Linear Algebra and its Applications 436 (2012) 237–242

Contents lists available at SciVerse ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

Bounded and unbounded polynomials and multilinear forms: Characterizing continuity ☆

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ARTICLE INFO

Article history:
Received 21 December 2010
Accepted 30 June 2011
Available online 27 July 2011

Submitted by R.A. Brualdi

AMS classification:
46E10
46G25

Keywords:
Lineability
Continuous polynomial
Non-continuous polynomial

ABSTRACT

In this paper we prove a characterization of continuity for polynomials on a normed space. Namely, we prove that a polynomial is continuous if and only if it maps compact sets into compact sets. We also provide a partial answer to the question as to whether a polynomial is continuous if and only if it transforms connected sets into connected sets. These results motivate the natural question as to how many non-continuous polynomials there are on an infinite dimensional normed space. A problem on the *lineability* of the sets of non-continuous polynomials and multilinear mappings on infinite dimensional normed spaces is answered.

1. Introduction and notation

It is well-known (see [16, Theorem 2]) that a mapping $f : \mathbb{R} \to \mathbb{R}$ is continuous if and only if it satisfies the following two conditions:

(1) f maps compact sets into compact sets.
(2) f maps connected sets into connected sets.

☆ J.L. Gámez-Merino, G.A. Muñoz-Fernández and J.B. Seoane-Sepúlveda were supported by the Spanish Ministry of Science and Innovation, grant MTM2009-07848. D. Pellegrino was supported by CNPq Grant 620108/2008-9 (Edital Casadinho).

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doi:10.1016/j.laa.2011.06.050

At the other end of the scale, it is possible to construct $2^{\mathfrak{c}}$-dimensional spaces of everywhere discontinuous functions in $\mathbb{R}^{\mathbb{R}}$ satisfying only one of the above conditions (see [11]). However, the same situation does not hold for the case of polynomials on a normed space. Actually, condition (1) characterizes the continuity of a polynomial on a normed space, which is proved in Section 2. As we will also see in Section 2, we study when condition (2) above characterizes continuity for polynomials on normed spaces, problem which will be solved partly.

Finally, Section 3 is devoted to the construction of linear spaces of maximal dimension of non-bounded polynomials between normed spaces.

For convenience we recall the basic definitions and standard results needed to discuss polynomials on normed spaces. A map $P : E \to F$ is an *n-homogeneous polynomial* if there is an n-linear mapping $L : E^n \to F$ for which $P(x) = L(x, \dots, x)$ for all $x \in E$. In this case it is convenient to write $P = \widehat{L}$. According to a well-known algebraic result, for every n-homogeneous polynomial $P : E \to F$ there exists a unique symmetric n-linear mapping $L : E^n \to F$ such that $P = \widehat{L}$. When that happens, L is called the polar of P.

We let $\mathcal{P}_a({}^nE; F)$, $\mathcal{L}_a({}^nE; F)$ and $\mathcal{L}_a^s({}^nE; F)$ denote respectively the linear spaces of all n-homogeneous polynomials from E into F, the n-linear mappings from E into F and the symmetric n-linear mappings from E into F. More generally, a map $P : E \to F$ is a *polynomial of degree at most n* if

$$P = P_0 + P_1 + \cdots + P_n,$$

where $P_k \in \mathcal{P}_a({}^kE; F)$ $(1 \leqslant k \leqslant n)$, and $P_0 : E \to F$ is a constant function. The polynomials of degree at most n between the normed spaces E and F are denoted by $\mathcal{P}_{n,a}(E; F)$.

