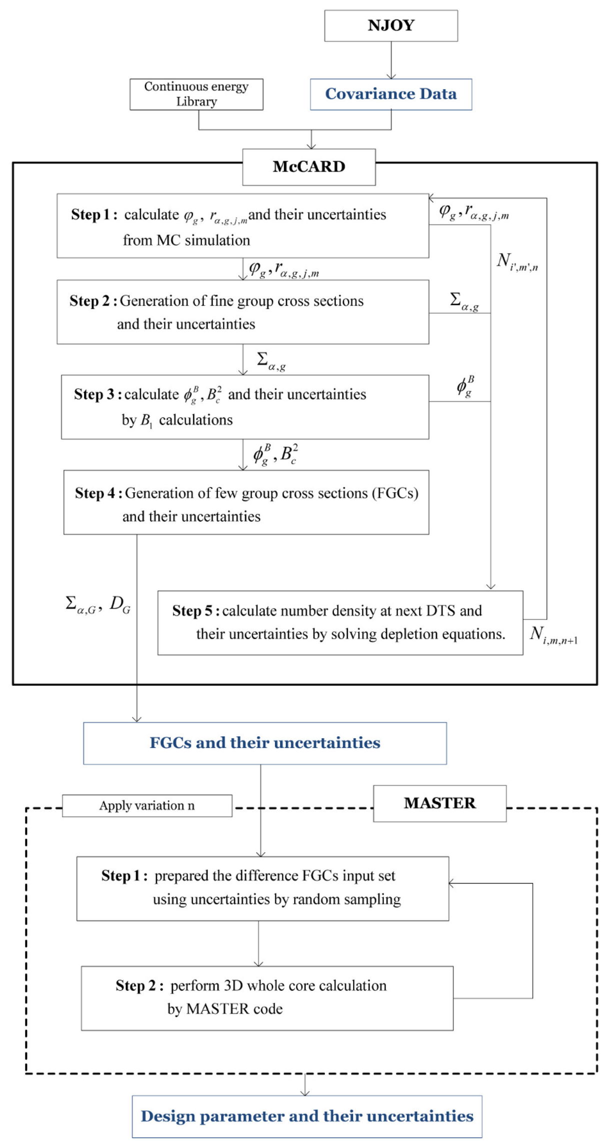

Fig. 1. The Computational Flow Chart for Uncertainty Propagation Analysis by McCARD/MASTER System

Table 1. Functional Relation between the Input (I) and the Output (O) Variables for Each Step

Step no.	Functional relation between IO
1	$\varphi_g = \varphi_g(\sigma_{\alpha',g',i'}, N_{i',m',n})$ $r_{\alpha,g,j,m} = r_{\alpha,g,j,m}(\sigma_{\alpha,g,i}, N_{i,m,n})$
2	$\Sigma_{\alpha,g} = \Sigma_{\alpha,g}(N_{j',m',n}, r_{\alpha,g,j',m'}, \varphi_g)$
3	$\phi_g^B = \phi_g^B(\cdot\cdot, \Sigma_{\alpha',g'}, \cdot\cdot)$ $B_c = B_c(\cdot\cdot, \Sigma_{\alpha',g'}, \cdot\cdot)$
4	$\Sigma_{\alpha,G} = \Sigma_{\alpha,G}(\Sigma_{\alpha,G}, \phi_g^B)$ $D_G = D_G(B_c, \phi_g^B, J_g^B)$
5	$N_{i,m,n+1} = N_{i,m,n+1}(r_{j,m}, N_{j,m,n})$ $r_{j,m} = r_{j,m}(\varphi_g, \phi_g^B, r_{\alpha,g,j,m})$

To start with, let's note the definition of cov[$\Sigma^{p_1}_{\alpha,G}$, $\Sigma^{p_2}_{\alpha',G'}$]:

$$\mathrm{cov}[\Sigma^{p_1}_{\alpha,G},\ \Sigma^{p_2}_{\alpha',G'}] = \lim_{K\to\infty}\frac{1}{K-1}\sum_{\kappa=1}^{K}\left(\Sigma^{p_1,\kappa}_{\alpha,G} - \overline{\Sigma^{p_1}_{\alpha,G}}\right)\left(\Sigma^{p_2,\kappa}_{\alpha',G'} - \overline{\Sigma^{p_2}_{\alpha',G'}}\right) \quad (10)$$

