Mixed convection of ferrofluids in a lid driven cavity with two rotating cylinders

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A B S T R A C T

Mixed convection of ferrofluid filled lid driven cavity in the presence of two rotating cylinders were numerically investigated by using the finite element method. The cavity is heated from below, cooled from driven wall and rotating cylinder surfaces and side vertical walls of the cavity are assumed to be adiabatic. A magnetic dipole source is placed below the bottom wall of the cavity. The study is performed for various values of Reynolds numbers (100 ≤ Re ≤ 1000), angular rotational speed of the cylinders (−400 ≤ U ≤ 400), magnetic dipole strengths (0 ≤ γ ≤ 500), angular velocity ratios of the cylinders (0.25 ≤ Di/Dj ≤ 4) and diameter ratios of the cylinders (0.5 ≤ Di/Dj ≤ 2). It is observed that flow patterns and thermal transport within the cavity are affected by variation in Reynolds number and magnetic dipole strength. The results of this investigation revealed that cylinder angular velocities, ratio of the angular velocities and diameter ratios have profound effect on heat transfer enhancement within the cavity. Averaged heat transfer enhancements of 181.5 % is achieved for clockwise rotation of the cylinder at Re = 400 compared to motionless cylinder case. Increasing the angular velocity ratio from Ωj/Ωi = 0.25 to Ωj/Ωi = 4 brings about 91.7 % of heat transfer enhancement.

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1. Introduction

The interaction between the shear driven flow and natural convection effect is quite complex and has many engineering applications such as solidification, food processing, MEMs, nuclear reactors, coating etc. Mixed convection in a lid driven cavity is a benchmark problem for the study of interaction between the shear driven flow and free convection. In order to control the heat transfer and fluid flow within the cavity, active and passive methods were used. Some attempts can be mentioned as a) using magnetic field or electrical field b) using an obstruction within the cavity [1–11] c) using surface corrugation and its geometrical parameters [12,13]. The methods can be combined to have a large number of control parameters [14,15].

Recently, due to its importance in many industrial applications such as MEMs, coolers of nuclear reactors, purification of molten metals magnetic field interaction with fluid flow and heat transfer were extensively studied. An external magnetic field can be used to control the convection inside the cavity [16–18]. Stability of ferromagnetic fluid for a fluid layer heated from below and subjected to a uniform vertical magnetic field was studied by Finlayson [19], Kefayati [20] numerically studied the ferrofluid natural convection flow in a cavity with linearly temperature distribution using Lattice Boltzmann method. He observed that heat transfer decreases as the nanoscale ferromagnetic particle volume fraction increases. Rahman et al. [4] numerically investigated the conjugate effect of Joule heating and magnetic force for an obstructed lid-driven cavity saturated with an electrically conducting fluid using finite element method. Sheikholeslami and Ganji [21] numerically investigated the influence of an external magnetic field on ferrofluid flow and heat transfer in a semi annulus enclosure with sinusoidal hot wall using Control Volume based Finite Element Method. They showed that for low Rayleigh number, heat transfer enhances as the Hartmann number increases and magnetic number decreases. Al-Salem et al. [22] studied the effects of moving lid direction on MHD mixed convection in a cavity with linearly heated bottom wall using finite volume method. They observed that direction of lid is more effective on heat transfer and fluid flow in the cavity and heat transfer is decreased with increasing of magnetic...
field parameter. Oztop et al. [23] studied the mixed convection with a magnetic field in a lid-driven cavity heated by a corner heater. They showed that heat transfer deteriorates as the Hartmann number increases and magnetic field can be used to control heat transfer and fluid flow.

