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On Mathematics in the History of Sub-Saharan Africa[1]

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FOR DIRK STRUIK ON HIS 100TH BIRTHDAY[2]

This article presents an overview of research findings and of sources on or related to mathematics in the history of Africa south of the Sahara. Topics such as counting and numeration systems, mathematical games and puzzles, geometry, graphs, and continental and international connections are included. Attention is paid to the objectives of research in the history of mathematics in Africa, to methodology, to the relationship with ethnomathematical research, and to the uses of research findings in mathematics education. Some possible directions for further research are also identified. © 1994 Academic Press, Inc.

Cet article présente un panoramique des résultats de recherche et des sources d'information relatifs à l'histoire de la mathématique en Afrique sub-saharienne. Il comporte des thèmes tels que le comptage et les systèmes de numération, les jeux et les puzzles mathématiques, la géométrie, les graphes, et les connexions continentales et internationales. L'accent est mis sur les objectifs de la recherche sur l'histoire de la mathématique en Afrique, sa méthodologie, sa relation avec la recherche ethnomathématique, et les usages des résultats de la recherche dans l'enseignement de la mathématique. Quelques directions possibles pour approfondir la recherche sont également identifiées. © 1994 Academic Press, Inc.

Este artigo apresenta uma vista panorâmica dos resultados de investigação e das fontes de informação relacionados com a história da matemática na África ao Sul do Sahara. Incluem-se tópicos tais como contagem e sistemas de numeração, jogos e puzzles matemáticos, geometria, grafos e conexões continentais e internacionais. Presta-se atenção aos objectivos da investigação respeitante à história da matemática em África, à metodologia, à relação com pesquisa etnomatemática e aos usos dos resultados de investigação, na educação matemática. Identificam-se também algumas direcções possíveis para continuar a investigação. © 1994 Academic Press, Inc.

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INTRODUCTION

In her classical study *Africa Counts: Number and Pattern in African Culture* [318] (reviewed in [309]), Claudia Zaslavsky presented an overview of the available

[1] The present article is an updated version of [120].

[2] During a three-hour television interview broadcast by VPRO Television (Hilversum, The Netherlands) on August 1, 1993, he presented a retrospective on his life, the 20th century, mathematics, and the historiography of mathematics; once more he showed his interest in the history of mathematics in Africa by explaining the geometry involved in the production of a basket from Mozambique.

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literature on mathematics in the history of sub-Saharan Africa. She discussed written, spoken, and gesture counting, number symbolism, concepts of time, numbers and money, weights and measures, record-keeping (sticks and strings), mathematical games, magic squares, graphs, and geometric forms, while Donald Crowe contributed a chapter on geometric symmetries in African art.

Since the publication of Zaslavsky's overview, many scholars, students, teachers and laymen alike—both in Africa and abroad—have become interested in the mathematical heritage of sub-Saharan Africa. The African Mathematical Union (AMU) formed its Commission on the History of Mathematics in Africa (AMUCHMA) in 1986 in order to stimulate research in the history of mathematics in Africa in general, and to promote the dissemination of research findings and the exchange of information in this field. In 1987 the AMUCHMA began publishing a newsletter in English, French, and Arabic.[3]

In this paper, an overview of research findings and of sources on or related to the history of mathematics in Africa south of the Sahara is presented, particularly studies that have appeared since the publication of *Africa Counts*. Topics such as counting and numeration systems, numerology, mathematical games and puzzles, geometry, graphs, Islamic influences on mathematical developments, international connections, and the history of mathematics curricula will be included. Attention will also be paid to the objectives of research in the history of mathematics in Africa, to methodology, to the relationship with ethnomathematical research, and to the uses of research findings in mathematics education. Some possible directions for further research are also identified.

WHY STUDY THE HISTORY OF MATHEMATICS IN SUB-SAHARAN AFRICA?

There are many reasons which make the general study of the history of mathematics both necessary and attractive (see, e.g., [279]). One of the most important reasons according to Dirk J. Struik—and certainly a valid one in the case of Africa—is that it "helps [us] to understand [the] cultural heritage, not only through the applications mathematics has had and still has to astronomy, physics and other sciences, but also because of the relation[s] . . . to such varied fields as art, religion, philosophy and the crafts" [279, 26]. Aside from Struik's general arguments, there exist important additional considerations which make the study of the history of mathematics in sub-Saharan Africa even more indispensable.

Most histories of mathematics devote only a few pages to ancient Egypt and to northern Africa during the 'Middle Ages.' Generally they ignore the history of mathematics in sub-Saharan Africa and give the impression either that this history is not knowable/traceable, or even stronger still, that there was no mathematics at all south of the Sahara (cf. the critics of [196, 225]). "Even the Africanity of

[3] Readers interested in receiving the *AMUCHMA Newsletter* may send their request to the author.

Egyptian mathematics is often denied" [271, 2]. Prejudice and narrow conceptions both of 'history' (cf., e.g., [172]) and of 'mathematics' form the basis of such Eurocentric views (cf., e.g., [161, 163]).

These factors only worsen the situation in mathematics education, in which African countries face the problem of low levels of attainment. Not only is "math anxiety" widespread, but many children (and teachers too!) experience mathematics as a rather strange and useless subject, imported from outside Africa. One of the causes of this is that the goals, contents, and methods of mathematics education are not sufficiently adapted to the cultures and needs of the African peoples (cf., e.g., [82, 346; 83; 158]). Today's existing African educational system is "unadapted and elitist" and "favours foreign consumption without generating a culture that is both compatible with the original civilization and truly promising" [173, 4].[4] To overcome these obstacles the mathematical heritage of African peoples has to be valued and African mathematical traditions need to be embedded into the curriculum [6, 64, 66, 99, 101, 102, 107, 112, 182, 183, 210, 225, 270, 271]. Since the scientific legacy of African cultures south of the Sahara is little known, research in this area constitutes a challenge to which a response is urgently necessary [225, 4]. In addition, African-Americans and minorities of African descent all over the world have experienced a growing desire to learn about their cultural–mathematical heritage [35, 91, 252, 318]. More generally, both in highly industrialized and in Third World countries it has become increasingly recognized that it is necessary to *multiculturalize* the mathematics curriculum in order to improve its quality, to augment the cultural confidence of all pupils, and to combat racial and cultural prejudice [8, 14, 29, 30, 161, 207, 220, 330, 333].