The superscript κ is the same index used for denoting the different FGCs input data sets into MASTER core calculation $\mathbf{X}^\kappa$, which is formed by $\Sigma^{p_1,\kappa}_{\alpha,G}$ and $\Sigma^{p_2,\kappa}_{\alpha',G'}$. The bar sign is used to imply the mean value of the quantity under it. Thus $\overline{\Sigma^{p}_{\alpha,G}}$ ($p = p_1, p_2$) is the mean value of $\Sigma^{p,\kappa}_{\alpha,G}$ defined as:

$$\overline{\Sigma^{p}_{\alpha,G}} = \lim_{K\to\infty}\frac{1}{K}\sum_{\kappa=1}^{K}\Sigma^{p,\kappa}_{\alpha,G}\,. \quad (11)$$

In order to estimate cov[$\Sigma^{p_1}_{\alpha,G}$, $\Sigma^{p_2}_{\alpha',G'}$] by Eq. (10), one needs to compute the fluctuation of $\Sigma^{p,\kappa}_{\alpha,G}$ about its mean, $\overline{\Sigma^{p}_{\alpha,G}}$. This can be done by using the I/O functional relation of step (iv) of the B_1 theory-augmented MC method, which may be derived from Table 1 as:

$$\Sigma^{p,\kappa}_{\alpha,G} \equiv \Sigma^{p,\kappa}_{\alpha,G}(\Sigma^{p,\kappa}_{\alpha,g},\phi^{p,\kappa}_{g})\,. \quad (12)$$

The first order Taylor expansion of $\Sigma^{p,\kappa}_{\alpha,G}$ about the mean values of its dependent variables leads one to find:

$$\Sigma^{p,\kappa}_{\alpha,G} - \overline{\Sigma^{p,\kappa}_{\alpha,G}} \cong \sum_{g\in G}\left\{\left.\frac{\partial\Sigma^{p,\kappa}_{\alpha,G}}{\partial\Sigma^{p,\kappa}_{\alpha,g}}\right|_0\cdot\left(\Sigma^{p,\kappa}_{\alpha,g} - \overline{\Sigma^{p,\kappa}_{\alpha,g}}\right) + \left.\frac{\partial\Sigma^{p,\kappa}_{\alpha,G}}{\partial\phi^{p,\kappa}_{g}}\right|_0\cdot\left(\phi^{p,\kappa}_{g} - \overline{\phi^{p,\kappa}_{g}}\right)\right\}. \quad (13)$$

The symbol "$|_0$" implies that the derivative of $\Sigma^{p,\kappa}_{\alpha,G}$ is evaluated with all its dependent variables being equal to their respective mean values. The use of Eq. (13) in Eq. (10) results in:

$$\begin{aligned}\mathrm{cov}[\Sigma^{p_1}_{\alpha,G},\Sigma^{p_2}_{\alpha',G'}] &= \sum_{g\in G}\sum_{g'\in G'}\mathrm{cov}[\Sigma^{p_1}_{\alpha,g},\ \Sigma^{p_2}_{\alpha',g'}]\cdot\left.\frac{\partial\Sigma^{p_1}_{\alpha,G}}{\partial\Sigma^{p_1}_{\alpha,g}}\right|_0\cdot\left.\frac{\partial\Sigma^{p_2}_{\alpha',G'}}{\partial\Sigma^{p_2}_{\alpha',g'}}\right|_0 \\ &+ \sum_{g\in G}\sum_{g'\in G'}\mathrm{cov}[\phi^{B,p_1}_{g},\phi^{B,p_2}_{g'}]\cdot\left.\frac{\partial\Sigma^{p_1}_{\alpha,G}}{\partial\phi^{B,p_1}_{g}}\right|_0\cdot\left.\frac{\partial\Sigma^{p_2}_{\alpha',G'}}{\partial\phi^{B,p_2}_{g'}}\right|_0 \\ &+ \sum_{g\in G}\sum_{g'\in G'}\mathrm{cov}[\Sigma^{p_1}_{\alpha,g},\ \phi^{B,p_2}_{g'}]\cdot\left.\frac{\partial\Sigma^{p_1}_{\alpha,G}}{\partial\Sigma^{p_1}_{\alpha,g}}\right|_0\cdot\left.\frac{\partial\Sigma^{p_2}_{\alpha',G'}}{\partial\phi^{B,p_2}_{g'}}\right|_0 \\ &+ \sum_{g\in G}\sum_{g'\in G'}\mathrm{cov}[\phi^{B,p_1}_{g},\ \Sigma^{p_2}_{\alpha',g'}]\cdot\left.\frac{\partial\Sigma^{p_1}_{\alpha,G}}{\partial\phi^{B,p_1}_{g}}\right|_0\cdot\left.\frac{\partial\Sigma^{p_2}_{\alpha',G'}}{\partial\Sigma^{p_2}_{\alpha',g'}}\right|_0\,,\end{aligned} \quad (14)$$