Polynomials on a finite dimensional normed space are always continuous; however, the same thing does not happen for infinite dimensional normed spaces. Boundedness is a characteristic property of continuous polynomials on a normed space. In particular, $P \in \mathcal{P}_{n,a}(E; F)$ is continuous if and only if P is bounded on the unit ball of E (denoted by B_E). This is standard and particularly well-known for homogeneous polynomials (see for instance [10, Proposition 1.11]). For the non-homogeneous case, a complexification procedure lets us focus our attention on polynomials defined on a complex normed space. Let P be a polynomial of degree at most n on the complex normed space E. We define the homogenization of P by

$$Q(x, \lambda) = \begin{cases} \lambda^n P\left(\frac{x}{\lambda}\right) & \text{if } \lambda \neq 0, \\ 0 & \text{if } \lambda = 0, \end{cases}$$

for every $(x, \lambda) \in E \oplus \mathbb{C}$. It is a simple exercise to prove that Q is a homogeneous polynomial on $E \oplus \mathbb{C}$. Let $E \oplus_\infty \mathbb{C}$ stand for $E \oplus \mathbb{C}$ endowed with the norm $\|(x, \lambda)\|_\infty = \max\{\|x\|, |\lambda|\}$. Now if P is bounded on B_E, by the Maximum Modulus Principle

$$\begin{aligned} \sup\{\|Q(x, \lambda)\| : \|(x, \lambda)\|_\infty \leqslant 1\} &= \sup\left\{\left\|\lambda^n P\left(\frac{x}{\lambda}\right)\right\| : \|x\| \leqslant 1,\ |\lambda| \leqslant 1\right\} \\ &= \sup\left\{\left\|P\left(\frac{x}{\lambda}\right)\right\| : \|x\| \leqslant 1,\ |\lambda| = 1\right\} \\ &= \sup\left\{\|P(x)\| : \|x\| \leqslant 1\right\}. \end{aligned}$$

Hence Q is bounded on $E \oplus_\infty \mathbb{C}$, and therefore continuous. This implies that P is also continuous since P is a restriction of Q. Conversely, if P is continuous, Q is clearly continuous for all $(x, \lambda) \in E \otimes_\infty \mathbb{C}$ with $\lambda \neq 0$. Thus Q is continuous in $E \otimes_\infty \mathbb{C}$ (see again [10, Proposition 1.11]) and bounded in $\mathsf{B}_{E \otimes_\infty \mathbb{C}}$. Therefore P must be bounded too in B_E.

If $P : E \to F$ and $L : E^n \to F$ are, respectively, a continuous polynomial of degree at most n and a continuous n-linear mapping we define

$$\begin{aligned} &\|P\| = \sup\{\|P(x)\| :\ \|x\| \leqslant 1\}, \\ &\|L\| = \sup\{\|L(x_1, \dots, x_n)\| :\ \|x_1\| \leqslant 1, \dots, \|x_n\| \leqslant 1\}. \end{aligned}$$

We let $\mathcal{P}(^nE; F)$, $\mathcal{P}_n(E; F)$, $\mathcal{L}(^nE; F)$ and $\mathcal{L}^s(^nE; F)$ denote, respectively, the normed spaces of the continuous n-homogeneous polynomials from E into F, the continuous polynomials of degree at most n from E into F, the continuous n-linear mappings from E into F, and the continuous symmetric n-linear mappings from E into F.

In general the results on the continuity of scalar-valued polynomials and multilinear forms can be easily extended to vector-valued polynomials and multilinear mappings. For this reason we are working from now on with scalar-valued polynomials and multilinear forms. If $\mathbb{K}$ is the real or complex field we use the notations $\mathcal{P}(^nE)$, $\mathcal{P}_n(E)$, $\mathcal{L}(^nE)$ and $\mathcal{L}^s(^nE)$ in place of $\mathcal{P}(^nE; \mathbb{K})$, $\mathcal{P}_n(E; \mathbb{K})$, $\mathcal{L}(^nE; \mathbb{K})$, and $\mathcal{L}^s(^nE; \mathbb{K})$ respectively. All the results in this paper will be considered in a real setting.

2. A characterization of continuity for polynomials

In this section we will consider both conditions (1) and (2) given in the Introduction, in the frame of polynomials on normed spaces. Let us begin with proving that, actually, condition (1) characterizes the continuity of polynomials on any normed space.

