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Corrections to $\sin(2\beta)$ from CP asymmetries in $B^0 \to (\pi^0, \rho^0, \eta, \eta', \omega, \phi) K_S$ decays

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Abstract

Neglecting smaller amplitudes the time-dependent CP asymmetry in penguin-dominated $b \to sq\bar{q}$ transitions (such as $B \to \phi K_S$) is expected to equal $\pm \sin(2\beta)$, an expectation not borne out by the present average experimental data. I compute and discuss the correction due to the smaller amplitudes in the framework of QCD factorization.

1. Introduction

The angle β of the unitarity triangle has been determined to $\sin(2\beta) = 0.725 \pm 0.037$ [1] from time-dependent CP asymmetries in $b \to c\bar{c}s$ transitions. If sub-leading decay amplitudes can be neglected as argued in [2], time-dependent CP asymmetries in penguin-dominated $b \to sq\bar{q}$ transitions should also take the value $\pm \sin(2\beta)$. There now exist various measurements [3], which *on average* point to the significantly smaller value 0.43 ± 0.07. It is not inconceivable that flavour-specific new flavour-violating interactions cause anomalous effects in $b \to s$ transitions without resulting in inconsistencies with other measurements. This would be a rather spectacular resolution of the apparent discrepancy. But before this conclusion can be drawn, a thorough study of the sub-leading decay amplitudes is necessary to ascertain the Standard Model expectation. This is undertaken here in the framework of QCD factorization [4].

The analysis is based on the next-to-leading order (NLO) factorization calculations performed in [5], where numerical values of the time-dependent CP asymmetries for the ϕK_S and $\eta' K_S$ final states have already been given. In this Letter I include a larger set of final states (see also the recent work [6–8]), and consider a more detailed error estimation that includes a scan of the theoretical parameter space [9]. I also discuss constraints on the sub-leading decay amplitudes that do not rely on factorization but are inspired by it. Another method to constrain the differences

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of time-dependent CP asymmetries in $b \to c\bar{c}s$ and $b \to s\bar{s}q$ transitions based on systematic approximations to the strong interactions relies on the assumption of SU(3) flavour symmetry. This results in bounds on the magnitude of this difference, but the sign cannot be determined [10]. An estimate in a model of (long-distance?) final state interactions is given in [6].

The time-dependent CP asymmetry in decays to CP eigenstates is given by

$$\frac{\mathrm{Br}(\bar{B}^0(t) \to f) - \mathrm{Br}(B^0(t) \to f)}{\mathrm{Br}(\bar{B}^0(t) \to f) + \mathrm{Br}(B^0(t) \to f)} \equiv S_f \sin(\Delta m_B t) - C_f \cos(\Delta m_B t), \tag{1}$$

with Δm_B the $B^0\bar{B}^0$ mass difference. The $\bar{B}$ decay amplitude involves two weak couplings $V_{pb}V_{ps}^*$ and two strong interaction amplitudes a_f^p. I write

$$A(\bar{B} \to f) = V_{cb}V_{cs}^* a_f^c + V_{ub}V_{us}^* a_f^u \propto 1 + e^{-i\gamma} d_f, \tag{2}$$

where

$$d_f = \epsilon_{\mathrm{KM}} \frac{a_f^u}{a_f^c} \equiv \epsilon_{\mathrm{KM}} \hat{d}_f \quad \text{with} \quad \epsilon_{\mathrm{KM}} = \left| \frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*} \right| \sim 0.025. \tag{3}$$

A standard calculation now gives

$$\Delta S_f \equiv -\eta_f S_f - \sin(2\beta) = \frac{2\,\mathrm{Re}(d_f)\cos(2\beta)\sin\gamma + |d_f|^2(\sin(2\beta + 2\gamma) - \sin(2\beta))}{1 + 2\,\mathrm{Re}(d_f)\cos\gamma + |d_f|^2}, \tag{4}$$

$$A_{\mathrm{CP},f} \equiv -C_f = \frac{2\,\mathrm{Im}(d_f)\sin\gamma}{1 + 2\,\mathrm{Re}(d_f)\cos\gamma + |d_f|^2}. \tag{5}$$

