Multi-Scale Edge Detection Based on Triangulation Interpolation 2-EMD

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Abstract

In order to improve the speed of two-dimensional empirical mode decomposition (2-D EMD)’s procedure and improve the quality of edges extract from intrinsic mode function (IMF) image, an algorithm based on triangulation interpolation two-dimensional empirical mode decomposition to detect image edges at multiple scales is proposed. This algorithm is realized using the faster interpolation method of Delaunay triangulation and cubic spline functions for surface interpolation, then extract edges from the first IMF image that was processed by multi-scale wavelet decomposition. Through the simulation experiment, results demonstrate that this method can not only detect image edge precisely, but also effectively restrain noise. Finally, the proposed algorithm is confirmed feasible and valid.

Keywords: 2-D EMD; triangulation interpolation; multi-scale analysis; edge detection

1. Abstract

Edge is the most basic characteristics of the target image, edge detection is one of the key technologies in image processing, which played an important role in image detection, targeting, image recognition and other areas. In image processing, classic edge detection operators including Sobel operator, Roberts operator, Prewitt operator, Gauss - Laplace operator (Laplacian of Gaussian) and so on. Since these marginal operators use the first-order derivative or second-order derivative directional variation of the near around edge, it is extremely sensitive to noise. Wavelet analysis were widely used in image processing, this method has multi-scale, multi-resolution features, but the choice of wavelet basis is important, limited length of wavelet basis functions caused the energy leakage, which lead to the signal energy-frequency-time distribution is difficult to quantitative given, and the wavelet basis function...
does not have the self-adaptive, that is to a big difference of non-stationary signals, wavelet analysis is weak in the local.

Empirical mode decomposition (EMD) method \(^2\), can decomposed one-dimensional signal into a variety of internal oscillation modes, called intrinsic mode functions (Intrinsic Mode Function, IMF), and can calculate the meaningful diversity instantaneous frequency. In fact empirical mode decomposition is an adaptive wavelet transform, it is a local signal decomposition method and stronger than the wavelet transform in frequency domain and time domain. In recent years, with the EMD widely used in signal processing, particular in 2003 French scholar J.C.Nunes \(^5\) extended one-dimensional EMD to two-dimensional, some foreign scholars have conducted the exploration of two-dimensional EMD applied in the field of image processing, and obtain some achievements in image enhancement \(^4\), texture analysis \(^5\), image compression \(^6-7\), and image fusion \(^8\) and so on. There have two reasons apply EMD for edge detection: first, the advantages of using this technology: empirical mode decomposition is a completely data-driven decomposition method and does not require any preset filter or wavelet function; Second, we can easily extend it to two-dimensional space. Reference \(^9\) obtained the IMF images through radial basis function (RBFs) interpolation of the two-dimensional EMD decomposition method, because image is a two-dimensional signal and have large amount of data, so RBF need to consume a lot of time in constructed and solved the linear equations. From the speed of whole process of two-dimensional EMD, triangulation interpolation faster over hundreds of times than radial basis function interpolation, the former is far superior to the latter. For the high-frequency image with more extreme points, interpolation effects between them are similar, so can consider triangulation interpolation \(^10\). Reference \(^11\) used threshold to detect the IMF image that decomposed by two-dimensional EMD, result edge image lost some of the edge detail.

From above analysis, an algorithm based on triangulation and cubic spline interpolation of two-dimensional EMD combined with wavelet transform is proposed, which improved the speed of two-dimensional EMD process, and then extract edges from the first IMF image through multi-scale wavelet decomposition. Finally, the proposed algorithm is confirmed feasible and valid.

2. Two-dimensional EMD algorithm based on triangulation interpolation

EMD is an adaptive signal decomposition method, the principle is a complex signal can be decomposed a series of limited empirical mode function (IMF) by filtering, each empirical mode function independent of each other, and include the different frequencies and different space local features of the original signal. EMD decomposition of the empirical mode functions must satisfy two conditions: \(1\) in the entire data set, the number of extreme points and zero points must be equal or differ at most no more than 1; \(2\) at any point, the average formed by the local maxima of the envelope and the local minimum of the envelope is zero.

2.1 Two-dimensional EMD algorithm

Extend one-dimensional EMD to two-dimensional and apply to multi-scale decomposition of the image, image details can be extracted through the screening process. For one image signal \(I(m,n)\), process of two-dimensional EMD as follows:

(1)External initialization, \(I_{res,1}(x,y) = I(m,n)\), input image signal is \(I_{res,1}(x,y)\), for formula \(I_{res,k}(x,y), (l=1\cdots L; k=1\cdots K)\), \(l\) is indicate the \(l\) IMF which selected from the images; \(k\) is indicate the \(k\) time screening; \(m,n\) is the size of the image.
(2) Calculate maximum and minimum points for \( I_{res_{l,k}} (x,y) \), \((l = 1 \ldots L; k = 1 \ldots K)\).