An obstruction can be used to control the heat transfer and fluid flow within a cavity. Khanafer and Aithal [2] numerically studied the mixed convection in a lid-driven cavity with a circular object inside by using finite element method. Their results showed that the Richardson number, cylinder diameter and the location of the cylinder have impact on the transport phenomena within the cavity. Islam et al. [7] have inserted an isothermally heated square blockage inside a square cavity and the effects of various different blockage sizes, concentric and eccentric placement of the blockage inside the cavity have been numerically investigated using finite volume method. The investigation of mixed or natural convection in enclosures with rotating cylinders was conducted by several researchers [24–28]. Hussain and Hussein [28] numerically analyzed the mixed convection in a cavity having a rotating cylinder with finite volume method. Their results showed that cylinder locations have important effects on convection within the cavity. Costa and Raimundo [29] numerically studied the mixed convection in a differentially heated square enclosure with an active rotating circular cylinder. The effects of the radius, rotation velocity and thermal conductivity and thermal capacity of the cylinder on the mixed convection was studied. Recently, Selimefendigil and Oztop [15] studied the convection in a vented cavity with a rotating cylinder. They observed that the length and size of the recirculation zones can be controlled with magnetic dipole strength and angular rotational speed of the cylinder.

In the present study, a lid driven square cavity heated from below with two rotating cylinders under the influence of a magnetic source were numerically simulated. The present study aims at investigating the effects of Reynolds number, angular speed of the cylinders, the ratio of the angular velocities, the ratio of the cylinder diameters and the strength of the magnetic dipole source on the fluid flow and heat transfer characteristics of the lid driven cavity heated from below. Using the ratio of the angular velocities and the ratio of the cylinder diameters as additional control parameters for mixed convection along with magnetic dipole source is the

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**Nomenclature**

- \( a, b \): location of the magnetic dipole
- \( B \): magnetic induction
- \( h \): local heat transfer coefficient, \( (\text{W/m}^2 \text{K}) \)
- \( H \): magnetic field
- \( k \): thermal conductivity, \( (\text{W/m K}) \)
- \( L \): length of the enclosure, (m)
- \( \text{Mn} \): Magnetic number, \( \mu_0 H^2 / \rho_0 \nu^2 \)
- \( n \): unit normal vector
- \( Nu \): local Nusselt number, \( hL/k \)
- \( p \): pressure, (Pa)
- \( Pr \): Prandtl number, \( \nu \lambda / \kappa \)
- \( Re \): Reynolds number, \( u_0 L / \nu \)
- \( T \): temperature, (K)
- \( u, v \): x-y velocity components, (m/s)
- \( x, y \): cartesian coordinates, (m)

**Greek characters**

- \( \alpha \): thermal diffusivity, \( (\text{m}^2/\text{s}) \)
- \( \beta \): non-dimensional temperature, \( T - T_c / T_h - T \)
- \( \nu \): kinematic viscosity, \( (\text{m}^2/\text{s}) \)
- \( \rho \): density of the fluid, \( (\text{kg/m}^3) \)
- \( \chi \): magnetic susceptibility
- \( \gamma \): strength of the dipole
- \( \Phi \): viscous dissipation
- \( \Omega \): nondimensional rotation velocity of cylinder, \( \omega L / 2u_0 \)

**Subscripts**

- \( c \): cold wall
- \( max \): maximum
- \( mean \): average
- \( h \): hot wall

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![Fig. 1. Schematic description of lid-driven cavity with rotating cylinders and boundary conditions.](image-url)
originality of this study. The present configuration may be encountered in many engineering applications such as coating, float glass production, solidification, microelectronic devices and food processing. Convective heat transfer control by using a magnetic field in a cavity with two rotating cylinders can be utilized in nuclear reactor fuel rods, drilling of oil wells and rotating tube-heat exchangers. The results of this study can be utilized to find the optimum flow and geometrical parameters to achieve effective heat transfer enhancement in those systems.