Thus, Zaslavsky's *Africa Counts* represents a pioneering work on mathematics and its history south of the Sahara. She offered her book as "a preliminary survey of a vast field awaiting investigation" [318, vi]. This task was not an easy one, considering "the inadequacy of easily accessible material . . ." she had to search "the literature of many disciplines—history, economics, ethnology, anthropology, archaeology, linguistics, art and oral tradition—. . ." [318, vi]. Zaslavsky's study deals with what she calls the *sociomathematics* of Africa: she considers "the applications of mathematics in the lives of African people, and, conversely, the influence that African institutions had upon the evolution of mathematics" [318, 7]. The concept of sociomathematics may be considered a forerunner of the concept of *ethnomathematics*. It is ethnomathematics as a discipline that studies mathematics (and mathematical education) as embedded in their cultural context.[5] (For the (possible) relationships between ethnomathematics and the history of mathematics, see [9] and (in the case of Africa) [112, 271]). The application of historical

[4] On mathematics education and the selection of élites, see [80, 3].

[5] In this sense "ethnomathematics" is closely related to the "sociology of mathematics" as founded by Struik [276, 280]. Cf. [123], where an analysis of several forerunners of the concept of ethnomathematics and related concepts is presented.

and ethnomathematical research methods has contributed, as will be shown, to a better knowledge and understanding of mathematics in the history of sub-Saharan Africa, as well as to an awareness of further mathematical elements in African traditions.

THE BEGINNINGS

As early evidence for (proto-) mathematical activity in Africa, Zaslavsky presented a bone, dated between 9000 and 6500 B.C., which was unearthed at Ishango (Zaire). The bone has what appear to be tallying marks on it, notches carved in groups. Its discoverer, De Heinzelin, interpreted the patterns of notches as an "arithmetical game of some sort, devised by a people who had a number system based on 10 as well as a knowledge of duplication and of prime numbers" [145, 110]. Marshack [200], however, explained the bone as an early lunar phase count. Their views, summarized in [318, 17–19], were reproduced in [88, 5–7]. Later, Marshack reevaluated the dating of the Ishango bone, setting it back from about 8,000 B.C. to 20,000 B.C. Zaslavsky raises the question "who but a woman keeping track of her cycles would need a lunar calendar?" and concludes that "women were undoubtedly the first mathematicians!" [334, 4].

Bogoshi *et al.* reported in 1987 on a still much older "mathematical artefact": "A small piece of the fibula of a baboon, marked with 29 clearly defined notches, may rank as the oldest mathematical artefact known. Discovered in the early seventies during an excavation of Border Cave in the Lebombo Mountains between South Africa and Swaziland, the bone has been dated to approximately 35,000 B.C." [31, 294]. They note that the bone "resembles calendar sticks still in use today by Bushmen clans in Namibia" [31, 294].

A research project looking for numerical representations in San (Bushmen) rock art has recently been started by Martinson [201, 202].[6] Lea and her students at the University of Botswana have also collected information from the surviving San hunters in Botswana. Her papers describe counting, measurement, time reckoning, classification, tracking, and some mathematical ideas in San technology and craft. The San developed very good visual discrimination skills and visual memory as needed for survival in the harsh environment of the Kalahari desert [185, 186, 188, 189, 190, 275].

NUMERATION SYSTEMS AND NUMBER SYMBOLISM

Zaslavsky's discussion of oral and gestural counting as well as written numeration systems drew heavily on the work of earlier investigators.[7] In the meantime other sources have also come to the fore, such as the studies [258, 262, 263] on

[6] Several rock drawings and engravings of southern Africa are also interesting from a geometrical point of view. See for example [69].

[7] [7] on Guinea Bissau; [12] on the Yoruba (Nigeria); [18, 53, 147] on the Kru (Liberia, Côte d'Ivoire); [181] on tally-systems; [198] on the Yoruba; [205] on Northern Nigeria; [229] on the Hima (Uganda); [253, 260 (overview), 284, 293, 311] on the Dabida (Kenya).

the Tchokwe (Angola), [144] on the Gun, Gen, and Bariba, and [300, 301] on the Bambara (Mali).

During recent years, a whole series of research projects on spoken and written numeration systems in Africa have been initiated. These include studies on:

* counting in traditional Ibibio and Efik societies (I.O. Enukoha, University of Calabar, Calabar, Nigeria);
* numeration among the Fulbe (Fulani) (S.O. Ale, Ahmadu Bello University, Bauchi, Nigeria; cf. [4]);
* pre-Islamic ways of counting (Y. Bello, Bayero University, Nigeria);
* mental arithmetic, algorithms, and counting among the different ethnic groups of Nigeria (Ahmadu Bello University, Zaria);[8]
* precolonial numeration systems in Burundi (J. Navez, University of Burundi, Bujumbara);
* methods of learning counting in Côte d'lvoire [296, 340];
* numeration systems used by the principal linguistic groups in Guinea (S. Oulare, University of Conakry);
* counting among the various ethnic groups in Kenya (J. Mutio, Kenyatta University, Nairobi);
* oral and possible graphic numeration systems from Zaire [214];
* numeration and geometric figures in Great Zimbabwe [215];
* traditional counting in Botswana (H. Lea, University of Botswana, Gaberone);
* number and pattern in selected cultures in Uganda (E. Segujja-Munagisa).