Here η_f denotes the CP eigenvalue of f. (All final states discussed below have $\eta_f = -1$.) The quantity ΔS_f is the central object of this Letter. One notes that (a) d_f is suppressed by a small ratio of CKM elements, ϵ_{KM}, leading to the expectation that $-\eta_f S_f \approx \sin(2\beta)$ (see above); (b) if d_f is small as expected, then to first order in d_f the two asymmetries S_f and C_f involve *independent* hadronic parameters, namely the dispersive and absorptive part of $\hat{d}_f = a_f^u/a_f^c$.

2. Anatomy of ΔS_f in factorization

The hadronic amplitudes a_f^p, $p = u, c$ are sums of "topological" amplitudes, referring to tree (T, C), QCD penguin (P^p), singlet penguin (S^p), electroweak penguin (P_{EW}^p, $P_{\mathrm{EW},C}^p$) and annihilation contributions. The relation to the "flavour" amplitudes used in QCD factorization [5] is $T \leftrightarrow \alpha_1$, $C \leftrightarrow \alpha_2$, $P^p \leftrightarrow \alpha_4^p + \beta_3^p$, $S^p \leftrightarrow \alpha_3^p + \beta_{S3}^p$, and $(P_{\mathrm{EW}}^p, P_{\mathrm{EW},C}^p) \leftrightarrow (\alpha_{3,\mathrm{EW}}^p, \alpha_{4,\mathrm{EW}}^p)$ with the difference that the α_i exclude form factors, decay constants and the CKM factors, while the topological amplitudes exclude only the CKM factor. In addition, a penguin amplitude such as P^c may be a sum of several α_4^p terms depending on the flavour flow to the final state. The expressions for all relevant decay amplitudes in terms of flavour amplitudes are collected in Appendix A of [5]. Schematically, for the strangeness-changing decays $\bar{B}^0 \to M\bar{K}^0$, the hadronic amplitude ratio is given by

$$d_f \sim \epsilon_{\mathrm{KM}} \frac{\{P^u, C, \ldots\}}{P^c + \cdots}, \tag{6}$$

where the dominant amplitudes have been indicated. Note that the amplitudes $P^u, C, \ldots$ depend on the final state f.

In the QCD factorization framework the topological amplitudes are computed in the form [4]

$$
\begin{aligned}
T, C, P^{c,u}, \ldots = \sum_{\text{terms}} C(\mu_h) \times \Big\{ & F^{BM_1} \times \underbrace{T^{\mathrm{I}}(\mu_h,\mu_s)}_{1+\alpha_s+\cdots} \star f_{M_2}\Phi_{M_2}(\mu_s) \\
& + f_B\Phi_B(\mu_s) \star \big[\underbrace{T^{\mathrm{II}}(\mu_h,\mu_I)}_{1+\cdots} \star \underbrace{J^{\mathrm{II}}(\mu_I,\mu_s)}_{\alpha_s+\cdots}\big] \star f_{M_1}\Phi_{M_1}(\mu_s) \star f_{M_2}\Phi_{M_2}(\mu_s)\Big\} \\
& + 1/m_b\text{-suppressed terms}
\end{aligned}
\tag{7}
$$

reducing the hadronic input to form factors F^{BM} and light-cone distribution amplitudes Φ_X. The underbraces indicate the order in perturbation theory to which the various short-distance kernels are computed at NLO. The numerical implementation of (7) also includes some $1/m_b$ power corrections from scalar penguin operators, and from an estimate of annihilation topologies. The accuracy of the treatment is generically limited by $\Lambda_{\mathrm{QCD}}/m_b \sim$ (10–20)% at the amplitude level.

