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Review

A review of shear strength models for rock joints subjected to constant normal stiffness

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ABSTRACT

The typical shear behaviour of rough joints has been studied under constant normal load/stress (CNL) boundary conditions, but recent studies have shown that this boundary condition may not replicate true practical situations. Constant normal stiffness (CNS) is more appropriate to describe the stress–strain response of field joints since the CNS boundary condition is more realistic than CNL. The practical implications of CNS are movements of unstable blocks in the roof or walls of an underground excavation, reinforced rock wedges sliding in a rock slope or foundation, and the vertical movement of rock-socketed concrete piles. In this paper, the highlights and limitations of the existing models used to predict the shear strength/behaviour of joints under CNS conditions are discussed in depth.

1. Introduction

An appropriate evaluation of the shear behaviour of rock joints is vital, for instance when analysing the stability of rock slopes, designing excavations in jointed rock, assessing the stability of concrete dam foundations, and designing rock-socked piles. In conventional studies, the shear behaviour of a joint is usually investigated in the laboratory under constant normal load/stress (CNL) boundary conditions where the normal stress remains constant and the surface of the joint dilates freely during shearing. The best example to illustrate a CNL condition is a slope stability problem where the rock block is sliding along the joint without any constraint. However, in engineering practice, the normal stress acting on the joint interface may vary during shearing, and dilation of the joint may be constrained by the confined environment formed across the interface, which often represents a constant normal stiffness (CNS) condition. The practical implications of this are movements of unstable blocks in the roof or walls of an underground excavation, reinforced rock wedges sliding in a rock slope or foundation, and the vertical movement of rock-socketed concrete piles, as illustrated in Figs. 1–3, respectively. Several researchers have insisted that a CNS boundary condition is more appropriate for many field situations (Heuze, 1979; Leichnitz, 1985; Johnston et al., 1987; Ohnishi and Dharmaratne, 1990; Saeb and Amadei, 1990; Skinas et al., 1990; Haberfield and Johnston, 1994; Kodikara and Johnston, 1994; Indraratna and Haque, 1997, 2000; Indraratna et al., 1999, 2010a, 2015; Seidel and Haberfield, 2002; Jiang et al., 2004; Thirukumaran et al., 2015). The CNS boundary condition is usually simulated by a spring with a CNS $K_n = d\sigma_n/d\delta_v$, where $d\sigma_n$ and $d\delta_v$ are the changes in normal stress and normal displacement, respectively. The value of this CNS K_n is externally controlled by applied reinforcement or the adjacent rock mass across the joint interface.

In addition to the boundary normal stiffness imposed by the surrounding rock mass, there are other parameters that may affect the shear behaviour of rock joints such as the joint surface roughness and strength, the level of initial normal stress acting on the joint interface, the presence of infill (gouge) material, and water in the joint interface. A considerable amount of work has been conducted to describe how these factors affect the shear behaviour of joints under CNL conditions, but only a few studies with limited experimental data and analysis on the shear behaviour of joints under CNS conditions are available as yet. Apart from this boundary effect, the shear behaviour of rough rock joints is complex because the stress–strain response is governed by non-uniform asperity damage and gouge material that accumulates on the joint interfaces. To date, only a few studies have been devoted to studying

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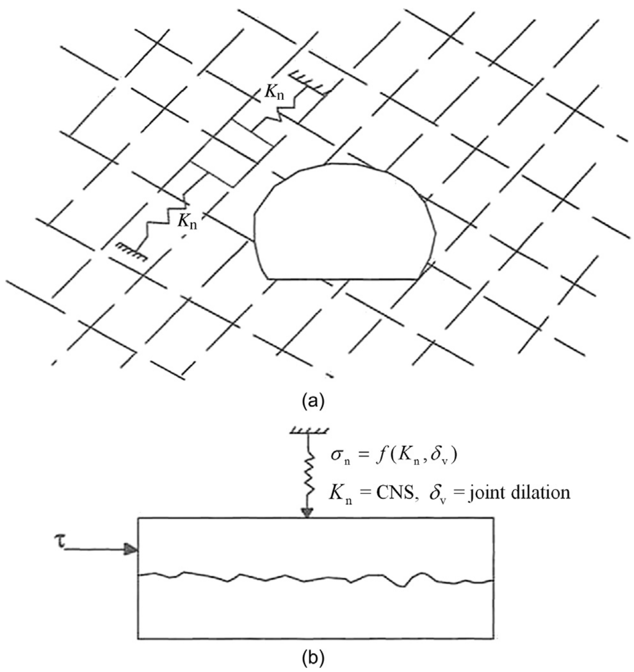

Fig. 1. Joint behaviour in the roof or walls of an underground excavation (after Indraratna et al., 1999). (a) Underground excavation in jointed rock. (b) Equivalent two-dimensional model for joint on the top of roof.

the evolution of asperity damage and production of gouge on the joint surface due to the technical difficulty of experimentally measuring the rate of asperity damage and the production and distribution of gouge material. Some studies have attempted to characterise the asperity deformation directly on the joint surface (Ladanyi and Archambault, 1970; Riss et al., 1997; Roko et al., 1997; Gentier et al., 2000; Homand et al., 2001; Grasselli et al., 2002; Yang et al., 2010; Indraratna et al., 2014; Tatone and Grasselli, 2015). Others indirectly appraised asperity deformation by assessing the joint dilation angle (Plesha, 1987; Hutson and Dowding, 1990; Leong and Randolph, 1992; Lee et al., 2001; Indraratna et al., 2015), or mobilised the friction angle (Barton, 1982), as well as provided insight into asperity deformation on the basis of numerical modelling (Karami and Stead, 2008; Asadi et al., 2012) during shearing. Nevertheless, incorporating the influence of asperity degradation and gouge accumulation to the model for rock joints is still a very challenging task that needs more advanced studies.

Unlike CNL boundary conditions, only a few methods have been proposed to model either the peak shear strength of rock joints or the complete shear behaviour of rough rock joints under CNS conditions (Heuze, 1979; Leichnitz, 1985; Saeb and Amadei, 1990, 1992; Skinas et al., 1990; Seidel and Haberfield, 2002; Indraratna et al., 1999; Indraratna and Haque, 2000; Indraratna et al., 2005, 2010b, 2015; Oliveira and Indraratna, 2010). The objective of this review paper is to study the importance of developed models and also identify the limitations for using these existing models in practical applications.