Theorem 2.1. *If E is a normed space and P is a polynomial on E then P is continuous if and only it transforms compact sets into compact sets.*

Proof. All continuous functions between topological spaces map compact sets into compact sets, so we just need to prove that if P maps compact sets into compact sets, then P is continuous. Actually, we only need to show that all polynomials mapping compact sets in compact sets are continuous at 0. If we prove that and $x_0 \in E$ is arbitrary, then the polynomial defined by $Q(x) = P(x + x_0)$ for all $x \in E$ also maps compact sets into compact sets. Being Q continuous at 0, we would also have that P is continuous at x_0. Actually, a more general statement can be proved: a polynomial is continuous if and only if it is continuous at 0.

Let us prove then that P is continuous at 0. Let (x_k) be a convergent sequence in $E \setminus \{0\}$ to 0 such that $\lim_{k\to\infty} P(x_k)$ does not exist or it is not equal to $P(0)$. Since the set $C = \{x_k : k \in \mathbb{N}\} \cup \{0\}$ is compact and $P(C)$ is compact too by hypothesis, we can assume without loss of generality that $(P(x_k))$ converges to $a \neq P(0)$ and that $P(x_k) \neq P(0)$ for all $k \in \mathbb{N}$.

Observe that only one of the following statements can hold:

(1) $P(x_k) \neq a$ for infinitely many k's.
(2) $P(x_k) = a$ for all but a finite number of k's.

For the first case we consider a subsequence (y_k) of (x_k) so that $P(y_k) \neq a$ for all $k \in \mathbb{N}$. Then $C^* = \{y_k : k \in \mathbb{N}\} \cup \{0\}$ is compact but $P(C^*)$ is not even closed since it does not contain its limit point a.

For the second case we may assume that $P(x_k) = a$ for all $k \in \mathbb{N}$. Now suppose $P = P_n + P_{n-1} + \cdots + P_1 + P_0$, where $P_j \in \mathcal{P}_a(^jE)$ and P_0 is a constant function taking the value $P(0)$. Then for each $k \in \mathbb{N}$, $P_j(x_k)$ cannot vanish for every $j = 1, \ldots, n$ (otherwise $P(x_k) = P(0)$). Therefore the one variable polynomial defined by $p_k(\lambda) := P(\lambda x_k)$, for all $\lambda \in \mathbb{R}$, is not constant, and hence it takes infinitely many values on every interval. Using the continuity of the polynomial p_k one can construct a sequence $(\lambda_k) \subset (0, 1]$ such that for each $k \in \mathbb{N}$ we have

$$|P(\lambda_k x_k) - P(x_k)| = |p_k(\lambda_k) - p_k(1)| < \frac{1}{k}$$

and

$$P(\lambda_k x_k) \notin \{P(0), a\}.$$

Notice that

$$|P(\lambda_k x_k) - a| \leqslant |P(\lambda_k x_k) - P(x_k)| + |P(x_k) - a| \longrightarrow 0 \quad \text{as } k \to \infty.$$

Finally, by letting $y_k = \lambda_k x_k$, we have that $(y_k) \subset E \setminus \{0\}$, $P(y_k) \neq P(0)$, $\lim_{k\to\infty} y_k = 0$, $\lim_{k\to\infty} P(y_k) = a$, and $P(y_k) \neq a$ for every $k \in \mathbb{N}$. This leads us to a contradiction as in the first case. $\square$

After checking that condition (1) from the Introduction characterizes continuity, a natural question arises now:

Is $P \in \mathcal{P}_{n,a}(E)$ continuous if and only if for every connected set $C \in E$, $P(C)$ is also connected for every infinite dimensional normed space E?