An important study—from the point of view of its contents and the methodological debate it initiated—is E. Kane's doctoral dissertation, "The spoken numeration systems of West-Atlantic groups and of the Mandé" [167]. Kane (Cheikh Anta Diop University, Dakar-Fann, Senegal) analyzed numeration systems in about 20 languages spoken in Senegal. He realized the necessity of basing his research on ethnomathematics and therefore tried to understand mathematical ideas in relation to the general culture in which they are embedded. Therefore he did preparatory research in four domains: African linguistics, history of numeration systems, ethnology, and African languages spoken in Senegal (as understood by interviewing many speakers of the same and different languages). His study showed that oral numeration systems, such as that of the Mandé, are susceptible to reform and evolution. Kane develops a methodology for the analysis of numeration systems that is adapted to the specificities of 'oral cultures.'

Mozambique's Instituto Superior Pedagógico has published a study of numeration systems and popular counting practices [125]. It includes an introduction to African systems of numeration [132] (see also [133, 128]), a survey of written and

[8] Shirley [269] presents an overview of B.Ed., M.Ed. and Ph.D. projects undertaken at the Faculty of Education of the Ahmadu Bello University (Zaria, Nigeria): [1, 4, 11, 81, 92, 146, 218, 236, 282]. Cf. also [6].

oral sources on numeration in 13 languages spoken in Mozambique, an analysis of popular counting methods [157] (see also [274]), comparative tables and maps about spoken numeration in Mozambique [199], and reflections on numeration and learning/teaching of arithmetic [72].

Zaslavsky dedicated a chapter to number symbolism and taboos on counting [318, 52–57]. The examples she gave are based, among other sources, on a study of number symbolism among the Ijo in Nigeria [312], where, for example, the number three is associated with men, the number four with women. Ojoade [235] published a paper on the number three in African lore, highlighting the sacredness, mysticism and taboos attached to it (cf. also [223]). In [237] objects of African art, mostly from the Yoruba (Nigeria), are analyzed as functions of the involved repetitions. The twofold objects evoke the standard dichotomies: good/bad, life/death; the threefold objects sometimes evoke a hierarchy; the fourfold objects may be associated with the directions in space. By systematically searching the ethnographic literature as well as novels, biographies, etc., it is likely that a good deal more information about number symbolism in African cultures will be found (see also [5]). For instance, the anthropological study of Thornton explains the significance of the number nine among the Iraqw of Tanzania [290, 96, 167, 183]. Number symbolism may have a rational basis as well. The Emakwa basket makers in northern Mozambique call odd numbers or odd quantities of plant strips 'ugly,' and they have good reasons for doing so as shown in [155]. (For an earlier discussion of 'even' and 'odd' numbers in basketry see [99].) Emakwa hunters always form groups with an even number of members [155]. [214] discusses the symbolic expression of numbers in Luba cosmogony (Zaire); for example, the significance of 'even' and 'odd' and the use of 'numbers of peace' such as 4, 12, 24, 48, and 96. Certainly, further collections of oral data are likely to shed new light on African numerology.

GAMES, RIDDLES, AND PUZZLES

Among the games with mathematical 'ingredients' referred to in [318, 102–136] are counting rhymes and rhythms, three-in-a-row-games, arrangements, games of chance, and board games. [329] gives more information on three-in-a-row-games in Africa. [257] (reviewed in [50]) presents the rules and a brief history of 'mancala' games (cf. [295]), also known as Ayo, Bao, Wari, Mweso, Ntchuva, etc. Zaslavsky [332] suggests that it could be important in the reconstruction of the history of mathematical thinking in Africa to investigate further mathematical aspects of traditional games. As a starting point, Zaslavsky mentions, along with [25] the following publications: [23, 38, 73, 174, 238]. *Jeux et jouets de l'Ouest Africain* [23] has been reviewed in [65]. To this list may be added *Omweso: A Game People Play in Uganda* [226], *Strategic Games in Cameroon and Their Mathematical Aspects* [209], *Wari and Solo: The African Calculation Game* [54], *Mankala Games in Zaire, Rwanda, and Burundi* [294], *Rules and Strategies of the Awélé Game* [20], and *African games of strategy, a Teaching Manual* [44]. Crane [44] reports on some of the most common types of African games involving strategy and mathematical principles, such as games of alignment (Shisima (Kenya), Achi (Ghana), Murabaraba (Lesotho)), 'struggle-for-territory' games (Sega (Egypt), Kei (Sierra Leone)), and 'Mancala' games, both two-row versions (Oware (Ghana), Awélé (Ivory Coast), Ayo and Okwe (Nigeria)) and four-row versions (Omweso (Uganda), Tshisolo (Zaire)). Bell and Cornelius [25] give some information on Achi (Ghana), Dara (Nigeria), and Sega (Egypt) and on 'Mancala' games. [221, 255] analyze the learning of strategies and tactics of the 'awélé' game. The important research of Doumbia and her colleagues at the Mathematical Research Institute of Abidjan (Côte d'Ivoire) has focused on traditional African games. It deals with classification and solution of mathematical problems posed by the games, and explores the possibilities of using these games in the mathematics classroom. Their studies [66, 67, 68] reveal that the rules of some games, such as Nigbé Alladian, indicate an empirical knowledge of probabilities, a finding that will certainly stimulate further research. Vergani (Open University, Lisbon) is preparing a monograph on the mathematical aspects of intellectual games in Angola. Mve Ondo (Omar Bongo University, Gabon) published a study on two 'calculation games,' i.e., 'mancala' games, Owani (Congo) and Songa (Cameroon, Gabon, Equatorial Guinea) [219]. The possible relationship between visual memory and concentration as necessary factors for success in many African games (cf. [241]) and the development of mathematical ideas also deserve further attention.