2. Existing shear strength models

2.1. Heuze's (1979) analytical model

Heuze (1979) emphasised that when a joint begins to dilate, it is partially restrained by external normal stiffness applied across the interface and thus the normal stress across the joint increases. Therefore, he used an analytical method to calculate the incremental

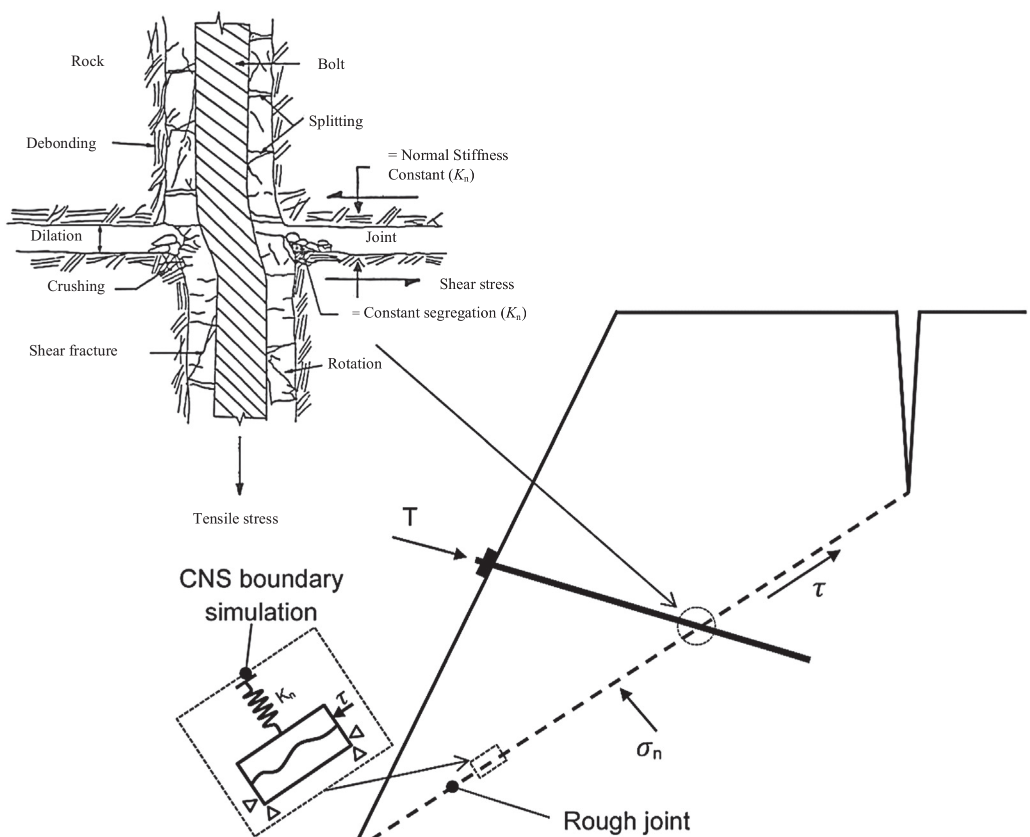

Fig. 2. Behaviour of joints in a reinforced rock slope (inspired after Indraratna and Haque, 2000).

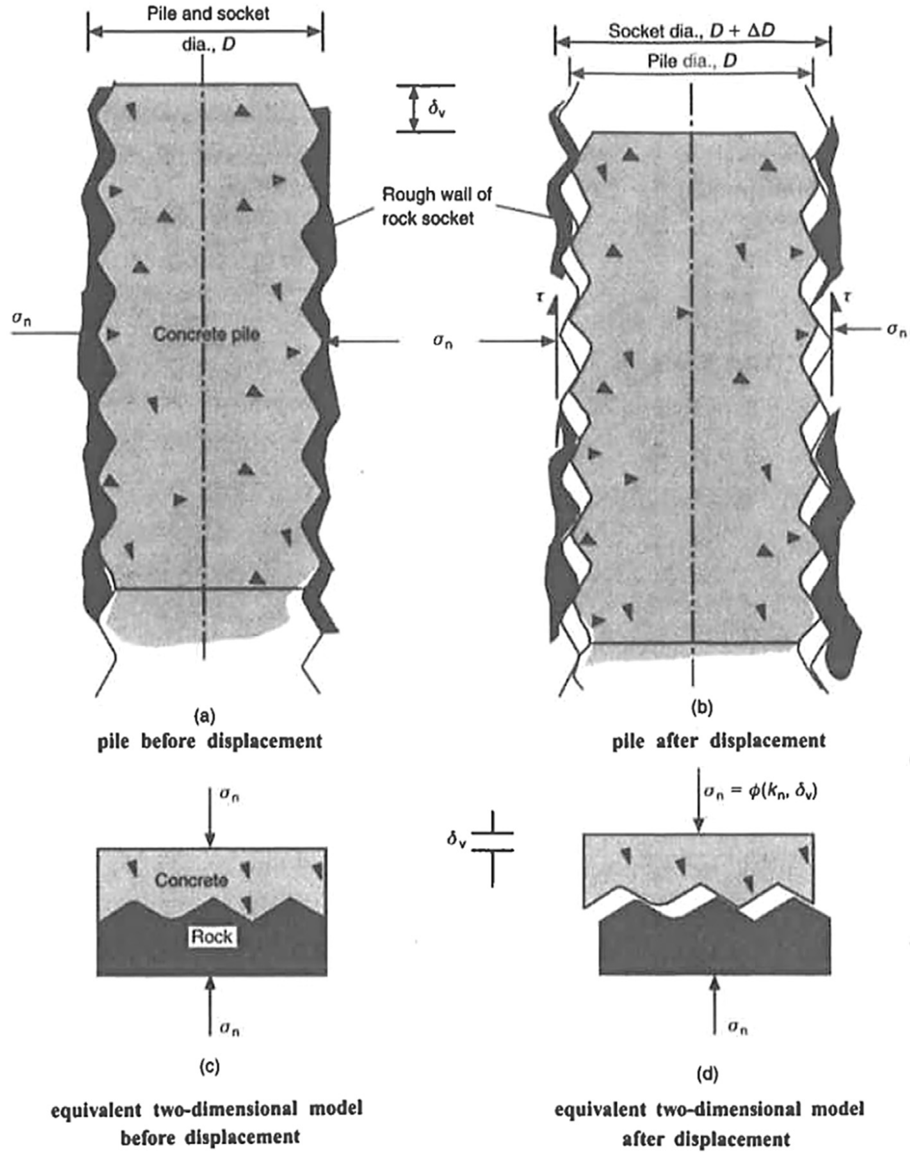

Fig. 3. Idealised displacement of pile socketed in rock (after Johnston et al., 1987).