2. Physical model and numerical study

The physical system is depicted in Fig. 1 along with the boundary conditions. The rotating cylinders of diameter $D_1$ and $D_2$ are located at the mid of the left and right half cavities. The top horizontal wall of the cavity is moving at constant speed of $u_w$ while on the other walls no-slip boundary condition was used. Cylinders are rotating with angular speeds of $\omega_1$ and $\omega_2$. Top horizontal wall of the enclosure is kept at constant hot temperature $T_h$ while lower horizontal wall is at cold constant temperature $T_c < T_h$. Other walls of the cavity and cylinder surfaces are insulated. A magnetic dipole source is placed below the bottom wall of the cavity and the fluid is assumed to be electrically non-conducting (ferrofluid does not induce electromagnetic current).

A term related to magnetic field was added to the momentum equation of an incompressible fluid with constant viscosity as:

$$\rho \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \rho \mathbf{g} (T - T_c) + \nabla \cdot (\mathbf{H} \mathbf{B}) + \mu \nabla^2 \mathbf{u} \quad (1)$$

where $\mathbf{H}$ and $\mathbf{B}$ represent the magnetic field and magnetic induction. The energy equation of an incompressible fluid can be written as:

$$\rho c \mathbf{u} \cdot \nabla T = k \nabla^2 T + \frac{\partial}{\partial T} \left( \mathbf{M} \cdot \nabla \mathbf{H} \right) \quad (2)$$

where $\mathbf{M}$ denotes the magnetization. Maxwell’s equation for a non-conducting fluid can be written as

$$\nabla \cdot \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0 \quad (3)$$

The constitutive relation between $\mathbf{B}, \mathbf{M}$ and $\mathbf{H}$ can be stated using the following formula as:

$$\mathbf{B} = \mu_0 (\mathbf{M} + \mathbf{H}) \quad (4)$$
Fig. 4. Effect of varying Reynolds numbers on the Nusselt number distributions along the bottom wall \( (\gamma = 100, Gr = 8.032 \times 10^6, \Omega_1 = \Omega_2 = 250, D_1 = D_2) \).

Fig. 5. Streamlines and isotherms for various angular velocity of the cylinders \( (\gamma = 100, Re = 100, Ri = 1.14, \Omega_1/\Omega_2 = 1, D_1 = D_2) \).
The magnetic field is induced with a magnetic dipole placed below the bottom wall of the enclosure. A magnetic scalar potential can be defined for the magnetostatic case as,

\[ V_m(x) = \frac{\gamma}{2\pi} \frac{x - c}{(x - c)^2 + (y - d)^2} \]

where \( \gamma, c \) and \( d \) represent the magnetic field strength and location of the magnetic dipole source. The term in the momentum equation is the force per unit volume when the spatially non-uniform magnetic field is employed to the magnetic fluid. The relation between the magnetization vector \( \mathbf{M} \) and magnetic field vector \( \mathbf{H} \) can be written as

\[ \mathbf{M} = \chi_m \mathbf{H} \]

where \( \chi_m \) is the total magnetic susceptibility

\[ \chi_m = \frac{\chi_0}{1 + \alpha T + T_0} \]

Finally, using the constitutive equation in (4), the body force in the momentum equation can be written as:

\[ f = \mu_0 \chi_m (1 + \chi_m) (\mathbf{H} \cdot \nabla) \mathbf{H} + \mu_0 \chi_m \mathbf{H} (\mathbf{H} \cdot \nabla)(1 + \chi_m) \]

The appropriate forms of the dimensional boundary conditions for the studied problem are:

- For the horizontal top wall:
  \[ u = u_m, v = 0, T = T_c \]

- For the horizontal bottom wall:
  \[ u = v = 0, T = T_h \]

- For the vertical walls:
  \[ u = v = 0, \ \frac{dT}{dx} = 0 \]

Fig. 6. Velocity distribution along the mid plane of the cavity for various angular velocity of the cylinders (\( \Omega = 100, \ Re = 100, \ Ri = 1.14, \ \Omega_1/\Omega_2 = 1, \ D_1 = D_2 \)).
On the cylinder surfaces:

\[ u = -\omega(y - y_0), \quad v = \omega(x - x_0), \quad \frac{\partial T}{\partial n} = 0 \]

The relevant nondimensional numbers are Reynolds number \( (Re = \frac{u_0 L}{v}) \), Magnetic number \( (Mn = \frac{\mu_0 H_0^2}{\rho_0 v^2}) \), Grashof number \( (Gr = g\beta(T_h - T_c)L^3/\nu^2) \), Richardson number \( (Ri = Gr/Re^2) \) and angular rotational speed of the cylinders \( (\Omega_1, \Omega_2) \), \( (\Omega = \omega L/2u_0) \).