(3) Fit surface for maximum and minimum points respectively, the two-dimensional image envelopes were: \( Up_{l,k} (x,y) \) and \( Low_{l,k} (x,y) \).

(4) Determine the mean of upper and lower envelopes: \( m_{l,k} (x,y) = \left[ Up_{l,k} (x,y) + Low_{l,k} (x,y) \right]/2 \).

(5) Subtracted mean envelope from input image and obtain: \( H_{l,k} (x,y) = I_{res_{l,k}} (x,y) - m_{l,k} (x,y) \).

(6) If \( H_{l,k} (x,y) \) meet the definition conditions of IMF, then \( H_{l,k} (x,y) \) is the \( l \) IMF was screened, verify if it meet the termination condition of screening:

\[
SD = \sum_{x,y} \sum_{i,j} \left[ I_{res_{i,j-1}} (x,y) - I_{res_{i,j}} (x,y) \right]^2 / \left[ I_{res_{i,j-1}} (x,y) \right]^2
\]

(1)

If not, let \( I_{res_{l,k+1}} (x,y) = H_{l,k} (x,y) \), and go to step (2), until it meet the termination condition, then \( k = K \), obtain the IMF \( inf_{i} (x,y) = H_{l,k} (x,y) \).

(7) Let \( I_{r} (x,y) = I_{res_{i}} (x,y) - inf_{i} (x,y) \), \( I_{r} (x,y) \) will be as a new decomposed image signal, so \( I_{res_{i+1}} (x,y) = I_{r} (x,y) \), \( I_{r} (x,y) \) represents the residual item.

(8) Repeat steps (2)-(7) until the residual item less than two extreme points or the number of decomposition meet the requirements of the IMF, and then end the decomposition process. At this point, the image can be expressed as:

\[
I(m,n) = \sum_{i=1}^{L} inf_{i} (m,n) + I_{r} (m,n)
\]

(2)

2.2 Delaunay triangulation and cubic spline interpolation algorithm

After found the extreme points, needed use two-dimensional interpolation to create the upper and lower envelopes. In order to improve the speed of two-dimensional empirical mode decomposition procedure, in this paper an algorithm Delaunay triangulation and cubic spline interpolation was adopt to construct the envelope for maximum and minimum points. This algorithm constructs an initial triangle \( T \) first, which contains all nodes that in the point set \( V \), considered the insert point \( v \), because all of the points \( v_{1}, v_{2}, \ldots, v_{N} \) were inserted before had composed \( DT(v_{1}, v_{2}, \ldots, v_{N}) \) is already a Delaunay triangulation, so just need to consider the changes caused by the insert point \( v \), make adjustments let \( DT(v_{1}, v_{2}, \ldots, v_{N}) \cup v \) to be a new Delaunay triangulation \( DT(v_{1}, v_{2}, \ldots, v_{N}) \) . For the set of points \( V = \{v_{1}, v_{2}, \ldots, v_{N}\} \), \( i = 1, 2, \ldots, N \), Delaunay triangulation algorithm as follows:

(1) Select a large triangle \( v_{1} - v_{2} - v_{3} \) which contains \( V \);

(2) Select a point \( v \) from \( V \) and delete it, \( v \) will fall into a triangle (or a triangle edge);

(3) Find all the circumcircle of the triangle contains \( v \), and then form a convex hull;

(4) Delete all the edges of the convex hull;

(5) Connected \( v \) with all vertices of the convex hull;

(6) Obtain the current Delaunay triangulation;

(7) If \( V \neq \emptyset \), then go to step (2); otherwise end.

Down Delaunay triangulation to the set of discrete points of space (the vertices of the triangle were the local extreme points of image), then can use cubic spline function interpolation to calculate the
envelope of the extreme points of image.

Down Delaunay triangulation and two-dimensional EMD decomposition with cubic spline interpolation to $256 \times 256$ Lena image, obtain four IMF images and residual image, results shown as figure1.

Figure1. 2-EMD of image Lena: (a) original image, (b) IMF1, (c) IMF2, (d) IMF3, (e) IMF4, (f) residual

Figure1. 2-EMD of image Lena

3. Multi-scale edge of IMF image

The basic idea of detect edge is find a local maximum gradient magnitude pixel in one image. From the nature of two-dimensional EMD decomposition, we known through two-dimensional EMD decomposition, image and noise will be distributed to the IMF sub-images according to the instantaneous frequency from high to low, edge information of the IMF sub-images will decreasing with scale increasing. As figure1 (b) shows, IMF1 contains a wealth of edge information. Therefore, in the actual edge detection, we only need to extract edges from IMF1, which will greatly reduce the time consumed. In order to improve the contrast of the image edge features, and improve the quality of edge image that extract from IMF1, we can first do histogram equalization to the original image then calculate IMF1, at the end extract the edge image from IMF1 use multi-scale wavelet decomposition.