normal stress as a function of external boundary stiffness and the net results of joint closure under compression and opening during shearing. The concept of this analytical model is illustrated in Fig. 4 and also briefly described hereafter. Based on an assumption of joint bi-dilation, the incremental normal stress ($\Delta\sigma_n$) is formed by a positive dilation $\Delta\delta_v$ which compresses the external constant stiffness spring, whereas the increment of normal stress in the system tends to stop the joint from opening due to normal stiffness in the joint itself (k_n). The equilibrium of $\Delta\delta_v$ is thus expressed from

$$d\delta_v = \frac{\partial\delta_v}{\partial\delta_h}d\delta_h + \frac{\partial\delta_v}{\partial\sigma_n}d\sigma_n \tag{1}$$

where $\partial\delta_v/\partial\delta_h = \tan i$ and $\partial\delta_v/\partial\sigma_n = -1/k_n$.

Thus, Heuze (1979) proposed the following equation to calculate the increment of normal stress under CNS:

$$d\sigma_n = \tan i\left(\frac{k_n K_n}{k_n + K_n}\right)d\delta_h \tag{2}$$

where δ_h is the shear displacement.

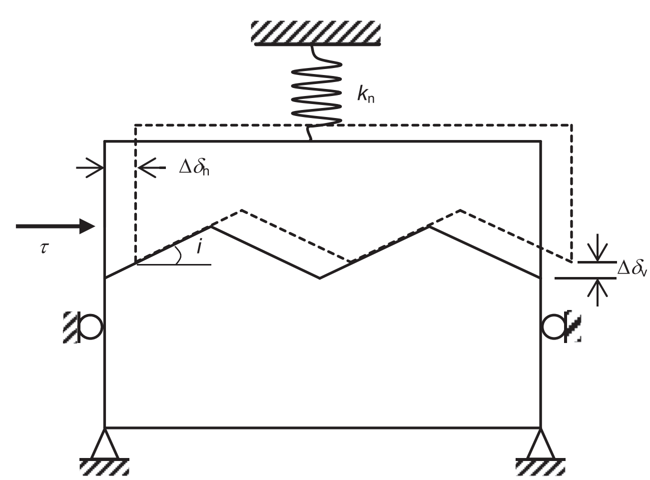

Fig. 4. Conceptual model of dilatant joints (after Heuze, 1979).

Eq. (2) can be used to calculate the change in normal stress under CNS. Hence, Heuze (1979) suggested a three-degree polynomial equation to describe the peak shear strength of a rough rock joint below the critical normal stress, beyond which no dilation occurred. Thus, the peak shear stress $(\tau_p)_{CNS}$ under CNS can be determined by

$$(\tau_p)_{CNS} = a\sigma_n + b\sigma_n^2 + c\sigma_n^3 \tag{3}$$

where $a = \tan\phi_p$, $b = 3C_p/\sigma_{cr}^2 - 2(\tan\phi_p - \tan\phi_r)/\sigma_{cr}$, and $c = -2C_p/\sigma_{cr}^3 + (\tan\phi_p - \tan\phi_r)/\sigma_{cr}^2$, in which σ_n is the normal stress, σ_{cr} is the critical normal stress, C_p is the apparent cohesion, ϕ_p is the peak friction angle equal to the addition of initial dilation angle (at $\sigma_n = 0$), and ϕ_r is the residual friction angle. When $\sigma_n > \sigma_{cr}$, the peak strength was simply given by $\tau_p = C_p + \sigma_n\tan\phi_r$, and the residual shear strength was given by $\tau_r = \sigma_n\tan\phi_r$.

2.2. Leichnitz's (1985) analytical model

From the results of CNS and CNL direct shear tests, Leichnitz (1985) showed that the shear force (S) and the normal displacement (v) are independent of a given stress path (i.e. either CNL or CNS stress path) and are functions of the shear displacement (u) and normal force (N) not of the stiffness; in other words, the shear force $S = \widehat{S}(u,N)$ and normal displacement $v = \widehat{v}(u,N)$. Hence, he proposed the following partial differential equations in order to predict the shear response under CNS:

$$\mathrm{d}S = \frac{\partial\widehat{S}}{\partial u}\mathrm{d}u + \frac{\partial\widehat{S}}{\partial N}\mathrm{d}N \tag{4}$$

$$\mathrm{d}v = \frac{\partial\widehat{v}}{\partial u}\mathrm{d}u + \frac{\partial\widehat{v}}{\partial N}\mathrm{d}N \tag{5}$$

Eqs. (4) and (5) can be written in the compact matrix form as

$$\begin{pmatrix} \mathrm{d}S \\ \mathrm{d}N \end{pmatrix} = \begin{pmatrix} \overset{*}{\delta} + \overset{*}{\mu}\overset{*}{v}\overset{*}{k} & \overset{*}{\mu}\overset{*}{k} \\ \overset{*}{v}\overset{*}{k} & \overset{*}{k} \end{pmatrix} \begin{pmatrix} \mathrm{d}u \\ \mathrm{d}v \end{pmatrix} \tag{6}$$

where $\overset{*}{\delta} = \partial\widehat{S}/\partial u$, $\overset{*}{\mu} = \partial\widehat{S}/\partial N$, $-\overset{*}{v} = \partial\widehat{v}/\partial u$ and $1/\overset{*}{k} = \partial\widehat{v}/\partial N$. Leichnitz (1985) called these parameters stiffness functions, and they can be calculated from the test results to predict the shear behaviour of joints under CNS.