Local Nusselt number is defined as

\[ Nu_x = -\left(\frac{\partial \theta}{\partial n}\right)_{wall} \quad (9) \]

where \( \theta \) and \( n \) represent the nondimensional temperature and surface normal component, respectively. Spatial averaged Nusselt number is obtained after integrating the local Nusselt number along the bottom wall of the cavity as,

\[ Nu = \frac{1}{H} \int_0^H Nu_x dx. \quad (10) \]

The solution of governing equations in Eqs. (1)–(4) is made by Galerkin weighted residual finite element formulation. The computational domain is discretized into triangular elements. Triangular Lagrange finite elements of different orders are used for each of the flow variables within the computational domain. Residuals for each of the conservation equation is obtained by substituting the approximations into the governing equations. To simplify the nonlinear terms in the momentum equations, Newton–Raphson iteration algorithm was used. Segregated parametric solvers are used for fluid flow and heat transfer variables. Biconjugate gradient stabilized iterative method solver (BICGSTab) is used for fluid flow and heat transfer modules of the commercial software COMSOL. The convergence of the solution is assumed when the relative error for each of the variables satisfy the following convergence criteria:

\[ \left| \frac{\Gamma^{n+1} - \Gamma^n}{\Gamma^{n+1}} \right| < 10^{-6} \quad (11) \]

The computational domain is divided into triangular elements and the mesh is finer near the walls of cavity and cylinder in order to resolve the high gradients in the thermal and velocity boundary layer. An optimal grid distribution with accurate results and

![Figure 7: Local Nusselt number distribution along the bottom wall of the cavity for different angular velocity of the cylinders](image-url)
minimal computational time is obtained by performing numerical experiments with various grid sizes. The results of averaged Nusselt number along the bottom wall for different grid sizes are shown in Table 2 ($\gamma = 500, \text{Ri} = 1.14, D_1 = D_2, \Omega_1 = \Omega_2 = 400$). G4 with 33796 nodes ensures grid independent solution and hence was used in the subsequent computations. The present solver is validated against the numerical results of Iwatsu et al. [30]. Table 1 shows the comparison results of averaged Nusselt number at the top wall of the lid driven cavity. Fig. 2 demonstrates the comparison of the isotherms between the present solver Iwatsu et al. [30] (dashed lines) and Khanafer and Chamkha [31] (solid lines). The comparison results show good overall agreement.

Fig. 8. Streamlines and isotherms for various magnetic dipole source strength ($\text{Re} = 100, \text{Ri} = 1.14, \Omega_1 = \Omega_2 = 200, D_1 = D_2$).

Fig. 9. Velocity distribution along the mid of the cavity for various magnetic dipole source strength ($\text{Re} = 100, \text{Ri} = 1.14, \Omega_1 = \Omega_2 = 200, D_1 = D_2$).
3. Results and discussion

In this numerical investigation, the influence of Reynolds number (100 ≤ Re ≤ 103), angular rotational speed of the cylinders (0 ≤ Ω ≤ 400), strength of the magnetic dipole source (0 ≤ γ ≤ 500), angular velocity ratios of the cylinders (0.25 ≤ Ω1/Ω2 ≤ 4) and diameter ratios of the cylinders (0.5 ≤ D1/D2 ≤ 2) on the flow field and heat transfer within the cavity was numerically studied.