Zaslavsky [318, 109–110] presents a riddle from the Kpelle (Liberia) about a man who has a leopard, a goat, and a pile of cassava leaves to be transported across a river, whereby certain conditions have to be satisfied: the boat can carry no more than one at a time, besides the man himself; the goat cannot be left alone with the leopard, and the goat will eat the cassava leaves if it is not guarded. How can he take them across the river? [16] places this river-crossing problem in a cross-cultural perspective and analyzes mathematical–logical aspects of story puzzles of this type from Algeria, Cape Verde Islands, Ethiopia, Liberia, Tanzania, and Zambia. More difficult to solve is an 'arithmetical puzzle' from the Valuchazi (eastern Angola and northwestern Zambia), recorded and analyzed by Kubik: "This . . . dilemma tale is about three women and three men who want to cross a river in order to attend a dance on the other side. With the river between them there is a boat with the capacity for taking only two people at one time. However, each of the men wishes to marry all the three women himself alone. Regarding the crossing, they would like to cross in pairs, each man with his female partner, but failing that any of the other men could claim all the women for himself. How are they crossing?" [179, 62]. In order to solve the problem or to explain the solution, auxiliary drawings are made in the sand. [23, 744–745] presents a series of riddles from West Africa. [86] describes riddles he learnt as a child in Uganda.

A 'topological' puzzle from the Bambala (Bakuba, Zaire) is recorded in [291, 90, 96]. It consists of two pieces of calabash and a string arranged as shown in Fig. 1. The player has to separate one of the calabash pieces from the string without cutting or untying the string. A more complicated puzzle from the Guinean forest is the game called *pèn* [23, 413]. It uses an instrument (see Fig. 2) produced

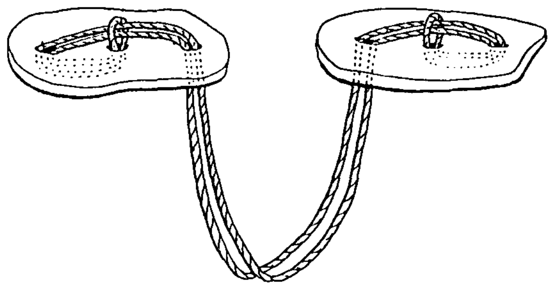

FIG. 1. Bambala 'topological' puzzle, drawn after [291, 90].

in the following way. A stick is perforated in the middle. A loop of a string is passed through the hole and both ends of the string must then pass through the loop. Perforated nuts are strung on the thread and thereafter the ends of the string are tied to the endpoints of the sticks. Now the player has to move the nuts from the big loop on the leftside to the big loop on the right side without cutting or untying the string. Other examples of 'topological' puzzles from West Africa are recorded in [23, 413–419].

GEOMETRY

(1) Art and Symmetries

Njock (University of Yaoundé, Cameroon) characterizes the relationship between African art and mathematics as follows: "Pure mathematics is the art of creating and imaginating. In this sense black art is mathematics" [225, 8].

Mathematicians have mostly been drawn to the analysis of symmetries in African art. Symmetries of repeated patterns may be classified on the basis of the 24 different possible types of patterns which can be used to cover a plane surface (the so-called 24 plane groups due to Federov). Of these, 7 admit translations in only one direction and are called strip patterns. The remaining 17, which admit

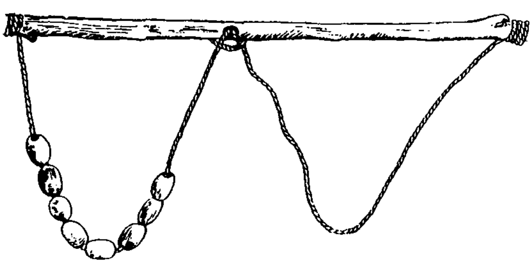

FIG. 2. West African game '*pèn*,' drawn after [23, 413].

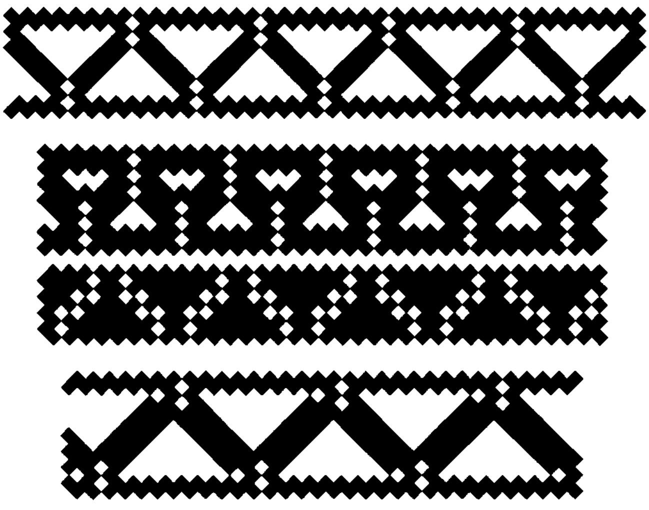

FIG. 3. '*Sipatsi*' strip patterns, drawn after [131].

two independent translations, are called plane patterns. In Chap. 14 of *Africa Counts*, Crowe applies this classification to decorative patterns that appear on the raffia pile cloths of the Bakuba (Zaire) (see also [45]), along with those on Benin bronzes and Yoruba adire cloths (Nigeria), showing that all 7 strip patterns occur as well as many of the other 17 plane patterns. Crowe continued this research and published a catalog of Benin patterns [47] and a symmetry analysis of the smoking pipes of Begho (Ghana) [48; cf. also 49]. In Washburn and Crowe [304] a number of patterns from African contexts are classified in the same way. The use of group theory in the analysis of symmetries in African art underscores and attests to the creative imagination of the artists and artisans involved and their capacity for abstraction (cf. [208]). More recently Washburn [305] showed how a symmetry analysis of the raffia patterns can differentiate patterns produced by the different Bakuba groups. She also takes into account the way in which the artists and artisans themselves classified and analyzed their symmetries. [131] explains the calculations done by basket weavers from Inhambane Province in Mozambique in order to produce strip patterns on their 'sipatsi' handbags. Their book also presents a catalog of more than 100 patterns (see, for example, Fig. 3). The creativity of the 'sipatsi' weavers—formerly only women—can also be seen from the fact that they invented strip patterns belonging to all of the seven different theoretically possible symmetry groups (see also [130, 297]).