2.3. Saeb and Amadei's (1990) graphical method

Goodman (1980) originally presented a graphical method of coupling closure and shear behaviour under constant normal stress or zero normal stiffness (CNL) for rough joints, while simultaneously analysing the dependent path where no dilatancy is permitted during shearing under normally controlled displacement or constant normal displacement, but it may be allowed during strict CNS conditions. However, Saeb and Amadei (1990) emphasised that constant or variable normal stiffness boundary conditions are more likely to exist across joint surfaces in-situ rather than CNL. They extended Goodman's (1980) method to predict the shear behaviour of rough joints under constant or variable normal stiffness boundary conditions by coupling the closure of joints at different shear displacements and shear behaviours under CNL. They used the curves in Fig. 5 to plot the variation in normal stress σ_n versus normal displacement v for different values of shear displacement u that is shown in Fig. 6. Every curve u_i in Fig. 6 was constructed using the values of σ_n and v at the points of intersection between the shear displacement lines u_i and the normal displacement versus shear displacement curves shown in Fig. 5c.

As can be seen from Fig. 6, the curve $u = u_0$ represents the closure of mated joints (interlocked joints) under uniaxial compressive loading shown in Fig. 5a, where each curve $u = u_i$ ($i = 1$–4) represents the behaviour of the joint under normal compressive loading after being separated by a shear displacement equal to u_i. With the response of joints shown in Fig. 5c, all the curves u_i ($i > 4$) coincided with the curve u_4 because there is no further dilation for shear displacement higher than u_4, so all the curves u_i approached the curve u_0 as σ_n increased.

Fig. 5 can be used to predict the shear behaviour of a joint for any load path. For example, Fig. 6 shows that the four distinct load paths that originated from point *A* were given by assuming that a normal stress $\sigma_n = 4A$ was applied prior to shearing. Under a CNS K_n, the joint may follow the path *AFGHI* but it would follow the path *ABCDE* under CNL ($K_n = 0$) or *AJKLM* when no change in joint normal displacement was allowed ($K_n \to \infty$). Finally, the path *ANPQR* corresponds to a joint in a rock mass with increasing applied normal stiffness. In Fig. 6, by recording the values of σ_n and u at the point of intersection of each path with curves u_i and then using Fig. 5b and c, the shear stress–shear displacement and dilation curves for $\sigma_n = 4A$ can be constructed; they are shown as dashed lines in Fig. 5.

2.4. Saeb and Amadei's (1992) analytical model

Saeb and Amadei (1992) stated that the total normal displacement of a joint v must be a function of the shear displacement u and the normal stress σ_n based on the previous graphical analysis. In order to describe this function, they proposed the following mathematical expression:

$$v = u\left(1 - \frac{\sigma_n}{\sigma_T}\right)^{k_2} \tan i_0 + \frac{\sigma_n V_m}{k_{ni}V_m - \sigma_n} \tag{7}$$

where σ_T is the transitional stress which is treated as an independent constant obtained from experimental results; i_0 is the initial dilation angle (the average inclination angle of asperities in contact); k_2 is the empirical constant with a value of 4, as suggested by Ladanyi and Archambault (1970); V_m is the maximum joint closure; and k_{ni} is the initial normal stiffness of the joint.

By differentiating Eq. (7) and then rearranging it, the following incremental formulation can be obtained:

$$\mathrm{d}\sigma_n = \frac{\mathrm{d}v - \left(1 - \frac{\sigma_n}{\sigma_T}\right)^{k_2} \tan i_0 \mathrm{d}u}{-\frac{uk_2}{\sigma_T}\left(1 - \frac{\sigma_n}{\sigma_T}\right)^{k_2 - 1} \tan i_0 + \frac{k_{ni}V_m^2}{(k_{ni}V_m - \sigma_n)^2}} \tag{8}$$

Eq. (8) relates the change in normal stress to the changes in normal and shear displacements, which can be rewritten in a more compact form as

$$\mathrm{d}\sigma_n = k_{nn}\mathrm{d}v + k_{nt}\mathrm{d}u \tag{9}$$

where $k_{nn} = \partial\sigma_n/\partial v$ and $k_{nt} = \partial\sigma_n/\partial u$. However, Eq. (9) is only valid for $\sigma_n/\sigma_T < 1$. In a similar way, they proposed the following expression for shear stress because it depends on normal and shear displacements:

$$\mathrm{d}\tau = k_{tn}\mathrm{d}v + k_{tt}\mathrm{d}u \tag{10}$$

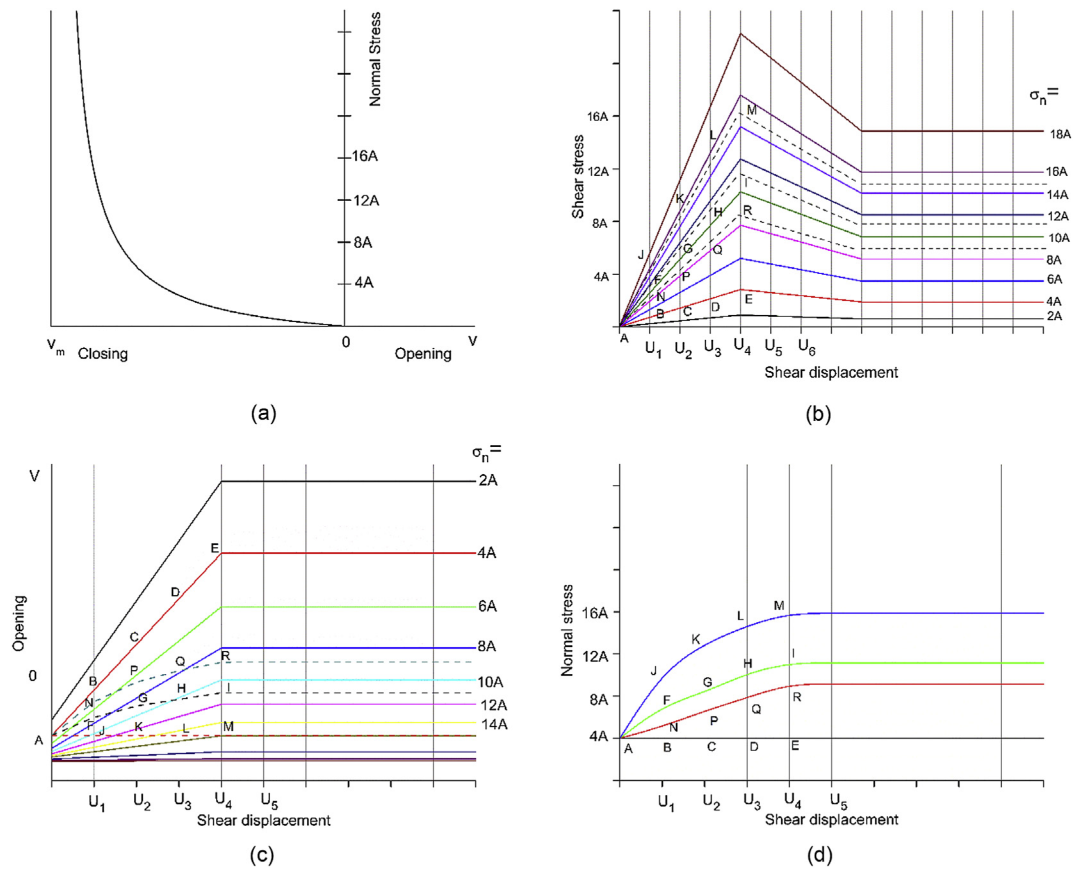

Fig. 5. The joint response curves for normal stresses ranging between 0 and 20A, where A is an arbitrary stress (after Saeb and Amadei, 1990).

where $k_{tn} = \partial\tau/\partial v$ and $k_{tt} = \partial\tau/\partial u$. In order to evaluate these shear stiffness, Saeb and Amadei (1992) used the two basic models which were recommended by Goodman (1976) to represent the joint shear stress–shear displacement behaviour under CNL. The following relations apply for both of these models:

$$\left.\begin{aligned} \tau &= k_s u \\ k_s &= \frac{\tau_p}{u_p} \end{aligned}\right\} \quad (u < u_p) \tag{11a}$$

$$\tau = \frac{\tau_p - \tau_r}{u_p - u_r} u + \frac{\tau_r u_p - \tau_p u_r}{u_p - u_r} \quad (u_p \le u < u_r) \tag{11b}$$

$$\tau = \tau_r \quad (u > u_r) \tag{11c}$$

where τ_p and τ_r are the peak and residual shear stresses, respectively; and u_p and u_r are the peak and residual shear displacements, respectively. Using the chain rule of differentiation and Eq. (11a), (11b), they obtained $k_{tn} = \partial\tau/\partial v$ and $k_{tt} = \partial\tau/\partial u$ over the three regions of u for both models. By combining Eqs. (9) and (10), Saeb and Amadei (1992) suggested a general incremental formulation for the behaviour of rock joints under shear and normal loading at constant or variable boundary normal stiffness conditions:

$$\begin{Bmatrix} d\sigma_n \\ d\tau \end{Bmatrix} = \begin{bmatrix} k_{nn} & k_{nt} \\ k_{tn} & k_{tt} \end{bmatrix} \begin{Bmatrix} dv \\ du \end{Bmatrix} \tag{12}$$

2.5. *Skinas et al.'s (1990) graphical model*

Skinas et al. (1990) indicated that modelling the complete shear behaviour of joints under CNS requires a method that can predict the variations of dilation under changing normal stresses and shear displacements. Fig. 7 graphically illustrates their approach. The right side of the figure contains two dilation curves that correspond to shearing under CNL, while the left side shows a trend of variation in normal stress with dilation for CNS. By assuming that point 1 on the dilation curve corresponds to σ_{ni} if the joint is sheared at a new position u_{i+1}, normal displacement will increase to a value of v_{i+1} depending on $\Delta\sigma_n$. This new point (2) will refer to another dilation curve that corresponds to σ_{ni+1} on the right side of the plot. The position of point 2 can be defined if the following conditions are satisfied:

$$v_{i+1} = v_i^* + (u_{i+1} - u_i)\tan d_n \tag{13}$$

$$\sigma_{ni+1} = \sigma_{ni} + K_n(v_{i+1} - v_i) \tag{14}$$

A linear iterative procedure can be applied to Eqs. (13) and (14) to calculate point 2, but to determine the increment of dilation under CNL conditions, Skinas et al. (1990) adopted the concept of mobilised dilation that was proposed by Barton (1982), thus $d_n = (1/M)JRC_{mob}^{u_i}\log_{10}(JCS/\sigma_{ni+1})$, where M is the damage coefficient, JRC_{mob} is the mobilised joint roughness coefficient, and JCS is the compressive strength of the joint wall. As Barton and Choubey (1977) suggested, the values of M at peak strength are equal to 1 and 2 for low and high normal stresses, respectively, although Skinas et al. (1990) stated that M can reach a value of 5 in

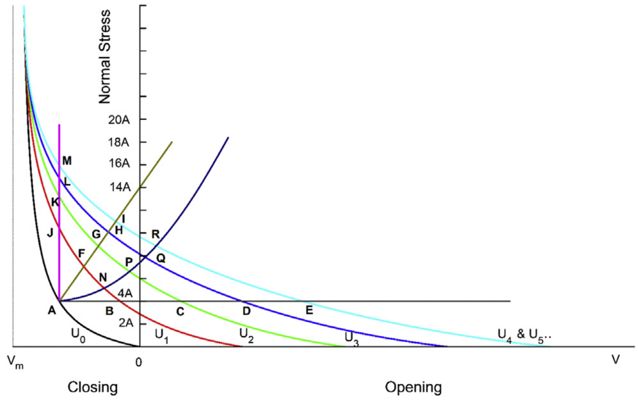

Fig. 6. Normal stress versus normal displacement at different shear displacement levels (after Saeb and Amadei, 1990).

the post-peak range at high normal stresses. Finally, they proposed the following equation to predict the mobilised shear stress at any stage of shearing under CNS:

$$\tau_{\text{mob}} = \sigma_{\text{n}i+1} \tan\left[JRC_{\text{mob}}^{u_i} \log_{10}\left(\frac{JCS}{\sigma_{\text{n}i+1}}\right) + \phi_{\text{r}}\right] \tag{15}$$

$$\tau = \sigma \tan(\phi_{\text{b}} + \theta) \tag{16}$$

where σ is the average normal stress applied to the joint, ϕ_{b} is the basic friction of the joint, and θ is the asperity inclination.