The actual uncertainties affect different observables to a different degree and must be estimated on a case-by-case basis. The "colour-allowed" amplitudes T, P^p_{EW} are rather certain, while the "colour-suppressed" amplitudes C, $P^p_{\mathrm{EW},C}$ receive contributions from spectator scattering (the second line of (7)) enhanced by large Wilson coefficients, and are inflicted by larger uncertainties. The QCD penguin amplitudes include uncertain annihilation contributions, although the ratio P^u/P^c is less affected. Finally, the singlet amplitude S^p involves several specific decay mechanisms [11], which are difficult to compute quantitatively, though none of them seems to be of particular importance for the CP asymmetries. Eq. (6) indicates that ΔS_f involves some of the less certain amplitudes.

The numerical analysis below takes into account all flavour amplitudes following [5], but it suffices to focus on a few dominant terms to understand the qualitative features of the result. Then, for the various final states, the relevant hadronic amplitude ratio is given by

$$
\begin{aligned}
&\pi^0K_S \quad \hat{d}_f \sim \frac{[-P^u]+[C]}{[-P^c]}, &&\rho^0K_S \quad \hat{d}_f \sim \frac{[P^u]-[C]}{[P^c]}, \\
&\eta'K_S \quad \hat{d}_f \sim \frac{[-P^u]-[C]}{[-P^c]}, &&\phi K_S \quad \hat{d}_f \sim \frac{[-P^u]}{[-P^c]}, \\
&\eta K_S \quad \hat{d}_f \sim \frac{[P^u]+[C]}{[P^c]}, &&\omega K_S \quad \hat{d}_f \sim \frac{[P^u]+[C]}{[P^c]}.
\end{aligned}
\tag{8}
$$

The convention here is that quantities in square brackets have positive real part. (Recall from (4) that ΔS_f mainly requires the real part of $\hat{d}_f$.) In factorization $\mathrm{Re}[P^u/P^c]$ is near unity, roughly independent of the particular final state, hence ΔS_f receives a nearly universal, small and *positive* contribution of about $2\epsilon_{\mathrm{KM}}\cos(2\beta)\sin\gamma \approx 0.03$. On the contrary the magnitudes and signs of the penguin amplitudes' real parts can be very different. Ignoring uncertainties, I find $|\mathrm{Re}[P^c]|$ in the proportions

$$
\begin{array}{c}
\pi^0K : \rho^0K : \eta'K : \phi K : \eta K : \omega K \\
1 \;\; : \; 0.5 \; : \; 2.2 \; : 0.8 : 0.5 : 0.5
\end{array}
\tag{9}
$$

Hence the influence of the colour-suppressed tree amplitude C determines the difference in ΔS_f between the different modes. For $(\pi^0, \eta, \omega)K_S$ the effect of C is constructive, but for $(\rho, \eta')K_S$ it is destructive. However, the magnitude of $\mathrm{Re}[P_c]$ is much larger for $\eta'K_S$ than for ρK_S, hence $\mathrm{Re}(\hat{d}_f)$ remains small and positive for the former final state, but becomes negative for the latter.

3. Factorization results

The result of the calculation of ΔS_f is shown in Table 1. The column labeled "ΔS_f (Theory)" uses the input parameters (CKM parameters, strong coupling, quark masses, form factors, decay constants, moments of light-cone