Unfortunately, this general question seems much deeper than it looks at first sight, although we can prove it for the particular case of real homogeneous polynomials of degree 1 and 2, as we see next:

Proposition 2.2. *Let E be a real Banach space and $P \in \mathcal{P}(^nE)$ with $n = 1, 2$. Then P is continuous if and only if it transforms connected sets into connected sets.*

Proof. If P is continuous, it obviously transforms connected sets into connected sets. Now suppose P is not continuous. Then there exists a sequence of non null vectors $\{x_n\}$ such that $\lim_{k\to\infty} x_k = 0$ but $\lim_{k\to\infty} P(x_k) = \infty$. We can also choose the x_k's so that $\{P(x_k)\}$ is an increasing sequence and $P(x_1) > 0$.

Now consider the connected set $C = \left(\bigcup_{k=1}^{\infty}[x_k, x_{k+1}]\right) \cup \{0\}$, where $[x_k, x_{k+1}]$ is the segment with endpoints x_k and x_{k+1} for every $k \in \mathbb{N}$. If $n = 1$, by linearity $P([x_k, x_{k+1}]) = [P(x_k), P(x_{k+1})]$ for all $k \in \mathbb{N}$. Hence $P(C) = [P(x_1), \infty) \cup \{0\}$ and since $P(x_1) > 0$, $P(C)$ is not connected. Furthermore, if $n = 2$ and $L \in \mathcal{L}^s(^2E)$ is the polar of P, we can assume that $L(x_k, x_{k+1}) \geqslant 0$. Indeed, we just need to replace x_k by $-x_k$ if necessary. It is important to notice that $P(x_k) = P(-x_k)$. Since

$$\begin{aligned} P(\lambda x_n + (1-\lambda)x_{k+1}) &= \lambda^2 P(x_k) + 2\lambda(1-\lambda)L(x_k, x_{k+1}) + (1-\lambda)^2 P(x_{k+1}) \\ &\geqslant \lambda^2 P(x_k) + (1-\lambda)^2 P(x_{k+1}) \geqslant \left[\lambda^2 + (1-\lambda)^2\right] P(x_k) \\ &\geqslant P(x_k), \end{aligned}$$

for every $\lambda \in [0, 1]$, we have that $P([x_k, x_{k+1}]) \subset [P(x_k), \infty)$. This, together with the fact that $\lim_{k\to\infty} P(x_k) = \infty$ imply that $P(C) = [P(x_1), \infty) \cup \{0\}$. Finally, since $P(x_1) > 0$, $P(C)$ is not connected. $\square$

Conjecture 2.3. *It is our belief that condition (2) also characterizes continuity for arbitrary polynomials on any infinite dimensional normed space.*

Remark 2.4. Although we do not know the answer to the previous conjecture, we do know that if $L \in \mathcal{L}_a(^nE)$ transforms connected sets in E^n into connected sets, then it is continuous. Indeed, using Proposition 2.2 with $n = 1$, it is easy to see that L is separately continuous, and hence continuous.

3. Non-bounded multilinear mappings and polynomials

After learning the characterizations obtained in the previous section (Theorem 2.1 and Proposition 2.2), this section is devoted to the relatively new notion of *lineability*, which will tie the paper together. This notion of lineability has the following motivation: Take a function with some special or pathological property. Coming up with a concrete example of such a function can be a difficult task. Actually, it may seem that if one succeeds in finding one example of such a function, one might think that there cannot be too many functions of that kind. Probably one cannot even find infinite dimensional vector spaces of such functions. This is, however, exactly what has happened. The search for large algebraic structures of functions with pathological properties has lately become somewhat of a new trend in mathematics. Let us recall that a set M of functions satisfying some pathological property is said to be *lineable* if $M \cup \{0\}$ contains an infinite dimensional vector space. More specifically, we will say that

M is *μ-lineable* if $M \cup \{0\}$ contains a vector space of dimension μ, where μ is a cardinal number. We refer to the interested reader to [11,12,1–9,13–15] for recent advances in this theory.