The effects of varying Reynolds number on the streamlines and isotherms are demonstrated in Fig. 3 (a)–(f) (γ = 100, Gr = 8.032 × 10⁵, Ω1 = 250, D1 = D2). Grashof number is fixed at Gr = 8.032 × 10⁵. Richardson number (Ri = Gr/Re²) which defines the ratio of natural convection to the forced convection due to the upper moving wall corresponds to Ri = 80.3, 3.21 and 0.80 for Reynolds number of Re = 100, 500 and 1000, respectively. At Re = 100, a multi-cellular structure is formed within the cavity. As the Reynolds number increases, the recirculating vortex adjacent to the upper wall gets larger in size and strength. The core of the upper vortex moves along the lid direction. The strength of the recirculating cell above the cylinders and below the upper vortex increases as the Reynolds number increases. The isotherms are also affected especially adjacent to the upper wall. The local and averaged Nusselt numbers are depicted in Fig. 4(a) and (b). Heat transfer is locally and in average enhanced. Heat transfer enhancement of 8.33% is obtained for Reynolds number of 1000 compared to case at Reynolds number of 100.

Fig. 5(a)–(f) demonstrate the effects of varying angular velocity of the cylinders (Ω) on the streamlines and isotherms for fixed values of (γ = 100, Re = 100, Ri = 1.14, Ω1/Ω2 = 1, D1 = D2). Ω = 0 corresponds to stationary cylinders which is depicted in Fig. 5(b) and (e). Two recirculating cells are formed between the cylinders and an additional vortex is also seen on the top of the second cylinder adjacent to the right vertical wall. Clockwise rotation of the cylinders are indicated by negative values of Ω. For Ω = −400, multi cellular structure adjacent to the cylinder surfaces disappear and only one recirculating vortex is formed between the cylinders (Fig. 5(a)). The cylinder rotation has a positive impact on the fluid motion from the hot bottom wall towards the upper wall. Small recirculating regions are also seen behind the cylinders. When the cylinder rotates in the clockwise direction which is indicated by positive values of Ω (Fig. 5(c) and (f)), a small vortex is seen on the left bottom corner of the cavity and the recirculating cell adjacent to upper wall disappears. The isotherms are also affected by the variation in the angular velocities of the cylinders. Steep temperature gradient is seen towards the right part of the bottom wall for clockwise rotation and towards the left part for counter clockwise rotation of the cylinders. Penetration of the thermal patterns in the vicinity of the cylinder surfaces are seen. The cylinder rotation brings about a positive effect on the energy transport. Fig. 6 shows that absolute values of the u and v velocities increase as the angular speeds of the cylinders increase due to the favorable effect of cylinder on convection. The local and averaged Nusselt number plots are shown in Fig. 7(a) and (b). Local

![Graph showing local Nusselt number](image1)

![Graph showing averaged Nusselt number](image2)

**Fig. 10.** Local and averaged Nusselt number distribution along the bottom wall of the cavity for various magnetic dipole source strength (Re = 100, Ri = 1.14, Ω1 = Ω2 = 200, D1 = D2).
Fig. 11. Effects of varying ratios of angular velocity of the cylinders on the streamlines and isotherms for clockwise rotation ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 = 200, D_1 = D_2$).

Fig. 12. Local and averaged Nusselt number for different ratios of angular velocity of the cylinders for clockwise rotation ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 = 200, D_1 = D_2$).
heat transfer along the bottom wall can be controlled with cylinder rotation angle. Averaged heat transfer increases as the cylinder rotation angle increases and heat transfer enhancements of 181.5% and 181.6% are achieved for $\Omega = -400$ and $\Omega = 400$ compared to motionless cylinder case.

The effects of varying magnetic dipole strength on the flow patterns and isotherms are demonstrated in Fig. 8(a)-(f) ($Re = 100, Ri = 1.14, \Omega_1 = \Omega_2 = 200, D_1 = D_2$). The cylinders are rotating in the counterclockwise direction. As the strength of the magnetic dipole is increased to $\gamma = 250$, the number of recirculating

![Diagram](image1.png)

**Fig. 13.** Effects of varying ratios of angular velocity of the cylinders on the streamlines and isotherms for counter-clockwise rotation ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 = -200, D_1 = D_2$).