Why do symmetries appear in human culture in general, and in African craftwork and art in particular? This question was addressed by Gerdes in a series of studies that analyze the origin of axial, double axial, and rotational symmetry of order 4 in African basketry (see [99, 103, 108, 112, 116, 121]). [115] shows how fivefold

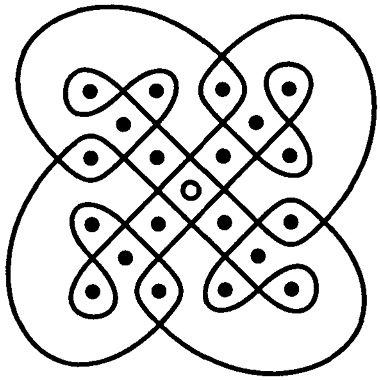

FIG. 4. Example of a monolinear '*sona*,' drawn after [243, 144; 90, 183].

symmetry emerged quite "naturally" when artisans were solving problems in basket weaving. The examples chosen from Mozambican cultures range from the weaving of handbags, hats, and baskets to the fabrication of brooms.

Zaslavsky [325] gives some examples of strip and plane patterns, and of bilateral and rotational symmetries, as these occur in African art, architecture, and design. Langdon [182, 183] describes the symmetries of 'adinkra' cloths (Ghana) and explores possibilities for using them in the classroom. From a similar perspective, Harris [143] describes and explores not only the print designs on plain woven cloths from Ghana but also symmetries on baskets from Botswana and 'buba' blouses from the Yoruba (Nigeria). Gerdes [122] presents a series of traditional African designs with fourfold symmetry and suggests ways of using them in a didactic context to discover/reinvent the Pythagorean Theorem.

(2) Networks, Graphs, or 'Sand-Drawings'

One section of *Africa Counts* [318, 105–109] was devoted to networks, based on Torday's findings regarding the Bushongo (Zaire) and Bastin's study of decorative art work done by the Tchokwe (Angola) [292, 21]. At the time Zaslavsky did not have access to the ethnographic information on such networks published in [22, 222–223; 142; 259]. Since the publication of *Africa Counts*, large ethnographic collections of networks have become available: [243] deals with 'sandgraphs' observed in the 1920s in the Kwandu-Kuvanga and Muxiku provinces of Angola; [90] considers 'sona' or 'sand-drawings' collected principally among the Tchokwe of northeastern Angola during the 1940s and 1950s; and [175–178] report on networks observed among the (Va)luchazi in northwestern Zambia during the 1970s.[9] In order to facilitate the memorization of their standardized 'sona' (see the example in Fig. 4), the drawing experts used the following mnemonic device. After cleaning

[9] Dirk Struik [277] was probably the first historian of mathematics to call attention to the geometry of this type of figure, referring to sand drawings from the Malekula Islands (Oceania).

and smoothing the ground, they first set out an orthogonal net of equidistant points with their fingertips. Next one or more lines are drawn that 'embrace' the points of the reference frame. By applying their method the drawing experts reduce the memorization of a whole drawing to that of two numbers (the dimensions of the reference frame) and a geometric algorithm (the rule of how to draw the embracing line(s)).

Most drawings of this type belong to a long tradition (cf. [254]). They refer to proverbs, fables, games, riddles, animals, etc., and play an important role in the transmission of knowledge and wisdom from one generation to the next. In Kubik's view the 'sona' "transmit empirical mathematical knowledge" [176, 450]. The 'sona' geometry is a "non-Euclidean geometry": "The forefathers of the eastern Angolan peoples discovered higher mathematics and a non-Euclidean geometry on an empirical basis applying their insights to the invention of these unique configurations" [177, 108]. Kubik calls attention to the symmetry of many 'sona,' the implicit rules for their construction and the rules for anchoring figures of the same type. The ethnographic publications of collections of 'sona' have attracted the attention of mathematicians. Ascher and Gerdes conducted research on the 'sona' independently of one another. Ascher [15, 17] deals with geometrical and topological aspects of 'sona,' in particular with symmetries, extension, enlargement through repetition, and isomorphy. Gerdes [108, 120–189; 124, Vol. 1] analyzes symmetry and monolinearity (i.e., a whole figure is made up of only one line) as cultural values, classes of 'sona' and corresponding geometrical algorithms for their construction, systematic construction of monolinear ground patterns, as well as chain and elimination rules for the construction of monolinear 'sona.' These studies further suggest that the 'drawing experts' who invented these rules probably knew why they are valid, i.e., they could prove in one or another way the truth of the theorems that these rules express. Gerdes also pursued reconstructions of lost symmetries and monolinearities by means of an analysis of possible drawing errors in reported 'sona' (for an introduction to his research findings, see [113, 117, 118]). Inspired by these historical research findings, Gerdes experimented with the possibilities of using the 'sona' in mathematics education in order to preserve and revive a rich scientific tradition that had been vanishing (see [104; 105; 108–110; 114; 119; 124, Vol. 2; 126]; cf. [252]).[10] He also initiated a mathematical exploration of the properties of some (extended) classes of 'sona' (see [108, 288–297; 124, Vol. 2]). In a similar way, Kubik's research stimulated a mathematical investigation by Jaritz on a particular class of 'sona' [160].