Table 1
Comparison of theoretical and experimental results for ΔS_f

Mode	ΔS_f (Theory)	ΔS_f [Range]	Experiment [3] (BaBar/Belle)
$\pi^0 K_S$	$0.07^{+0.05}_{-0.04}$	$[+0.02, 0.15]$	$-0.39^{+0.27}_{-0.29}$ $(-0.38^{+0.30}_{-0.33}/-0.43^{+0.60}_{-0.60})$
$\rho^0 K_S$	$-0.08^{+0.08}_{-0.12}$	$[-0.29, 0.02]$	–
$\eta' K_S$	$0.01^{+0.01}_{-0.01}$	$[+0.00, 0.03]$	$-0.30^{+0.11}_{-0.11}$ $(-0.43^{+0.14}_{-0.14}/-0.07^{+0.18}_{-0.18})$
ηK_S	$0.10^{+0.11}_{-0.07}$	$[-1.67, 0.27]$	–
ϕK_S	$0.02^{+0.01}_{-0.01}$	$[+0.01, 0.05]$	$-0.39^{+0.20}_{-0.20}$ $(-0.23^{+0.26}_{-0.25}/-0.67^{+0.34}_{-0.34})$
ωK_S	$0.13^{+0.08}_{-0.08}$	$[+0.01, 0.21]$	$-0.18^{+0.30}_{-0.32}$ $(-0.23^{+0.34}_{-0.38}/+0.02^{+0.65}_{-0.66})$

distribution amplitudes) summarized in Table 1 of [5]. In particular $|V_{ub}/V_{cb}| = 0.09 \pm 0.02$ and $\gamma = (70 \pm 20)^\circ$ is used. The uncertainty estimate is computed by adding in quadrature the individual parameter uncertainties. The central values are in good agreement with those given in [6], which also uses the input from [5]. For the final states $\rho^0 K_S$ and ωK_S they differ from those given in [7], where the leading order (naive factorization) approximation is employed, and the electroweak penguin amplitudes are neglected. The next-to-leading order correction included in the present calculation has a large impact on the branching fractions of penguin-dominated modes and is crucial for a successful comparison of QCD factorization results with data. Nonetheless, the NLO correction to ΔS_f is never larger than about 30%, since the amplitude enhancement partially cancels in the ratio $\hat{d}_f$. The NLO correction also eliminates the large renormalization scale uncertainty present at leading order.

The result displays the anticipated pattern. The variation of the central value from the nearly universal contribution of approximately ϵ_{KM} is due to $\mathrm{Re}[C/P^c]$, and the error comes primarily from this quantity. It is therefore dominated by the uncertainty in the hard-spectator scattering contribution to C, and the penguin annihilation contribution to P^c. In general one expects the prediction of the asymmetry S_f in factorization to be more accurate than the prediction of the direct CP asymmetry C_f, since S_f is determined by $\mathrm{Re}(a_f^u/a_f^c)$ which is large and calculated at next-to-leading order, while C_f is determined by $\mathrm{Im}(a_f^u/a_f^c)$, which is small and currently known only at leading order. The resultant error on ΔS_f is roughly of the size of ΔS_f itself. Since this is small, one arrives at accurate constraints, in particular for the final states $\eta' K_S$ and ϕK_S. It is striking that the theoretical prediction of ΔS_f is positive, with the exception of $\rho^0 K_S$, while the experimental data are all negative.

Quadratic addition of theoretical errors may not always lead to a conservative error estimate. Furthermore, the default parameters adopted in [5] do not lead to the best description of the data. As shown there, a different choice of a few parameters (defining certain "scenarios") results in a very good description of data—however, some observables, in particular the colour-suppressed tree amplitude C important to the present discussion, then take values outside the range estimated by quadratic error estimation. To allow for this possibility I perform a random scan of the allowed theory parameter space. For any observable I take the minimal and maximal value attained in this scan to define the predicted range of this observable. However, in doing so I discard all theoretical parameter sets which give CP-averaged branching fractions not compatible within 3 sigma with the experimental data, that is I require $8.5 < 10^6\,\mathrm{Br}(\pi^0 K^0) < 14.5$, $0.3 < 10^6\,\mathrm{Br}(\rho^0 K^0) < 9.9$, $5.3 < 10^6\,\mathrm{Br}(\phi K^0) < 11.9$, $2.9 < 10^6\,\mathrm{Br}(\omega K^0) < 8.3$, $10^6\,\mathrm{Br}(\eta K^0) < 6.0$. No further condition is imposed, neither from the corresponding charged decay modes, nor any other decay, or from direct CP asymmetries (since these depend on other hadronic parameters as mentioned above). Note that I also do not require the theoretical parameters to reproduce the $\eta' K^0$ branching fraction. The reason for this is that in [5] the singlet contribution F_2 to the $B \to \eta'$ form factor is set to zero simply for lack of better information. Since a non-zero F_2 can affect the branching fraction significantly [11], requiring the $\eta' K^0$ branching fraction to reproduce the data for $F_2 = 0$ would be overly restrictive on the remaining theory parameter space. Nevertheless, one finds that the distribution of $B^0 \to \eta' K^0$ branching fractions generated by the models that survive the other branching fraction restrictions has a (broad) maximum at 67×10^{-6} in nice agreement with experimental data.