![Diagram](image2.png)

**Fig. 14.** Local and averaged Nusselt number for different ratios of angular velocity of the cylinders for counter-clockwise rotation ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 = -200, D_1 = D_2$).
The inhomogeneous magnetic body force is responsible for this behavior. Further increment of dipole strength leads to diagonally elongation of recirculating vortices in the vicinity of the upper moving wall and the number of vortices near the bottom wall decreases (Fig. 8(c)). Steep temperature gradient is observed near the mid of the bottom wall. This effect is due to the spatial variation in the magnetization which is induced through temperature gradient. Velocity profiles along the mid of the cavity for different values of magnetic dipole strength are shown in Fig. 15 and Fig. 16.

**Fig. 15.** Streamlines and isotherms for various diameter ratios ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 - \Omega_2 = 200, D_2 = 0.2H$).

**Fig. 16.** Streamlines and isotherms for various diameter ratios ($\gamma = 100, Re = 100, Ri = 1.14, \Omega_1 - \Omega_2 = 200, D_1 = 0.2H$).
depicted in Fig. 9(a) and (b). The negative values of the x-component of the velocity corresponding to a recirculation zone. Magnetic dipole strength can be used to control the strength and size of the recirculation zone. The effects of dipole strength on the variation of local and averaged Nusselt numbers are demonstrated in Fig. 10(a) and (b) (Re = 100, Ri = 1.14, \( \Omega_1 = \Omega_2 = 200, D_1 = D_2 \)). Local enhancement (deterioration) of heat transfer is seen towards the right (left) end of the bottom wall for \( \gamma = 250 \) and \( \gamma = 500 \) while heat transfer is locally enhanced in the vicinity of mid of the bottom wall for only \( \gamma = 500 \). Averaged heat transfer first increases as the value of \( \gamma \) increases until \( \gamma = 250 \) and then decreases at \( \gamma = 500 \) due to the local deterioration of heat transfer until \( x = 0.3H \). Averaged heat transfer enhancements of 3.5 % and 8.5 % are obtained for \( \gamma = 500 \) and \( \gamma = 250 \) compared to case at \( \gamma = 0 \).

Streamlines and isotherms are shown in Fig. 11(a)–(f) for various ratios of angular velocities of the cylinders for counter-clockwise rotation (\( \gamma = 100, Re = 100, Ri = 1.14, \Omega_1 = 200, D_1 = D_2 \)). As the cylinder which is located on the right half rotates faster, the size and strength of the recirculation zones near the bottom wall decrease whereas the recirculating cell adjacent to the right cylinder gets larger with larger influence area. The temperature gradient becomes steeper along the bottom wall as the ratio of the velocities \( \Omega_2/\Omega_1 \) increase and isotherms show plum like behavior emanating from the cylinder located in the left half due to the favorable effect of rotation ratio on convection which results in better mixing and enhanced thermal transport. Fig. 12(a) and (b) shows the effect of ratio of cylinder rotations on the local and averaged heat transfer. Local and averaged heat transfer enhancement are achieved as the ratio of cylinder rotations increase. Averaged heat transfer enhancement of 92.8 % is obtained for \( \Omega_2/\Omega_1 = 4 \) compared to case at \( \Omega_2/\Omega_1 = 0.25 \) when the cylinders rotate in counter clockwise direction. The streamlines and isotherms for various ratios of angular rotations of the cylinders are demonstrated in Fig. 13(a)–(f) for clockwise rotation of the cylinders. The recirculating zone adjacent to the cylinders penetrate more into the bottom wall and the the recirculating cells near the top wall diminishes in size and strength with increasing values of ratio of the cylinder rotations. The isotherms are clustered more towards the right end of the bottom wall indicating heat transfer augmentation with ratio of the cylinder rotations. Local heat transfer enhancement towards the right end of the bottom wall is seen and peak values are attained at \( x = 0.8H \) as shown in Fig. 14(a) due to the favorable effect of ratio of
cylinder rotation on the thermal transport and better mixing for clockwise rotation. Averaged heat transfer enhancement of 91.7% is achieved for ratio of $D_2/D_1 = 4$ compared to case at $D_2/D_1 = 0.25$.