(2) Asperity shearing

Seidel and Haberfield (2002) noted that as shear displacement progresses, the contact area between two surfaces of a joint is restricted to one asperity face, which gradually reduces. Thus, normal stress increases as a consequence of the reduced contact area as well as the results of an applied external normal stiffness. A critical normal stress is then reached where the asperity can no longer withstand and individual asperity failure/damage takes place. In addition, numerical simulations and video records of direct shear tests convinced them that a rotational asperity failure occurred. This was in contrast to other models such as Patton (1966), which were based on planar failure surfaces. As a consequence, the shapes of the curved asperity failure led Seidel and Haberfield (2002) to use slope stability methods to model the shear failure/damage of asperities of soft rock. They adopted a closed form solution for the failure of a weightless slope with a slope angle ψ in a c-ϕ soil subjected to an inclined load, a method originally proposed by Sokolovsky (1960). The following equation was suggested by Seidel and Haberfield (2002) to predict the shear stress during asperity damage:

$$\tau = \frac{\sigma_{\text{n}}\left[\tan\left(\phi_{\text{b}} + \theta\right)\left(\tan\phi_{\text{b}} + \tan\beta^{*}\right) + \tan\left(\phi_{\text{b}} + \beta^{*}\right)\left(\tan\phi_{\text{b}} + \tan\theta\right)\right]}{\left(1 + \tan\phi_{\text{b}}\tan\beta^{*}\right)\left[\tan\left(\phi_{\text{b}} + \theta\right) + \tan\left(\phi_{\text{b}} + \beta^{*}\right)\right]} \tag{17}$$

where β^{*} is the asperity failure angle.

2.6. Seidel and Haberfield's (2002) theoretical model

Based on the comprehensive tests results on concrete/rock joints (Johnston et al., 1987; Johnston and Lam, 1989; Kodikara and Johnston, 1994; Haberfield and Johnston, 1994; Seidel and Haberfield, 1995; Haberfield and Seidel, 1999), Seidel and Haberfield (2002) proposed a theoretical model for predicting the pre-peak and post-peak shear behaviours of soft rock/rock joints and concrete/rock joint under CNS. The key hypothesis of their modelling was that natural joint profiles (with complex geometry) could be idealised as a series of simple triangular asperities, and thus the shear behaviour of the more complex profiles could be predicted from the models developed from triangular asperities. The tests on joint profiles with triangular asperities showed that shear behaviour involved two independent mechanisms, i.e. with initial sliding along the surface of the asperities and then simultaneous shearing through all the intact asperities (Seidel and Haberfield, 2002). To begin with, the development of a triangular asperity model by Seidel and Haberfield (2002) is briefly described as follows.

(1) Asperity sliding

In order to calculate the required average shear stress τ for asperity sliding, Seidel and Haberfield (2002) suggested the following equation, which is similar to the model of Patton (1966):

2.7. Models developed at University of Wollongong

Extensive research has been carried out at University of Wollongong (UOW), Australia on the shear behaviour of rock joints under CNS. In this section, models developed at UOW are described briefly hereafter. Indraratna et al. (1999) also emphasised that accurately modelling joint dilation could result in good predictions of the shear behaviour of joints under CNS, so they used the Fourier series to fit the exact joint dilation from the test results. The typical Fourier series used in their study to model the dilation of joints with triangular-shaped asperities are as follows:

$$(\delta_{\text{v}})_h = \frac{a_0}{2} + \sum_{n=1}^{n}\left[a_n \cos\left(\frac{2\pi nh}{T}\right) + b_n \sin\left(\frac{2\pi nh}{T}\right)\right] \tag{18}$$

where $(\delta_{\text{v}})_h$ is the joint dilation (normal displacement) with respect to the shear displacement h, T is the maximum shear displacement, n is the harmonic numbers related to the accuracy of fitting, and a_n and b_n are the Fourier coefficients that can be determined based on experimental data. By considering the energy balance principles, Indraratna and Haque (2000) suggested a new form of shear stress equation coupled with Fourier coefficients:

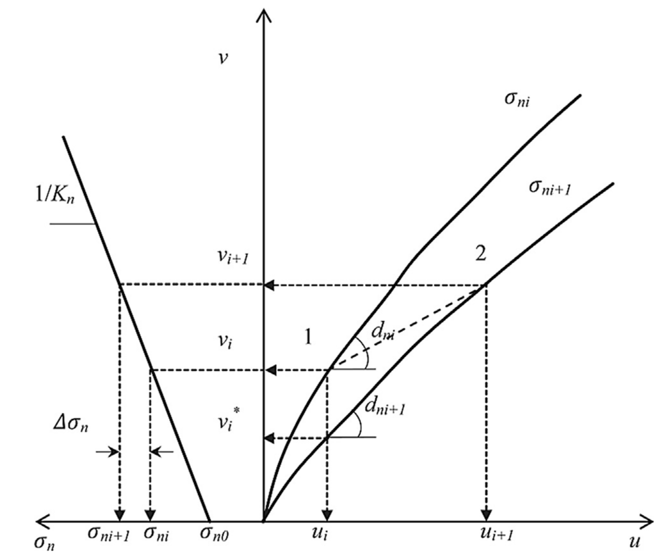

Fig. 7. The procedure for calculating dilation under CNS (modified from Skinas et al., 1990).

$$\tau_h = \left\{\sigma_{\mathrm{n}0} + \frac{K_{\mathrm{n}}}{A_{\mathrm{j}}}\left\{\frac{a_0}{2} + \sum_{n=1}^{n}\left[a_n \cos\left(\frac{2\pi nh}{T}\right) + b_n \sin\left(\frac{2\pi nh}{T}\right)\right]\right\}\right\} \cdot \frac{\tan\phi_{\mathrm{b}} + \tan i_0}{1 - \tan\phi_{\mathrm{b}} \tan i_h} \tag{19}$$

where τ_h is the shear stress at shear displacement h, i_h is the dilation angle at a shear displacement h, i_0 is the asperity angle, $\sigma_{\mathrm{n}0}$ is the initial normal stress, and A_{j} is the joint surface area.