If E is a normed space, in this section $\mathcal{NBL}(^nE)$, $\mathcal{NBL}^s(^nE)$, $\mathcal{NBP}(^nE)$ and $\mathcal{NBP}_n(E)$ represent, respectively, the set of non-bounded n-linear forms on E, the set of non-bounded symmetric n-linear forms on E, the set of non-bounded scalar-valued n-homogeneous polynomials on E and the set of non-bounded scalar-valued polynomials on E of degree at most n. Our results on the lineability of $\mathcal{NBL}(^nE)$, $\mathcal{NBL}^s(^nE)$, $\mathcal{NBP}(^nE)$ and $\mathcal{NBP}_n(E)$ rely on the lineability of the set of non-bounded scalar-valued functions defined on an infinite set I, denoted by $\mathcal{NBF}(I)$. The following set-theoretical lemma (see [3, Lemma 4.1]) will be needed for our main result in this section.

Lemma 3.1. *If $C_1, \ldots, C_m$ are m arbitrary, different, non-empty sets, then there exists $k \in \{1, \ldots, m\}$ such that for every $1 \leqslant j \leqslant m$ with $j \neq k$, we have that $C_k \setminus C_j \neq \varnothing$.*

Also, the next lemma (although of independent interest in itself) will be necessary.

Lemma 3.2. *If $I \subset \mathbb{R}$ is uncountable, then the set $\mathcal{NBF}(I)$ is $2^{\mathrm{card}(I)}$-lineable.*

Proof. For each non-void $C \subset I$ let $H_C \colon \mathbb{R} \times I^{\mathbb{N}} \to \mathbb{R}$ be defined by

$$H_C(x, x_1, \ldots, x_j, \ldots) = x \cdot \prod_{j=1}^{\infty} \chi_C(x_j).$$

If we fix a sequence $(x_n) \subset C$ then $H_C(x, x_1, \ldots, x_n, \ldots) = x$ for all $x \in \mathbb{R}$, and hence the H_C's are not bounded. Moreover, if $C_1, \ldots, C_m$ are m different subsets of I and $\sum_{k=1}^m \lambda_k H_{C_k}$ is a linear combination of the H_{C_k}'s ($1 \leqslant k \leqslant m$) with $\lambda_k \neq 0$ for all $k = 1, \ldots, m$ then, renaming the sets if necessary, from Lemma 3.1 it follows that for each $1 \leqslant j < m$ there exists $x_j \in C_m \setminus C_j$. Now let $v = (x, x_1, x_2, \ldots, x_{m-1}, x_{m-1}, \ldots) \in \mathbb{R} \times I^{\mathbb{N}}$ with $x \in \mathbb{R}$ arbitrary. Then

$$\sum_{k=1}^{m} \lambda_k H_{C_k}(v) = \sum_{k=1}^{m} \lambda_k \left[x \prod_{j=1}^{m-1} \chi_{C_k}(x_j) \right] = \lambda_m x,$$

for all $x \in \mathbb{R}$, which shows that $\sum_{k=1}^m \lambda_k H_{C_k}$ is not bounded.

Now if $\sum_{k=1}^m \lambda_k H_{C_k} \equiv 0$ and we set $v = (1, x_1, x_2, \ldots, x_{m-1}, x_{m-1}, \ldots) \in \mathbb{R} \times I^{\mathbb{N}}$, then

$$0 = \sum_{k=1}^{m} \lambda_k H_{C_k}(v) = \sum_{k=1}^{m} \lambda_k \left[\prod_{j=1}^{m-1} \chi_{C_k}(x_j) \right] = \lambda_m,$$

which contradicts the fact that $\lambda_k \neq 0$ for all $k = 1, \ldots, m$. Finally, since I is uncountable we can find a bijection $\Phi : I \leftrightarrow \mathbb{R} \times I^{\mathbb{N}}$. Then the set $\{H_C \circ \Phi : C \subset I\}$ has unbounded non trivial linear combinations and it is linearly independent with cardinality $2^{\mathrm{card}(I)}$, which concludes the proof. $\square$