The effects of varying diameter ratios on the streamlines and isotherms are shown in Fig. 15 for $D_1/D_2$ and in Fig. 16 for $D_2/D_1$ ($\gamma = 100, Re = 100, Ri = 114, \Omega_1 = \Omega_2 = 200$). When the ratio of the first cylinder (located at left half) to the second cylinder (located at right half) $(D_2/D_1)$ changes flow and thermal patterns are considerably affected. In Fig. 15, the radius of the second cylinder is kept at $D_2 = 0.2H$ while the radius of first cylinder changes. As the ratio increases, the gap between the cylinders reduces and strength of the convection increases in this region. When the radius of the first cylinder increases, more fluid adjacent to the first cylinder interacts with rotation which results in better thermal transport adjacent to the left part of the bottom wall. This is also seen in isotherm plots as shown in Fig. 15(d) as steep temperature gradient along the left part of the bottom wall. Fig. 16 shows streamlines and isotherms for various radius ratios when the radius of first cylinder is kept at $D_2 = 0.2H$ while changing the radius of the second cylinder. The cylinders are rotating in counter clock wise direction. As the diameter ratio increases, the strength of convection between the cylinder increase and the recirculating vortex adjacent to the cylinders disappear. The boundary layer towards the right end of the enclosure becomes thicker due to the rotation direction of the second cylinder which opposes convection in those region (Fig. 16(f)). Steep temperature gradient is seen in the mid of the bottom wall. Fig. 17 demonstrates the local and averaged Nusselt numbers for different diameter ratios. Local and averaged heat transfer enhancement is obtained as the ratio of the cylinder diameters increase. Increasing the first cylinder diameter is more effective for the local enhancement of heat transfer towards the left part of the bottom wall whereas increasing the second cylinder diameter leads to enhancement of heat transfer in the mid of the cavity. Averaged heat transfer enhancements of 88.9% and 62.4% are achieved for diameter ratios of $(D_1/D_2 = 4)$ and $(D_2/D_1 = 4)$ compared to cases at $(D_1/D_2 = 0.25)$ and $(D_2/D_1 = 0.25)$.

### 4. Conclusions

In this study, mixed convection of ferrofluid flow in a lid driven cavity in the presence of two rotating cylinders were numerically investigated in the presence of a magnetic dipole source. Following conclusions can be drawn from the numerical simulation results:

- The flow and thermal patterns are affected with Reynolds number and magnetic dipole source which adds an inhomogeneous magnetic body force. Local and averaged heat transfer enhancement are observed as the Reynolds number increases.
- The external magnetic field acts in a way to decrease the local heat transfer in some locations and increase it in some others.

Averaged heat transfer first increases then decreases as the value of the magnetic dipole strength increases from $\gamma = 0$ to $\gamma = 250$ and from $\gamma = 250$ to $\gamma = 500$.

- Cylinder angular velocities have profound effect on heat transfer enhancement within the cavity and heat transfer enhancements of 181.5% and 181.6% are achieved for $\Omega = -400$ and $\Omega = 400$ compared to motionless cylinder case.
- Ratio of the angular velocities and diameter ratios have favorable effect on thermal transport within the cavity. Increasing the angular velocity ratio from $D_2/D_1 = 0.25$ to $D_2/D_1 = 4$ brings about 91.7% of heat transfer enhancement. 88.9% of averaged heat transfer enhancement is observed for diameter ratio of $(D_1/D_2 = 2)$ compared to diameter ratio of $(D_1/D_2 = 0.5)$.

### References