Monolinear patterns also appear in other African contexts. For instance, [250, 90] displays a symmetric monolinear pattern on a Fulbe warrior's tunic from Senegal (see Fig. 5). The study of the types and spread of monolinear patterns throughout the African continent deserves further research (cf. [127]).

[10] [220, 308] presented suggestions on using sand-drawings in the classroom, based on the information available in [318].

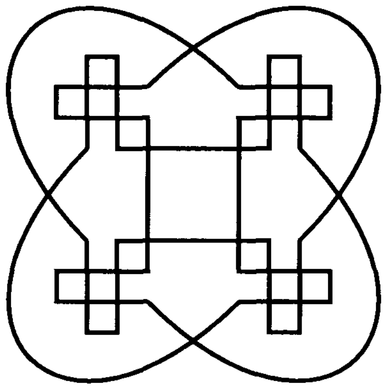

FIG. 5. Pattern of a Fulbe warrior's tunic, drawn after [250, 90].

(3) Geometry and Architecture

Chapter 13 of *Africa Counts* (see also [331]) is dedicated to geometric form in architecture. More information on the geometric shapes and on the ornamentation of traditional African buildings may be found in [55]. [10] presents a bibliography on African architecture, and [216], a booklet on African architecture, pays particular attention to shape and geometric form. Prussin has called attention to the fact that in West Africa the mathematician–scholar and the architectural designer–builder have often been the same person [250, 208]. She refers to the relationship between magic squares and the structure of domes, remarking that "a number of 'adinkra' [Ashanti, Ghana] stamp patterns, directly associated with Islam, were also used in the architectural setting" [250, 240]. For more information on the use of magic squares in West Africa, see [23, 746–748]. [39, 42, 206, 256] describe house decoration and mural painting in Southern Africa, in particular among the Ndebele. A publication of [227] gives a catalog of geometric patterns used on house walls in Lesotho (see the example in Fig. 6). According to [42], in West Africa it is mostly the women who decorate the walls of their houses with geometrical figures. Each year after the harvest, they gather to restore and paint their mud dwellings, which have been washed clean by the rains of the wet season. These studies may serve as a starting point for further research on geometry and ornamentation of buildings. [75, 76] explore possible relationships between modern fractal geometry and traditional settlement patterns in Africa. [99, 112, 121] describe the geometrical know-how used in laying out circular or rectangular house plans in Mozambique.

(4) Uncovering 'Hidden' Geometrical Ideas

Many 'mathematical' ideas and activities in African cultures are not explicitly mathematical. They are often intertwined with art, craft, riddles, games, graphic systems, and other traditions, so that the mathematics remains 'hidden' or 'im-

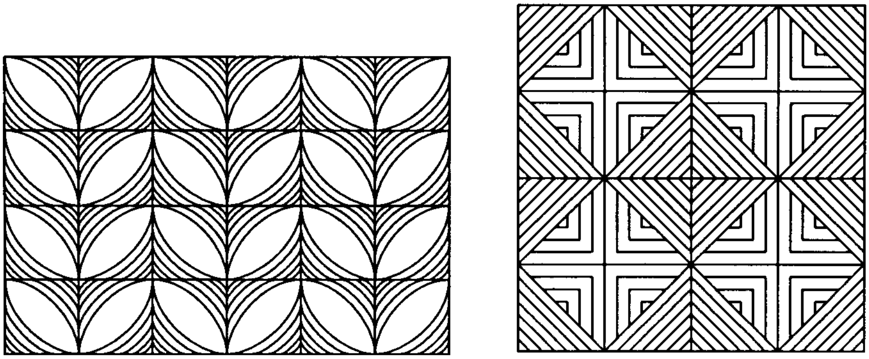

FIG. 6. Examples of '*litema*' patterns, drawn after [227].

plicit' (see [335]). How may this 'hidden' knowledge be uncovered? As some traditions are nowadays (becoming) obsolete, this 'uncovering' often requires a tentative reconstruction of knowledge as it existed in the past. Gerdes [99, 112, 121] explores the concept of 'hidden' mathematics and develops some methods of 'uncovering' and reconstructing 'hidden' geometrical thinking. One of these methods may be characterized as follows: when analyzing the geometrical forms of traditional objects—such as baskets, mats, pots, houses, fishtraps—the researcher poses the question: why do these material products possess the form they have? In order to answer this question, the researcher learns the usual production techniques and tries, at each stage of the production process, to vary the forms. Doing this, the researcher observes that the form generally represents certain practical advantages and that frequently it is the optimal or only possible solution of a production problem. By applying this method, it becomes possible to bring to the fore knowledge about the properties and relations of circles, angles, rectangles, squares, regular pentagons and hexagons, cones, pyramids, cylinders, symmetry, etc., that were probably involved in the invention of the production techniques under consideration.

One of the many areas of African culture which has not been studied as yet in the light of its inherent mathematical aspects is that of *string figures*. Elsewhere such an analysis has already begun: [242] studied the geometry of string figures among the Tapirapé Indians in Brazil and [213] analyzed string figures from the Navaho and other North American Indians and explored their potential for mathematics education. In the case of Africa south of the Sahara, the studies on string figures of Angola [193], Botswana [306], Central Africa [52], Ghana [139], Liberia [150], Nigeria (Yoruba) [240], Sierra Leone [149, 150], Southern Africa [141], Sudan [85, 151], Tanzania (Zanzibar) [150], and West Africa [23, 398–412; 195] may serve as a starting point.

In general, once the mathematical character or aspects of cultural elements are recognized, one may try to track the history of the mathematical thinking involved and its (possible) relationships to other cultural–mathematical 'threads' while exploring their educational and scientific potential.