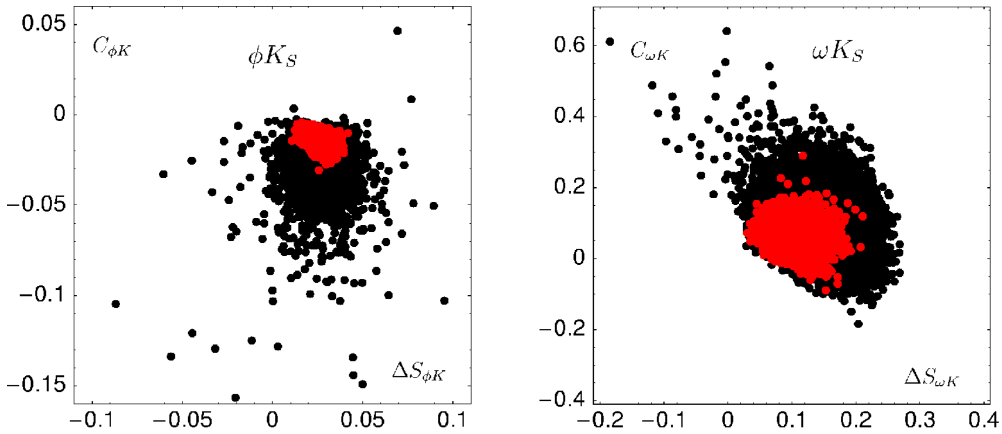

Fig. 1. Correlation between ΔS_f and C_f (direct CP asymmetry) for $f = \phi K^0$ (left) and $f = \omega K^0$ (right). Theory parameter models compatible with the experimental branching fractions (as described in the text) are in grey (red), all others in black. Based on a sample of 50000 input parameter models.

The resulting ranges for ΔS_f from a scan of 200000 theoretical parameter sets is shown in the column labeled "ΔS_f [Range]" in Table 1. It is seen that the ranges are in fact not much different from those obtained by adding parameter uncertainties in quadrature—except for the ηK_S final state, for which almost any value of S_f is possible. To understand this exception, one must know that similarly large ranges can appear also for other final states when no branching fraction restriction is imposed. These large values of ΔS_f originate from small regions of the parameter space, where by cancellations the leading penguin amplitude P_c becomes very small. This leads to large amplifications of C/P^c, and hence ΔS_f. Such small values of P^c always lead to very small branching fractions, hence they are excluded by observations except for the case of ηK_S, where no lower limit on the branching fraction exists at present.

The parameter scan contains more interesting pieces of information than the ranges of ΔS_f, since it allows to establish correlations between ΔS_f and input parameters, between the ΔS_f for different final states, etc. in the framework of QCD factorization. For instance, one finds that the "good" models prefer a strange quark mass around 80 MeV, smaller renormalization scales and a moderate annihilation contribution $\rho_A \approx 0.7 e^{i\phi_A}$ with $|\phi_A| < 70°$, all of which affects the magnitude of the dominant QCD penguin amplitude. Space does not permit a detailed discussion here, but Fig. 1 shows the correlation between ΔS_f and the direct CP asymmetry C_f (see (1), (5)) taking $f = \phi K^0$ and ωK^0 as examples. The distribution of points (each corresponding to one theoretical parameter set) does not reveal any particular correlation between the two observables, especially after the branching fraction restriction, as could have been guessed from the fact that they mainly involve independent hadronic parameters. The figure also shows that the requirement that the experimental branching fractions be reproduced within 3 sigma narrows the distribution considerably. Similar conclusions apply to all other final states.