Based on enormous CNS direct shear test results on unfilled and infilled joints (Indraratna and Haque, 1997; Indraratna et al., 1998, 1999, 2005, 2008), Indraratna et al. (2010b) proposed a new shear displacement criterion for soil-infilled idealised saw-toothed joint profiles incorporating the infilled squeezing mechanism:

$$\tau_h = \left[\sigma_{\mathrm{n}0} + \frac{K_{\mathrm{n}}}{A_{\mathrm{j}}}(\delta_{\mathrm{v}})_h\right] \frac{\tan\phi_{\mathrm{b}} + \tan i_0}{1 - \tan\phi_{\mathrm{b}} \tan i}\eta + \tan\left(\phi_{\mathrm{fill}} + i\right)(1-\eta) \tag{20}$$

where $i = \tan^{-1}(\mathrm{d}\delta_{\mathrm{v}}/\mathrm{d}h)$ is the dilation, η is the squeezing factor and ϕ_{fill} is the friction angle of infilled material. For unfilled joints, Eq. (20) is similar to Eq. (19). Oliveira and Indraratna (2010) updated Eq. (20) to incorporate natural joint surface roughness.

Most recently, Indraratna et al. (2015) proposed a conceptual variation of the dilation rate ($\dot{v}$) with the ratio of shear displacement to peak shear displacement ($\delta_h/\delta_{h\text{-peak}}$) for a joint subjected to direct shear under CNS (see Fig. 8). This variation in the rate of dilation was characterised by three major zones on the basis of $\delta_h/\delta_{h\text{-peak}}$ where each zone describes joint dilation under CNS. When $1 < \delta_h/\delta_{h\text{-peak}} \le c_0$, dilation will be postponed and in the region defined by $c_0 < \delta_h/\delta_{h\text{-peak}} \le 1$, the dilation rate increases. The last zone is $\delta_h/\delta_{h\text{-peak}} > 1$ where the dilation rate decreases as a result of degradation of the joint surface asperities. To analytically describe this variation in the rate of dilation for these three different zones, the following equations were suggested by Indraratna et al. (2015):

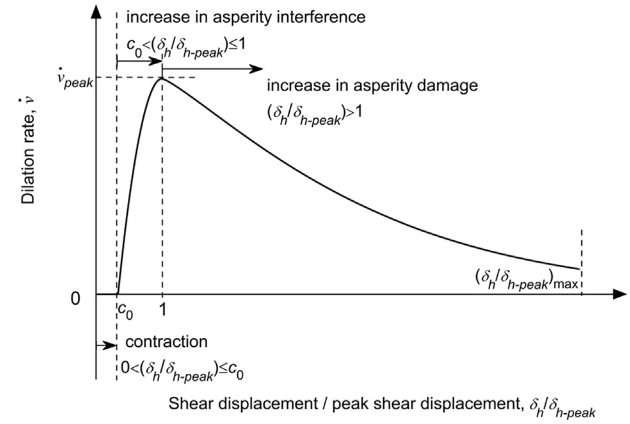

Fig. 8. A concept to model the variation of dilation rate with shear displacement (after Indraratna et al., 2015).

$$\dot{v} = \begin{cases} 0 \quad \left(0 < \delta_h \big/ \delta_{h-\mathrm{peak}} \le c_0\right) \\ \dot{v}_{\mathrm{peak}}\left[1 - \dfrac{1}{(c_0-1)^2}\left(\dfrac{\delta_h}{\delta_{h-\mathrm{peak}}} - 1\right)^2\right] \quad \left(c_0 < \delta_h \big/ \delta_{h-\mathrm{peak}} \le 1\right) \\ \dot{v}_{\mathrm{peak}} \exp\left\{-\left[c_1\left(\dfrac{\delta_h}{\delta_{h-\mathrm{peak}}} - 1\right)\right]^{c_2}\right\} \quad \left(\delta_h \big/ \delta_{h-\mathrm{peak}} > 1\right) \end{cases} \tag{21}$$

where c_0 is the ratio of $\delta_h/\delta_{h\text{-peak}}$ at which dilation is assumed to begin, c_1 and c_2 are the decay constants, and $\dot{v}_{\mathrm{peak}}$ is the peak dilation rate which can be calculated by $\dot{v}_{\mathrm{peak}} = \tan\beta/[1 - K_{\mathrm{n}}(-\alpha \sec^2\beta + \lambda)]$, in which $\alpha = \delta_{h\text{-peak}} JRC a\pi/180 M\sigma_{\mathrm{n}0}\ln 10$, $\beta = (1/M) JRC \log_{10}(JCS/\sigma_{\mathrm{n}0})$, $\lambda = k_{\mathrm{ni}}V_{\mathrm{m}}^2/(k_{\mathrm{ni}}V_{\mathrm{m}} + \sigma_{\mathrm{n}0})^2$, where the damage coefficient M was either 1 or 2 for shearing under low or high normal stress, respectively; and V_{m} is the maximum closure of the joint.

By adopting the concept of mobilised roughness as proposed by Barton (1982), Indraratna et al. (2015) proposed the following equation to calculate the mobilised shear stress τ_{mob} for CNS:

$$\tau_{\mathrm{mob}} = \left(\sigma_{\mathrm{n}0} + K_{\mathrm{n}}\int_0^{\delta_h} \dot{v}\,\mathrm{d}\delta_h\right)\frac{\tan\phi_{\mathrm{b}} + \dot{v}}{1 - \dot{v}\tan\phi_{\mathrm{b}}} \tag{22}$$

Eq. (22) can only be used to predict the shear behaviour of a joint when the asperities begin to mobilise at the joint interface, so Eq. (22) does not describe the shear behaviour within a small range of strain when shearing begins. Therefore, Indraratna et al. (2015) assumed the shear behaviour was elastic for the initial small range of shear displacement and then they proposed the following equation to calculate the complete shear stress–shear displacement behaviour of joint under CNS:

$$\tau = \begin{cases} k_{\mathrm{s}}\delta_h & (k_{\mathrm{s}}\delta_h < \tau_{\mathrm{mob}}) \\ \tau_{\mathrm{mob}} & (\text{otherwise}) \end{cases} \tag{23}$$

where k_{s} is the shear stiffness of the joint.