STUDIES IN RELATED DISCIPLINES

As the history of mathematics in Africa should not be considered in isolation from the development of culture in general or be dissociated from the evolution of art, cosmology, philosophy, natural sciences, medicine, graphic systems, and technology in particular, the historiography of African mathematics should take into account research findings from other disciplines. The following overview covers some of the pertinent literature.

Pappademos [239] presents an outline of Africa's role in the history of physics. Weule [307] studied early forms of mechanics, based on his fieldwork in eastern Africa. Lynch and Robbins [197] analyze evidence from Namoratunga, a megalithic site in northwestern Kenya, suggesting that a prehistoric calendar based on detailed astronomical knowledge was in use in eastern Africa (ca.300 B.C.). Dundas [74] describes precolonial time-reckoning among the Wachagga (Kilamanjaro region): the year is divided into 12 months; each month has 30 days, which are divided into six periods of five days each. The 'topology of time' among the Iraqw of Tanzania is analysed in [290]. Concepts of time, time-reckoning, and cosmology of the Kagura (eastern Africa), of the Dwala (Cameroon), and of the Tiv (Nigeria) have been described and analyzed by Beidelman [24] and Bohannan [32] (cf. also [33, 165]). Lacroix [180] discusses the time expressions in some West African languages. Traditional African calendars constitute one of the research themes of the 'Thought Systems in Black Africa' study group in Paris [19, 53]. Obenga [230] reviews the literature on astronomical knowledge in ancient Egypt, among the Borana (Ethiopia), Dogon (see also [136; 315; 137, 9–13], Lobi, Bambara (West Africa), Vili (Congo), Fang (Cameroon, Equatorial Guinee, Gabon), and Mbochi (Congo, cf. [231]). Keller's study [171] deals with the astronomical views of the Isubu in Cameroon. [2; 3; 34; 164, Vol. 2, 268–274; 194] may also be consulted on astronomy and on calendars in Africa.

An early tentative attempt to analyze craftwork and technology in eastern Africa in historical perspective is Stuhlmann's study [281]. Studies of this type are, however, relatively rare. As Thomas-Emeagwali stresses, despite the significance of the history of technology there is a relative dearth of writings on the issue in African historiography in general and in Nigerian historiography in particular. Her evaluation of the role of oral historiography in the reconstruction of the history of technology is important from a methodological point of view. One of the major problems with which the historian of technology is confronted in the course of fieldwork is the understandable reluctance of practitioners to divulge technological secrets so that they can maintain some measure of competitive power over their potential and actual rivals [286, 69]. Today technicians and craftsmen are often unaware of the precise scientific and engineering principles involved when they use ancient techniques to produce material objects (iron technology, textile production, basketry, woodwork). Therefore, according to Thomas-Emeagwali, the researcher "must be able to identify the underlying principles at play in the process" [286, 70]. Studies of traditional medicine, biologically based warfare, the control of water-based diseases, glassmaking technology and metallurgy in Nigeria, gold mining in precolonial Zimbabwe, and diamond mining in Sierra Leone are included in [288]. [287, 289] include studies on traditional medicine, religion and science, textile technologies, metal technology, mechanics, military technology and engineering, microbiology, traditional food processing, and gender and technology in Nigeria.

Studies on the multiple and varied interactions between African languages and the learning of mathematics present important data on mathematical concepts and ideas in African cutures. See, for example, the reports of the seminars organized by CASME [37], UNESCO [298], and IREM Niamey [154], and documents such as [184, 228, 314]. Wallman [303] realized a study on the communication of measurement in Lesotho. Mashinda [203] analyzed logical and linguistic problems encountered by pupils in Zaire. Zepp [336] studied how bilinguals understand logical connectives in English and Sesotho, and Zepp [338] analyzed inclusive disjunction in West African languages. Gay and Welmers [94] analyzed mathematics and logic in the Kpelle language (Liberia). (For an overview, [341] may be consulted.)

Since Gay and Cole's classical study on mathematical learning and mathematical capacities among the Kpelle of Liberia [93], researchers from Cornell University and the University of Rochester (U.S.A.) have pursued a series of psychological studies on mathematical knowledge and capacities in West Africa: Petitto [244, 245] on unschooled tailors and merchants in Côte d'Ivoire, Posner [247, 248] on Baoulé and Dioula children in Côte d'Ivoire, and Posner and Baroody [249] on number conservation. In particular Ginsburg [135] and Petitto and Ginsburg [246] compared their research findings on mental arithmetic in West Africa with those obtained in North America. Etuk [84] examined Piaget's theory with Yoruba children (Nigeria). Other psychologists have shown interest in the cultural context underlying the understanding of pictorial spatial relationships (pictorial depth perception), model-making and drawing in Kenya [57, 59], Nigeria [224], and Zimbabwe [60, 191, 192]. Cross-cultural psychologists have also examined symmetry and pattern reproduction in Ghana [159], Kenya [26], and Zambia [56, 58] (see also [283]) and in orientation and rotation of symmetry in drawings [266, 268].

SUB-SAHARAN AFRICA, NORTH AFRICA, AND THE OUTSIDE WORLD

The relationships between the development of mathematics in sub-Saharan Africa and the development of mathematics in ancient Egypt, in both Hellenistic and Islamic northern Africa, and across the Indian and Atlantic Oceans also deserve further study.

In connection with ancient Egypt, many open questions still exist. For example, do relationships exist between the duplication and symmetry patterns in pre-Bantu rock paintings in Mozambique, the 'binary combinatorics' of Pedi augurs in Transvaal (South Africa, cf. [164, 559–564]), elements of a binary structure in some African numeration systems and in the Akan weight measurement (Ghana, Côte d'Ivoire, cf. [222]), the Mbosi duplication (Congo, cf. [230]), etc., and the Egyptian duplication method (cf. [134]) for multiplication? Another important open question concerns the role of "black Africa" in the development of "classical Greek society" and its mathematics (cf. Bernal's study *Black Athena* [27, 28]. [233] reanalyzes the mathematical documents of Ancient Egypt and stresses their importance in the development of philosophy in Africa.