4. Discussion

Given the important role of ΔS_f in the detection of anomalous $b \to s$ flavour transitions, one may question the assumptions that go into the factorization approach or attempt to find independent validations. Also, given the current experimental status, it would already be interesting to know that ΔS_f should be positive, no matter its precise value. Can one establish $\Delta S_f > 0$ (except for ρK_S) with little assumptions on hadronic physics?

Recall from (4) that ΔS_f is roughly

$$2\epsilon_{\mathrm{KM}}\cos(2\beta)\sin\gamma\,\mathrm{Re}\left(\frac{a_f^u}{a_f^c}\right). \tag{10}$$

Large enhancements relative to the factorization predictions require an enhancement of the hadronic amplitude ratio. The first option is a strong suppression of a_f^c, but this is excluded by the branching fraction measurements (see also the discussion in the previous section). The second option is an enhancement of a_f^u by a factor of several. Can this be excluded, or can at least the sign of $\mathrm{Re}(a_f^u/a_f^c)$ be determined?

The only approach to non-leptonic decays other than factorization based on a small expansion parameter uses SU(3) flavour symmetry to relate amplitudes of final states belonging to the same SU(3) multiplet. In applications of the method to ΔS_f one uses the branching fractions of $b\to d$ transitions to bound $|d_f|$ of the related $b\to s$ transitions [10]. The best possible limit in this method is $|d_f|<\lambda^2\approx 0.05$ (λ the Wolfenstein parameter), so the *theoretical* limit of this method is $|\Delta S_f|\lesssim 0.07$. In practice, depending on the values of the $b\to d$ branching fractions and the final state f, the bound is considerably weaker, although the region of interesting values (indicated by the factorization results) may eventually be approached for some final states. Note that the sign of ΔS_f is not determined by this method.[1] (See, however, the last reference of [10], where additional information is supplied through a general amplitude fit based on SU(3) *and* the further assumption that some amplitudes can be neglected.)

A limited amount of information can be obtained from final states related to the given one by isospin symmetry, or from other observables related to the given final state. As already mentioned above, the measurement of the direct CP asymmetry (C_f) is of limited use if it is small, since it constrains the imaginary part of a_f^u/a_f^c rather than the real part. On the other hand, a very large direct CP asymmetry (for ϕK_S, $\eta' K_S$, $\pi^0 K_S$) would suggest that $\mathrm{Re}(d_f)$ could also be large, but this is not rigorous. It would certainly imply large violations of factorization, and hence cast doubt on the results in Table 1. No such large direct CP asymmetries have been observed to date for the final states discussed here.

The asymmetry S_f is more closely related to ratios of CP-averaged branching fractions, which also depend mainly on real parts of amplitude ratios. In the following I consider the pairs $(M\bar{K}^0, MK^-)$, including the charged partners of M for $M=\pi,\rho$. The decay amplitudes can be parameterized as

$$\begin{aligned}
&A\bigl(M^-\bar{K}^0\bigr)=P+e^{-i\gamma}P^u,\\
&\sqrt{2}A\bigl(M^0K^-\bigr)=\bigl[P+P^{\mathrm{EW}}\bigr]+e^{-i\gamma}\bigl[T+C+P^u\bigr],\\
&A\bigl(M^+K^-\bigr)=\bigl[P+P^{C,\mathrm{EW}}\bigr]+e^{-i\gamma}\bigl[T+P^u\bigr],\\
&\sqrt{2}A\bigl(M^0\bar{K}^0\bigr)=\bigl[-P+P^{\mathrm{EW}}-P^{C,\mathrm{EW}}\bigr]+e^{-i\gamma}\bigl[C-P^u\bigr]
\end{aligned} \tag{11}$$