3. Discussion

Most of the existing modelling techniques emphasized that by modelling the dilation of a joint under CNS, the complete shear behaviour of the joint under a CNS stress path can be described. The approaches used to model the dilation of a joint under CNS were to some degree complex compared to the modelling technique used in CNL, because the normal stresses in a joint change continuously during shearing. Highlights and limitations of existing modelling techniques are given in Table 1. A few attempts were made graphically to estimate joint dilation under CNS by using the existing dilation data from CNL conditions (e.g. Saeb and Amadei, 1990; Skinas et al., 1990). In order to use these graphical techniques to predict the shear behaviour of a joint under CNS at a given initial normal stress, a large number of CNL direct shear results would be required. Whereas the conceptual modelling approaches can be used analytically to predict the shear behaviour of a joint under CNS, their constitutive equations consisted of many material constants that may reduce the efficiency of the analysis (e.g. Leichnitz, 1985).

The modelling methods proposed by Leichnitz (1985), Saeb and Amadei (1990, 1992) and Skinas et al. (1990) were based on an assumption that the shear behaviour of a rock joint is independent of the stress history, but this assumption may not always be applicable, because increasing normal stress during shearing may cause different levels of asperity damage along the joint interface (e.g. Indraratna and Haque, 2000). Although, the models proposed by Heuze (1979), Indraratna and Haque (2000), Seidel and Haberfield (2002) and Indraratna et al. (2010b) included a dependence on the CNS stress path, they may not represent the true behaviour of natural joints, because these models were only validated for synthetic joint surfaces with regular shaped asperities. Furthermore, as Indraratna et al. (2010a) suggested, in practice, a considerable number of Fourier coefficients are needed to accurately predict the shear behaviour of rock joints, and this is often cumbersome. The most recent model developed by Indraratna et al. (2015) has incorporated asperities damage under CNS conditions. While the value of CNS (K_n = 0.56 MPa/mm) used in their study is reasonable for a sedimentary jointed rock, a different range of K_n values may be required for stiffer rock types. Although their analytical model was only validated for three different types of rough joints (JRC = 7.3, 10.4 and 15.3) with a range of JRC/σ_{n0} = 41–164, further model validation is needed for different values of JRC and a wider range of JRC/σ_{n0} in order to use them effectively in practical applications.

Table 1
Highlights and limitations of existing modelling techniques.

Modelling techniques	Investigator(s)	Highlights	Limitations
Graphical method	Saeb and Amadei (1990)	Original graphical method of Goodman (1980) was extended to predict the complete shear behaviour of the joint under CNS condition	A large volume of experimental results under CNL were required to predict the shear behaviour of joint under CNS at a given initial normal stress; The shear behaviour of the joint was assumed as independent of the stress history
	Skinas et al. (1990)	A new graphical method was used to calculate mobilised joint dilation under CNS and predict the complete joint shear behaviour; Surface roughness and compressive strength (JRC and JCS) of the natural joint were included	CNL stress history and test results were used to develop the model; The joint shear behaviour was assumed as independent of the stress history
Conceptual approach	Heuze (1979)	Joint shear behaviour was considered as dependent of the CNS stress path; Bi-dilation concept was used to calculate the peak CNS dilation angle	Applicable for joints with triangular or saw-tooth asperity profiles; Only the peak shear strength of joint can be predicted
	Leichnitz (1985)	Constitutive equations were proposed with a well-defined stiffness matrix	Shear behaviour of joint was considered as independent of the stress history; Constitutive equations consisted of numerous material constants; The roughness of joint surface was not incorporated in the model
	Saeb and Amadei (1992)	Constitutive equations similar to Leichnitz's (1985) equations were proposed with a revised stiffness matrix, in which a simplified joint surface roughness parameter was included	Shear behaviour of joint was considered as independent of the stress history; Joint surface roughness was simplified using an initial asperity angle; Model was validated with limited test data
	Indraratna and Haque (2000)	Shear behaviour of joint was considered as dependent of the CNS stress path; Fourier series was used to obtain the optimum joint dilation	Model was validated with the test results obtained for regular joint profiles (triangular or saw-tooth shaped); A large number of Fourier coefficients were required to predict the complete joint shear behaviour
	Seidel and Haberfield (2002)	Shear behaviour of joint was considered as dependent of the CNS stress path; Different values of joint stiffness (i.e. concrete/rock or rock/rock joint interfaces) were incorporated	Joint surface roughness was simplified using an initial asperity angle; Natural joint surfaces were not tested; Validated with limited experimental data
	Indraratna et al. (2010b)	Shear behaviour of joint was considered as dependent of the CNS stress path; Natural joint profile and infill material were considered; Fourier series was used to obtain the optimum joint dilation	A large number of Fourier coefficients were required to predict the complete joint shear behaviour; Validated with limited experimental data
	Indraratna et al. (2015)	Shear behaviour of joint was considered as dependent of the CNS stress path; A new simplified (conceptual) joint dilation model was introduced to predict the complete shear behaviour of joint; The natural joint surface roughness and its compressive strength (JRC and JCS) were considered in the model; Asperity degradation under CNS was incorporated in the model	Validated with limited experimental data

4. Conclusions

This paper mainly reviewed the existing shear strength models of rock joint under CNS boundary conditions. Particular attention was given to modelling techniques as well as the highlights and limitations of developed models. Although several methods were used to model the shear behaviour of rock joint under CNS, the main objective of these methods was to predict joint dilation under CNS. The accuracy of model predictions generally depends on how well one could predict the normal displacement (dilation or compression) while taking into account the asperity geometry, strength, and degradation as the normal stress changes or asperity contact area reduces. This has been a difficult question for long time in rock mechanics. A very few existing models, although they have not addressed this comprehensively, have advanced this aspect further to make predictions more rational. Finally, a large data base of CNS direct shear test results is needed to check the validation of existing models before recommending their use in rock engineering applications.

Conflict of interest

We wish to confirm that there are no known conflicts of interest associated with this publication and there has been no significant financial support for this work that could have influenced its outcome.

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