To estimate the derivatives in Eq. (14), one notes the definition for the FGCs $\Sigma^{p,\kappa}_{\alpha,G}$ s given by:

$$\Sigma^{p}_{\alpha,G} = \frac{\sum_{g'\in G}\Sigma^{p}_{\alpha,g'}\overline{\phi^{B,p}_{g'}}}{\sum_{g'\in G}\overline{\phi^{B,p}_{g'}}};\quad p=p_1,p_2 \quad (15)$$

The derivatives in Eq. (14) are then readily obtained from Eq. (15) by :

$$\left.\frac{\partial\Sigma^{p}_{\alpha,G}}{\partial\Sigma^{p}_{\alpha,g}}\right|_0 = \frac{\overline{\phi^{B,p}_{g}}}{\sum_{g'\in G}\overline{\phi^{B,p}_{g'}}};\quad p=p_1,p_2 \quad (16)$$

$$\left.\frac{\partial\Sigma^{p}_{\alpha,G}}{\partial\phi^{B,p}_{g}}\right|_0 = \frac{\overline{\Sigma^{p}_{\alpha,g}} - \overline{\Sigma^{p}_{\alpha,G}}}{\sum_{g'\in G}\overline{\phi^{B,p}_{g'}}};\quad p=p_1,p_2\,. \quad (17)$$

Equation (14) contains four unknown covariances: cov[$\Sigma^{p_1}_{\alpha,g}$, $\Sigma^{p_2}_{\alpha',g'}$], cov[$\phi^{B,p_1}_{g}$, $\phi^{B,p_2}_{g'}$], cov[$\Sigma^{p_1}_{\alpha,g}$, $\phi^{B,p_2}_{g'}$], and cov[ϕ^{B,p_1}_{g}, $\Sigma^{p_2}_{\alpha',g'}$]. They can be calculated in the same way as above by utilizing the I/O relations at steps (ii) and (iii). One may find that the computation of cov[$\Sigma^{p_1}_{\alpha,g}$, $\Sigma^{p_2}_{\alpha',g'}$] requires determining cov[$r^{p_1}_{\alpha,g,j,m}$, $r^{p_2}_{\alpha',g',j',m'}$], cov[$\varphi^{p_1}_{g}$, $\varphi^{p_2}_{g'}$], cov[$\varphi^{p_1}_{g}$, $r^{p_2}_{\alpha',g',j',m'}$], and cov[$r^{p_1}_{\alpha,g,j,m}$, $\varphi^{p_2}_{g'}$] while those of the three remaining covariances, cov[ϕ^{B,p_1}_{g}, $\phi^{B,p_2}_{g'}$], cov[$\Sigma^{p_1}_{\alpha,g}$, $\phi^{B,p_2}_{g'}$], and [ϕ^{B,p_1}_{g}, $\Sigma^{p_2}_{\alpha',g'}$], require only cov[$\Sigma^{p_1}_{\alpha,g}$, $\Sigma^{p_2}_{\alpha',g'}$]. Thus, calculation of the four unknown covariances in Eq. (14) reduces to determining four covariances; cov[$r^{p_1}_{\alpha,g,j,m}$, $r^{p_2}_{\alpha',g',j',m'}$], cov[$\varphi^{p_1}_{g}$, $\varphi^{p_2}_{g'}$], cov[$\varphi^{p_1}_{g}$, $r^{p_2}_{\alpha',g',j',m'}$], and cov[$r^{p_1}_{\alpha,g,j,m}$, $\varphi^{p_2}_{g'}$]. These can be calculated by using cov[$\sigma_{\alpha'',g'',i''}$, $\sigma_{\alpha''',g''',i'''}$], because it can be shown from the first step I/O relation that they are related to it as follows:

$$\mathrm{cov}[r^{p_1}_{\alpha,g,j,m},r^{p_2}_{\alpha',g',j',m'}] = \sum_{i'',\alpha'',g''}\sum_{i''',\alpha''',g'''}\mathrm{cov}[\sigma_{\alpha'',g'',i''},\sigma_{\alpha''',g''',i'''}]\cdot\left.\frac{\partial r^{p_1}_{\alpha,g,j,m}}{\partial\sigma_{\alpha'',g'',i''}}\right|_0\cdot\left.\frac{\partial r^{p_2}_{\alpha',g',j',m'}}{\partial\sigma_{\alpha''',g'',i'''}}\right|_0 \quad (18)$$

$$\mathrm{cov}[\varphi^{p_1}_{g},\varphi^{p_2}_{g'}] = \sum_{i'',\alpha'',g''}\sum_{i''',\alpha''',g'''}\mathrm{cov}[\sigma_{\alpha'',g'',i''},\sigma_{\alpha''',g'',i'''}]\cdot\left.\frac{\partial\varphi^{p_1}_{g}}{\partial\sigma_{\alpha'',g'',i''}}\right|_0\cdot\left.\frac{\partial\varphi^{p_2}_{g'}}{\partial\sigma_{\alpha''',g'',i'''}}\right|_0 \quad (19)$$

$$\mathrm{cov}[\varphi^{p_1}_{g},r^{p_2}_{\alpha',g',j',m'}] = \sum_{i'',\alpha'',g''}\sum_{i''',\alpha''',g'''}\mathrm{cov}[\sigma_{\alpha'',g'',i''},\sigma_{\alpha''',g'',i'''}]\cdot\left.\frac{\partial\varphi^{p_1}_{g}}{\partial\sigma_{\alpha'',g'',i''}}\right|_0\cdot\left.\frac{\partial r^{p_2}_{\alpha',g',j',m'}}{\partial\sigma_{\alpha''',g'',i'''}}\right|_0 \quad (20)$$

$$\mathrm{cov}[r^{p_1}_{\alpha,g,j,m},\varphi^{p_2}_{g'}] = \sum_{i'',\alpha'',g''}\sum_{i''',\alpha''',g'''}\mathrm{cov}[\sigma_{\alpha'',g'',i''},\sigma_{\alpha''',g'',i'''}]\cdot\left.\frac{\partial r^{p_1}_{\alpha,g,j,m}}{\partial\sigma_{\alpha'',g'',i''}}\right|_0\cdot\left.\frac{\partial\varphi^{p_2}_{g'}}{\partial\sigma_{\alpha''',g'',i'''}}\right|_0 \quad (21)$$

cov[$\sigma_{\alpha'',g'',i''}$, $\sigma_{\alpha''',g''',i'''}$] is computed by processing the covariance files of nuclear data files by the ERRORR module of the NJOY [11] or ERRORJ code [12].

3. NUMERICAL RESULTS AND DISCUSSION

The extended application of the uncertainty propagation analysis method described above enables one to estimate the covariances between the FGCs of any pair of FAs comprising the YGN4 core. As shown in Fig. 1, we used the covariances estimated by it to randomly sample a total of 10,000 FGCs input sets into MASTER-based nuclear design computations and to calculate the uncertainties of the nuclear design parameters such as k_{eff}, power peaking factor, etc. of beginning of life (BOL) YGN4 core at HZP and ARO caused by the uncertain FGCs input data by the DSM. Figure 2 shows a quarter of the YGN4 core. The core consists of a total of 177 FAs, each of which contains 236 fuel rods and 5 guide tubes arranged in a 16x16 square lattice. There are nine different types of FAs, depending on their enrichment, the numbers of high and low-enriched UO_2 fuel rods, and the number of gadolinia burnable poison rods as shown in Fig 3. Because of the presence of the axial cutback design in gadolinia-shimmed FAs, there are a total of 14 different FAs from the standpoint of FA FGCs generation.