for $M=\pi,\rho$ (assuming isospin symmetry), and

$$\begin{aligned}
&A\bigl(MK^-\bigr)=\bigl[P+P^{C,\mathrm{EW}}\bigr]+e^{-i\gamma}\bigl[T+C+P^u\bigr],\\
&A\bigl(M\bar{K}^0\bigr)=P+e^{-i\gamma}\bigl[C+P^u\bigr]
\end{aligned} \tag{12}$$

[1] It may be noted that the application of the SU(3) approach to final states containing η, η', ω and ϕ requires additional assumptions beyond SU(3). In the SU(3) limit these mesons would be pure octet or singlet states, but reality is far from this limit, in particular in the case of ω and ϕ, which are believed to be pure up–down and strange quark states, respectively. In [10] this SU(3) breaking singlet–octet mixing effect is taken into account by assuming that the operator matrix elements with the physical meson states are related to those with the putative SU(3) states by a single mixing angle. This is an assumption that cannot be justified in any controlled approximation [12]. Rather one must introduce a separate mixing angle for every operator. The existence of large mixing for ω–ϕ and η–η' should be taken as an indication that a SU(3) treatment might be unreliable, since for every operator a separate, presumably large, mixing angle must be introduced. Phenomenological evidence related to the matrix elements of current operators may indicate that this SU(3) breaking effect is nearly universal and could be described by a single mixing angle in the quark–flavour-basis [13], but little is known about the matrix elements of the effective weak Hamiltonian.

Table 2
Estimates of the *real part* of the amplitude ratios x in scenario IV of [5]

Modes	t	c	p^u	p^{EW}	$p^{C,\text{EW}}$
πK	−0.13	−0.06	0.02	0.13	0.03
ρK	0.27	0.13	0.01	−0.29	−0.07
$\eta' K$	−0.03	−0.01	0.02	–	0.01
ηK	0.34	0.14	0.02	–	−0.05
ϕK	0.01	0.00	0.02	–	0.01
ωK	0.23	0.11	0.02	–	−0.08

for the remaining $M = \eta^{(\prime)}, \phi, \omega$. The notation is chosen so that it indicates the dominant contribution to each amplitude; the dependence of P, $P^u, \ldots$ on M is not spelled out. In (12) the CKM-suppressed penguin amplitude P^u is redundant and could be absorbed into C. For $M = \phi$ the "tree" amplitudes T, C are actually annihilation amplitudes and thus very small, provided ϕ is a pure $s\bar{s}$ state, as will be assumed here. It is clear from (12) that nothing can be learned from the charged decay for $M = \eta^{(\prime)}, \phi, \omega$ without additional assumptions, since it involves two new amplitudes (the colour-suppressed electroweak penguin $P^{C,\text{EW}}$, and T). However, I shall now expand the ratios of CP-averaged branching fractions under the premise that certain amplitude ratios are small. To this end, note that T, C, P^u which multiply $e^{-i\gamma}$ are proportional to ϵ_{KM}, while the electroweak penguin amplitudes are suppressed by the electromagnetic coupling. Defining $x \equiv X/P$ and counting $\epsilon_{\text{KM}} \sim \lambda^2$ with λ a counting parameter of order $1/5$, the natural magnitudes of the amplitude ratios are t, $p^{\text{EW}} \sim \lambda$, and c, p^u, $p^{C,\text{EW}} \sim \lambda^2$. Estimates of the real parts of the amplitude ratios are given in Table 2 using the scenario IV of [5] as input. In the following discussion, c and p^u are allowed to be enhanced to order λ.