3.1 Steps of the algorithm

(1) Do histogram equalization to original image;
(2) Apply two-dimensional EMD decomposition, obtain IMF1 sub-image;
(3) Select the decomposition scale $J = 3$, down dyadic wavelet transform to IMF1, find the modulo values and angle pieces of each scale and detect wavelet coefficients modulus maxima point of various scales along the angular direction, obtain the edge image $P_j f(x, y)$ in each scale;
(4) Use adaptive threshold remove false edges;
(5) For each edge pixel at scale $j$, search the $3 \times 3$ matching region which may be edge pixel of the image at scale $j - 1$, if the points appear in the matching region and all of them will be marked as candidate edge point, so can get the candidate edge image $C_j^{-1} (x, y)$ at scale $j - 1$, and non-candidate edge points will be marked as zero;
(6) Link the non-zero pixel which similar to modulus and phase angle in image $C^{j-1}(x, y)$ at scale $j-1$, and delete the isolated chain whose length less than threshold $T^{j-1}$, obtain single-pixel-wide edge image $E^{j-1}(x, y)$;

(7) $j = j - 1$; if $j > 1$ go to step (5); otherwise take the next steps;

(8) While $j = 1$, $E^1(x, y)$ will be the final edge image.

3.2 Adaptive threshold [12]

In order to remove the noise and false edges caused by uneven grayscale, must set a threshold. If use a same threshold to entire image, the faint edge of the image will be removed while remove the noise, which will affect the detect results. So threshold value determined as follow: use 32 × 32 windows, scanned possible edges of the image, calculated threshold through wavelet coefficients in the window, formula as follow:

$$T = T_0 + a_0 \times \sum_{i,j} C_{i,j}$$  \hspace{1cm} (3)

Where $T$ is the threshold, $T_0$ is the initial value, $C_{i,j}$ is the wavelet coefficients corresponding with current window, $a_0$ is a scaling factor, used to determine the impact level of the wavelet coefficients at current window, the value of $T_0$, $a_0$ can be adjusted according to actual situation, in this paper $T_0$ is 5, $a_0$ is 0.001.

4. Simulation results and analysis

Select a 256 × 256 Lena image for the experiment to verify this algorithm. In the experiment, used Canny operator, wavelet transform, two-dimensional EMD algorithm, algorithm of this article respectively, the results show in figure2. Figure2 (a) is original image, (b), (c), (d) are the detect results of figure2 (a) used Canny operator, wavelet transform, reference[11] method, (e) is result of this article method; Figure2 (f) is added Gaussian noise image, $\sigma^2 = 0.005$, (g), (h), (i) are the detect results of figure2 (f) used Canny operator, wavelet transform, reference[11] method, (k) is the result of this article method.

Detect results indicate that Canny operator edge detection method can not extract some details, and vulnerable to noise. Edge detection based on wavelet transform is from the perspective of multi-resolution, but it is more complex at the wavelet basis function selected, and use different wavelet the decomposition results will be different, so the edge result is also inconsistent, in addition, to local signal wavelet analysis is weak, and easily lead to energy leakage, which will affect the edge detection results. In reference [11], through the threshold detection method, part of the edges will be missed, edge details are lack. This article’s algorithm not only good at noise suppressed, but also detected full edges of the image, and have a good processing to non-linear signal, so part of the texture detail features will more clearly, such as Lena image’s facial outline is more complete, and the head decoration is more realistic.
5 Conclusion

An algorithm based on triangulation interpolation two-dimensional empirical mode decomposition combined with wavelet transform to detect image edge is proposed. This algorithm contains two-dimensional EMD decomposition and edge detection two parts. Two-dimensional EMD used triangulation and cubic spline interpolation for the surface fitting which reduced the time computation, fastened the image decomposition. Use two-dimensional EMD extract different spatial frequency components or different spatial scales, then through the background and characterization of these object, we can focus on one or a few IMFs, rather than the entire image. This article down the wavelet transform on the first IMF image, not only can extract accurate image edges, but also inhibited most of noise. Clearly, two-dimensional empirical mode decomposition is provides us a new and promising parameter-free decomposition and image edge feature extraction method.

Thanks

The Foundation of National Natural Science (60872064)

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