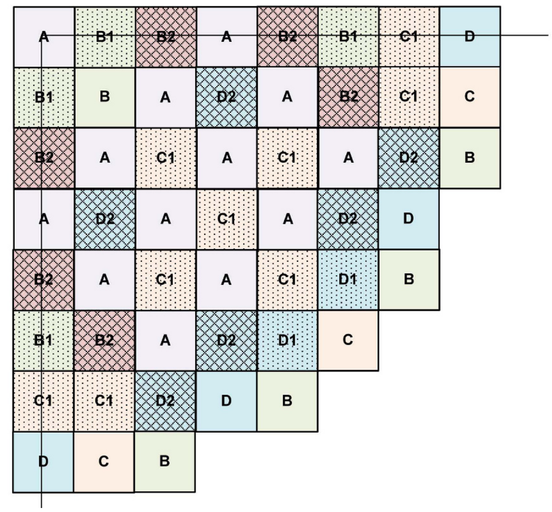

Fig. 2. The FA Loading Pattern for the Initial Core of YGN4

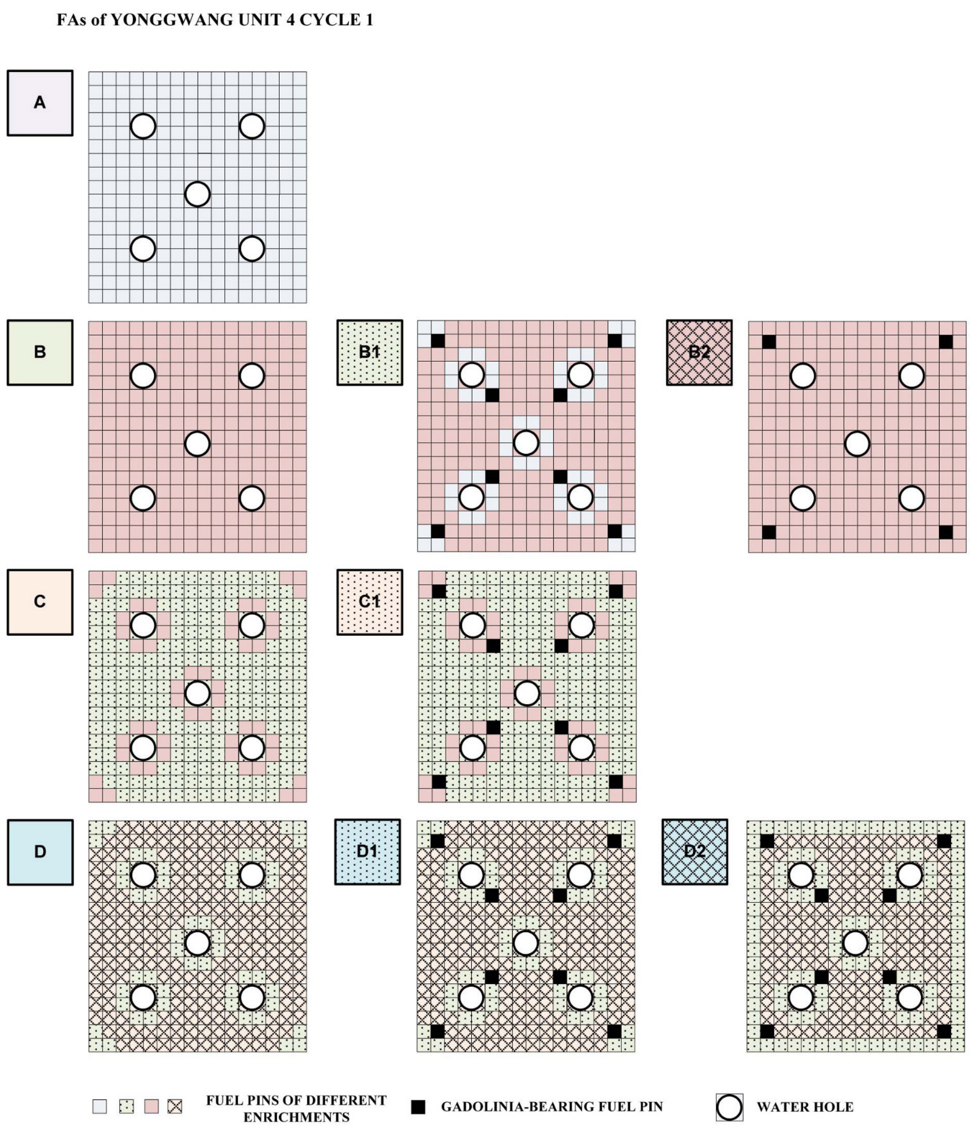

Fig. 3. Configuration of FAs for YGN4

3.1 Few Group Constants Generation and Estimation of their Uncertainties for YGN4 FA

The FGCs generation and uncertainty analysis module of McCARD was used to generate homogenized two group constants of 14 FAs and their uncertainties. The nuclear cross section ($\sigma_{\alpha' g' i'}$) and the covariance (cov[$\sigma_{\alpha'' g'' i''}$, $\sigma_{\alpha''' g''' i'''}$]) data inputted into the McCARD calculations for the two group constants and their uncertainties were obtained from the ENDF/B-VII.1 library. The covariance data files of only the major uranium isotopes U-235 and U-238 were used. Tables 2 shows the homogenized two group constants of FA type A0 and their percentile relative errors (% RE). The %RE is defined as $100 \times \sigma[X]/X$ in which X and $\sigma[X]$ denote any member of the two group constants and its standard deviation (SD), respectively. Table 2 contains contributions of the statistical and U-235 and U-238 cross section uncertainties to % RE of the individual two group constants. As mentioned in the introduction, the MC computations for generation of the two group constants of FAs are made on the basis of 200 active cycles with 10,000 neutron histories per cycle. It is noted that the statistical contribution to % RE of the two group constants under this condition of the MC calculations is negligibly smaller than the contribution of the cross section uncertainties of the two uranium isotopes. For example, Table 