Turning first to $f = \eta^{(\prime)}K, \phi K, \omega K$, Eq. (12) implies that even when an enhancement of the amplitudes c, p^u by a factor of several to order λ is allowed, they do not appear in the ratio of CP-averaged branching fractions at first order in λ. Thus, with an accuracy of a few percent,

$$R(f) \equiv \frac{\tau_{B^0}\,\text{Br}(M^0 K^-)}{\tau_{B^+}\,\text{Br}(M^0 \bar{K}^0)} \approx 1 + 2\cos\gamma\,\text{Re}(t). \tag{13}$$

Hence $\text{Re}(t)$ can be determined from data, if $R(f)$ is sufficiently different from 1 (to justify the neglect of the order λ^2 terms), but $\Delta S_f \propto \text{Re}(c + p^u)$. The colour-allowed tree amplitude T is believed to be well-predicted in factorization, and has a small absorptive part. Assuming this, an accurate measurement of $R(f)$ for $f = \eta^{(\prime)}K, \omega K$ provides an estimate of $\text{Re}(P)$, of which the sign should be reliable. Making the same assumption for C constrains the contribution from $\text{Re}(c)$ to ΔS_f, but in this case the assumption is already questionable. The contribution from $\text{Re}(p^u)$ is not constrained as long as it is of order λ. However, one may argue that *if* $\text{Re}(p^u)$ is enhanced to order λ by whatever mechanism, then—*probably*—the absorptive part $\text{Im}(p^u)$, and hence the direct CP asymmetry, will also be of order λ. Similar arguments can be applied to the πK and ρK system (11). To linear order in λ

$$R(f) \approx \left|\frac{1 + p^{\text{EW}}}{1 - p^{\text{EW}}}\right|^2 \big(1 + 2\cos\gamma\,\text{Re}(t + 2c)\big). \tag{14}$$

The electroweak penguin amplitudes are now important. For ρK the corresponding prefactor reduces the branching fraction ratio by a factor of three. In fact, the contribution is so large that the linear approximation becomes inapplicable to the ρK final state. For πK, the complete set of three branching fraction ratios can be used in principle to determine the real parts of t, c and p^{EW} simultaneously with a relative uncertainty of order λ in the linear approximation. However, the current experimental πK data does not lead to useful results.

I conclude from this discussion that it is very difficult to constrain ΔS_f independent of theoretical assumptions using only experimental data (other than the measurement of ΔS_f itself). With some plausible dynamical assumptions bounds can be derived using SU(3), or the real parts and signs of amplitudes related to the quantities of interest can be determined and compared to the factorization calculations, thus providing cross-checks.

5. Conclusion

QCD factorization calculations of the time-dependent CP asymmetry in hadronic $b \to s$ transitions yield only small corrections to the expectation $-\eta_f S_f \approx \sin(2\beta)$. With the exception of the $\rho^0 K_S$ final state the correction ΔS_f is positive, slightly strengthening the discrepancy with the current average experimental data. The effect and theoretical uncertainty is particularly small for the two final states ϕK_S and $\eta' K_S$ already analyzed in [5]; the calculation of ΔS_f for the final states $\rho^0 K_S$ and ηK_S, however, is more susceptible to errors because of amplitude cancellations. The final-state dependence of ΔS_f is ascribed to the colour-suppressed tree amplitude.

It appears difficult to constrain ΔS_f theory-independently by other observables. In particular, the direct CP asymmetries or the charged decays corresponding to $f = M K_S$ probe hadronic quantities other than those relevant to ΔS_f, if these observables take values in the expected range. Large deviations from expectations such as large direct CP asymmetries would clearly indicate a defect in our understanding of hadronic physics, but even then the quantitative implications for S_f would be unclear. A hadronic interpretation of large ΔS_f would probably involve an unknown long-distance effect that discriminates strongly between the up- and charm-penguin amplitude resulting in an enhancement of the up-penguin amplitude. No model is known to me that could plausibly produce such an effect.

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