We are now ready to state and prove the main (and general) lineability result in this section:

Theorem 3.3. *If $n \in \mathbb{N}$ and E is a normed space of infinite dimension λ then the sets $\mathcal{NBL}(^nE)$, $\mathcal{NBL}^s(^nE)$, $\mathcal{NBP}(^nE)$ and $\mathcal{NBP}_n(E)$ are 2^{λ}-lineable.*

Proof. In order to prove the lineability of $\mathcal{NBL}^s(^nE)$, let $\{e_i : i \in I\}$ be a normalized basis for E with $\mathrm{card}(I) = \lambda$. By Lemma 3.2 there exist 2^{λ} linearly independent mappings $\{f_j : j \in J\}$ (with $\mathrm{card}(J) = 2^{\lambda}$) generating a linear space of unbounded real valued functions on I. For each $j \in J$ we define a multilinear mapping $L_j : {}^nE \to \mathbb{R}$ by

$$L_j(e_{i_1}, \ldots, e_{i_n}) = f_j(i_1) + \cdots + f_j(i_n),$$

for all choices of $(i_1, \ldots, i_n) \in I^n$ and consider the set $\{\bar{L}_j : j \in J\}$, where $\bar{L}_j$ is the symmetrization of L_j for all $j \in J$. If $\sum_{k=1}^{m} \lambda_k \bar{L}_{j_k} \equiv 0$ with $\lambda_1, \ldots, \lambda_m \in \mathbb{R}$, then for every $i \in I$ we have

$$n\left(\sum_{k=1}^{m} \lambda_k f_{j_k}(i)\right) = \sum_{k=1}^{m} \lambda_k L_{j_k}(e_i, \overset{(n)}{\ldots}, e_i) = \sum_{k=1}^{m} \lambda_k \bar{L}_{j_k}(e_i, \overset{(n)}{\ldots}, e_i) = 0,$$

from which $\sum_{k=1}^{m} \lambda_k f_{j_k}(i) = 0$ for every $i \in I$. In other words $\sum_{k=1}^{m} \lambda_k f_{j_k} \equiv 0$ and therefore $\lambda_k = 0$ for all $1 \leqslant k \leqslant m$ since the f_{j_k}'s are linearly independent.

On the other hand, if $\lambda_k \neq 0$ for $k = 1, \ldots, m$, then

$$\left|\sum_{k=1}^{m} \lambda_k \bar{L}_{j_k}(e_i, \overset{(n)}{\ldots}, e_i)\right| = n\left|\sum_{k=1}^{m} \lambda_k f_{j_k}(i)\right|.$$

Hence $\sum_{k=1}^{m} \lambda_k \bar{L}_{j_k}$ is not bounded since $\sum_{k=1}^{m} \lambda_k f_{j_k}$ is not bounded either. This shows that $\mathcal{NBL}^s(^mE)$ is 2^λ-lineable and therefore $\mathcal{NBL}(^mE)$ is also 2^λ-lineable since $\mathcal{NBL}^s(^mE) \subset \mathcal{NBL}(^mE)$.

As another corollary to the fact that $\mathcal{NBL}^s(^nE)$ is 2^λ-lineable, we deduce that $\mathcal{NBP}(^nE)$ is also 2^λ-lineable since the algebraic spaces $\mathcal{L}_a^s(^nE)$ and $\mathcal{P}_a(^nE)$ are isomorphic.

Finally, $\mathcal{NBP}_n(E)$ is 2^λ-lineable since $\mathcal{NBP}(^nE) \subset \mathcal{NBP}_n(E)$ and we have just seen that $\mathcal{NBP}(^nE)$ is 2^λ-lineable. $\square$

Acknowledgement

The authors express their gratitude to Prof. D. Azagra for fruitful conversations and for pointing out Proposition 2.2. The authors also wish to thank the Referee, whose insightful remarks improved the text.

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