2 shows the maximum % RE value is 2.20% at fast group diffusion constant (D_1). Though not explicitly listed in Table 2, the statistical uncertainty contribution to % RE of D_1 is only 0.01%. Table 3 shows the covariance matrix of FGCs between two FA types, A0 and B0. The correlation coefficients between the two group constants of FA type A0 and those of FA type B0, corr[$\Sigma^{A0}_{\alpha,G}$, $\Sigma^{B0}_{\alpha',G'}$], appear to be similar to those between the same types, corr[$\Sigma^{A0}_{\alpha,G}$, $\Sigma^{A0}_{\alpha',G'}$] and corr[$\Sigma^{B0}_{\alpha,G}$, $\Sigma^{B0}_{\alpha',G'}$].

Table 2. Homogenized Two Group Cross Sections of FA Type B0 and Their Percentile Relative Errors (% RE)

Two Group Constants	$\Sigma_{\alpha,G}$, cm^{-1}	% RE[a)]
$\Sigma_{a,1}$	8.106×10^{-3}	1.15
$\Sigma_{a,2}$	6.109×10^{-2}	0.47
$\nu\Sigma_{f,1}$	4.941×10^{-3}	1.17
$\nu\Sigma_{f,2}$	8.972×10^{-2}	0.87
$\Sigma_{s,1\rightarrow 2}$	1.762×10^{-2}	0.38
D_1	1.440×10^{0}	2.20
D_2	4.374×10^{-1}	0.65

a) $\%RE = 100 \times \sigma(\Sigma_{\alpha,G}) / \Sigma_{\alpha,G}$

3.2 Uncertainties of Nuclear Design Parameters of BOL Core of YGN4

Figure 4 shows a distribution of k_{eff} from a total of 10,000 MASTER computations with 10,000 randomly sampled two group constants sets. Table 4 presents the mean values of k_{eff} and their uncertainties that were obtained with and without consideration of the correlation between the two group constants of different FA types. k_{eff} is 1.7803 ±0.00927 with the correlation taken into account, while it

Table 3. Correlation Coefficients Matrix of FGCs between FA Type A0 and FA Type B0

		A0 FA						B0 FA					
		$\Sigma_{a,1}$	$\Sigma_{f,1}$	$\nu\Sigma_{f,1}$	$\Sigma_{a,2}$	$\Sigma_{f,2}$	$\nu\Sigma_{f,2}$	$\Sigma_{a,1}$	$\Sigma_{f,1}$	$\nu\Sigma_{f,1}$	$\Sigma_{a,2}$	$\Sigma_{f,2}$	$\nu\Sigma_{f,2}$
A0 FA	$\Sigma_{a,1}$	1.00	-0.54	-0.49	0.46	-0.26	-0.04	0.95	-0.48	-0.41	0.39	-0.38	-0.09
	$\Sigma_{f,1}$	-0.54	1.00	0.90	-0.07	0.21	0.03	-0.42	1.00	0.86	-0.02	0.36	0.08
	$\nu\Sigma_{f,1}$	-0.49	0.90	1.00	-0.06	0.26	0.11	-0.38	0.93	1.00	-0.01	0.37	0.15
	$\Sigma_{a,2}$	0.46	-0.07	-0.06	1.00	0.31	0.13	0.46	-0.07	-0.05	0.99	0.06	0.09
	$\Sigma_{f,2}$	-0.26	0.21	0.26	0.31	1.00	0.90	-0.41	0.44	0.50	0.37	0.86	0.89
	$\nu\Sigma_{f,2}$	-0.04	0.03	0.11	0.13	0.90	1.00	-0.07	0.07	0.22	0.20	0.63	0.95
B0 FA	$\Sigma_{a,1}$	0.95	-0.42	-0.38	0.46	-0.41	-0.07	1.00	-0.33	-0.29	0.41	-0.43	-0.11
	$\Sigma_{f,1}$	-0.48	1.00	0.93	-0.07	0.44	0.07	-0.33	1.00	0.84	0.01	0.35	0.07
	$\nu\Sigma_{f,1}$	-0.41	0.86	1.00	-0.05	0.50	0.22	-0.29	0.84	1.00	0.02	0.43	0.23
	$\Sigma_{a,2}$	0.39	-0.02	-0.01	0.99	0.37	0.20	0.41	0.01	0.02	1.00	0.15	0.16
	$\Sigma_{f,2}$	-0.38	0.36	0.37	0.06	0.86	0.63	-0.43	0.35	0.43	0.15	1.00	0.89
	$\nu\Sigma_{f,2}$	-0.09	0.08	0.15	0.09	0.89	0.95	-0.11	0.07	0.23	0.16	0.89	1.00

Table 4. Uncertainties of k_{eff} for YGN4

Case		k_{eff} Mean Value	k_{eff} Standard deviation
2-Step McCARD/MASTER core analysis	No correlation between FAs	1.07812	0.00370
	Correlation between FAs	1.07803	0.00927
S/U Analysis by McCARD		1.07317	0.00830

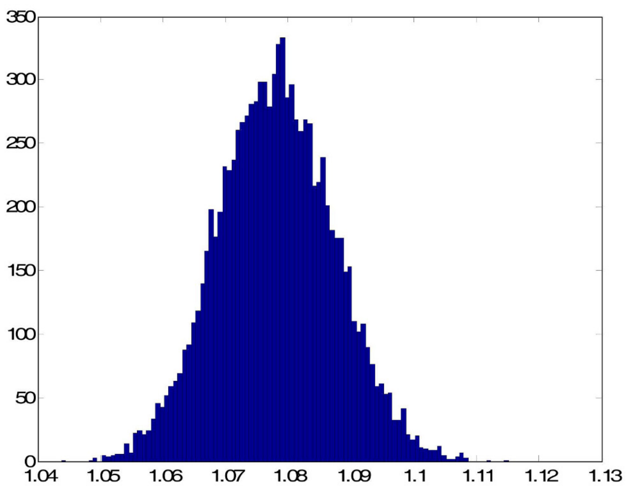

Fig. 4. Distribution of k_{eff} from 10,000 Samples

FA					B0	C0	D0
P_{McCARD}					0.553	0.863	1.028
RSD(%)					2.4	2.5	2.7
			B0	D0	D2	C1	C1
			0.553	0.998	1.103	1.169	1.219
			2.4	2.3	1.8	1.8	1.8
		C0	D1	D2	A0	B2	B1
		0.647	1.054	1.216	0.837	1.225	1.064
		2.5	2.1	1.5	0.6	0.6	0.7
	B0	D1	C1	A0	C1	A0	B2
	0.553	1.054	1.118	0.805	1.166	0.854	1.230
	2.4	2.1	1.3	0.6	0.8	1.2	1.2
	D0	D2	A0	C1	A0	D2	A0
	0.998	1.216	0.805	1.131	0.814	1.300	0.875
	2.3	1.5	0.6	1.1	1.8	2.1	2.4
B0	D2	A0	C1	A0	C1	A0	B2
0.553	1.103	0.837	1.166	0.814	1.202	0.892	1.273
2.4	1.8	0.6	0.8	1.8	2.5	3.1	3.1
C0	C1	B2	A0	D2	A0	B0	B1
0.863	1.169	1.225	0.854	1.300	0.892	1.335	1.100
2.5	1.8	0.6	1.2	2.1	3.1	3.5	3.7
D0	C1	B1	B2	A0	B2	B1	A0
1.028	1.219	1.064	1.230	0.875	1.273	1.100	0.879
2.7	1.8	0.7	1.2	2.4	3.1	3.7	4.1

FA = Type of Fuel Assembly
P_{McCARD} = mean value of McCARD/MASTER power
RSD(%) = Relative Stadard Deviation of McCARD/MASTER power

Fig. 5. Assembly-wise Power Distribution in the Initial Core of YGN4 and its Uncertainties

is 1.07812±0.00370 without. Thus the correlation effect of the two group constants of different FA types on k_{eff} is estimated to be roughly 550 pcm. In order to compare these results from the DSM by the 2-step McCARD/MASTER system with by direct MC calculations, the uncertainty of k_{eff} was estimated by using the sensitivities of k_{eff} with respect to nuclear cross section ($\sigma_{\alpha',g',i'}$) from the direct whole core McCARD calculation and the covariance ($\mathrm{cov}[\sigma_{\alpha'',g'',i''}, \sigma_{\alpha''',g''',i'''}]$) data [13, 14]. In the direct whole core McCARD calculation, the uncertainty analysis for k_{eff} was performed with 1,000 active cycles of 10,000 neutron histories per cycle with the ENDF/B-VII.1 covariance data. The last row of Table 4 shows the results of the direct McCARD calculations for k_{eff} and its uncertainty. It is noted that the uncertainties of k_{eff} from the whole core McCARD calculation and the direct sampling calculation by the two-step McCARD/MASTER system are roughly similar.

Figure 5 shows the uncertainties of normalized assembly-wise power distribution of the YGN4 core. The maximum relative standard deviation (RSD) is 4.1% for FA type A0 at the center of the core. Fig. 6 shows the axial power distribution and its uncertainties of YGN4 core. The maximum RSD is 0.71 % at the core boundary while the minimum RSD is 0.01% at about 85cm from the bottom of the core.

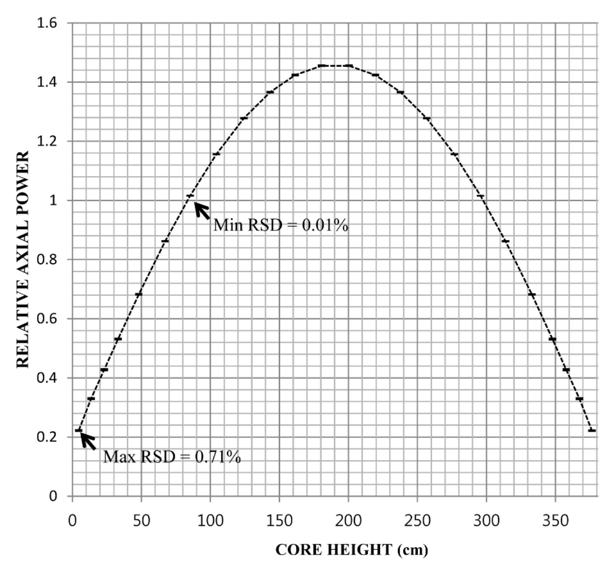

Fig. 6. Axial Power Distribution in the Initial core of YGN4 and its Uncertainties

Table 5. Uncertainties of Peak Values for YGN4

Case	Three-dimensional pin peaking factor (F_q)	Integrated radial pin peaking factor (F_r)
Design Limit or Requirement	< 2.58	< 1.55
Mean value and SD	1.98±0.06	1.36±0.04

Table 5 shows the peak values and their uncertainties. The three dimensional pin peaking factor, F_q, is 1.98±0.06, while the integrated radial pin peaking factor, F_r, is 1.36±0.04. It is noted that the estimated power peaking factors including their uncertainties meet the design limits or requirements.

4. CONCLUSIONS

In this paper we adopted the DSM as a way to quantify uncertainties of nuclear design parameters of reactor cores by the McCARD/MASTER two-step neutronics design system. One may adopt the so-called sensitivity and analysis (S/U) method or the perturbation-theory based method as an alternative way to do so. As in the case of the DSM, however, one must recognize that the S/U method also requires estimating the covariances between the FGCs of all the possible pairings of FAs comprising the core. Therefore, one of the major issues in the two approaches is how to estimate the required covariances between the uncertain FGCs of FAs. Because of this, the extended application of the uncertainty propagation analysis method [2] designed originally for calculating the uncertainties of the FA homogenized FGCs to estimation of the covariances between the FGCs of any pair of FAs offers a worthy and useful way to quantify uncertainty of the two-step nuclear core design analyses of current practice.

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