On the basis of mathematical ideas involved in the invention and use of basketry techniques (e.g., woven pyramidal funnels in Mozambique and Zaire) and mat making (e.g., circular 'spiral' mats), and taking into account the cultural linkages between ancient Egypt and the rest of "black Africa" (cf. [62]), it is possible to formulate new hypotheses as to how the ancient Egyptian formulas for the area of a circle [100] and for the volume of a truncated pyramid [99, 112] may have been found.

Throughout history there have been many and varied contacts between sub-Saharan Africa and North Africa. Kani's paper "Arithmetic in the pre-Colonial Central Sudan" [169] considers ^{c}*Ilm al-Hisab* (arithmetic) as part of the Islamic sciences introduced some time after the 11th century in Nigeria, first in Kanem-Borno and later, probably in the 15th century, in Hausaland. Arithmetic, as taught in both secular and *Islamiyya* schools, was used in the courts (calculation of inheritance) and in collecting and distributing *zakat* (poor-dues), as well as in business and land surveying. Scholars of Hausaland and Borno consulted Coptic solar calendars in determining their economic activities, especially agricultural ones. Kani concludes his paper with the following remarks: "Despite the availability of a great deal of literature on medicine, astrology, arithmetic and other related sciences, written in Arabic, Fulfulde, Hausa and other languages, little effort has been made to systematically study these sciences within the historical perspective. The intellectual output of the ^{c}*Ulama* (scholars) in this area has been wrongly classified by our contemporary historians and social scientists under the rubric of 'mysticism.' A serious investigation into the literary output of the scholars of western and central Sudan, however, may reveal the fact that these scholars had explored agricultural, medicinal, astronomical and mathematical sciences long before the advent of colonial rule" [169, 38] (see also [170]). Zaslavsky [318, 138–151] discusses the work of Muhammed ibn Muhammed from Katsina (now northern Nigeria) on chronograms and magic squares. Muhammed ibn Muhammed, who had been a pupil of Muhammed Alwali of Bagirmi, made a pilgrimage to Mecca in 1730 before he died in Cairo in 1741. Kani [168] discussed the work of Muhammed ibn Muhammed al Katsinawi on magic squares and numerological patterns. In 1990 a manuscript written by this same author was found in Marrakesh (Morocco) [Djebbar, personal communication]. Prussin [250, 76, 147] refers to the use of magic squares in amulets among the Fulbe and in Niger, Benin, and Timbuktu (Mali). Thomas-Emeagwali [285] reflects on the development of science in the Islamic world and its diffusion into Nigeria before 1903. Diop [61, 167] refers to the study of formal logic in Timbuktu and Lapousterle (Bamako, Mali) and is preparing a study on the contents of three mathematical manuscripts, written in Arabic, that belong to the Ahmad Baba Library in Timbuktu. One of the three manuscripts, whose calligraphy is typical of sub-Saharan Africa, seems to have been written by a mathematician from Mali, al-Arwani. The other two contain references to medieval mathematicians from the Maghreb. Systematic searches in libraries and archives will probably lead to the discovery of more mathematical manuscripts from Muslim scholars south of the Sahara. Other sources written in Arabic on mathematical activities south of the Sahara may also exist. An example from the eastern coast of Africa is provided by the comments of the late 15th-century traveller, Ahmad Ibn-Madjid, who reported on the counting of the Wac-Wac, probably not a Bantu but a Khoi-San people, living at that time in southern and central Mozambique (see [41]).

The possible mutual links between the development of mathematics in sub-Saharan Africa and across the Indian Ocean still remain to be studied (for general analysis of the historical relations across the Indian Ocean, see [299]). What about the possible influence of mathematical ideas of slaves from the African continent, and from India and Indonesia, on the development of mathematics on Madagascar, Mauritius, and other islands?

SLAVE TRADE AND COLONIZATION

What kind of mathematics was brought to the Americas by the slaves? Which mathematical ideas have survived in one way or another? Mancala and perhaps other games with mathematical ingredients are played in the Caribbean and may be compared with their ancestors in Africa. Ferreira [89, 2] refers to a study by one of his students on geometric symbology in Ubanda and Candomblé in Brazil. This research area remains almost virgin.

In [87], a study of Thomas Fuller (1710–1790), the African slave and calculating prodigy who was shipped to America in 1724, the authors suggest that ethnomathematical research may complement the analysis of written sources. Fuller's exceptional abilities can be understood only through closer examination of the cultural context that stimulated their development. Like the professional knowledge of the Tchokwe smiths, the knowledge of the Tchokwe drawing experts (*akwa kuta sona*) was mostly secret. When a blacksmith or a drawing expert was taken prisoner and sold as a slave, his specific professional knowledge could easily disappear completely from his village or region. Thus the slave trade was extremely destructive to the development of the existing mathematical traditions and their latent potential, since it broke down the continuity of transmission and deprived Africa of leading bearers of mathematical knowledge and skill, such as Thomas Fuller. Recent ethnomathematical research in Nigeria, as summarized by [273], shows the survival, nonetheless, of a rich tradition of mental calculations among illiterate people. It would be interesting and valuable to search for new records with data on the geographical or ethnic origin of Thomas Fuller, and to correlate this information with ethnomathematical and historical research on the same region or ethnic group.

One possible way in which the slave trade influenced the development of arithmetical knowledge in Africa has been described as follows by Clarkson in 1788: "Perhaps brought to the front or produced by the necessity of competing with English traders armed with pencil and paper, many of the old-time slave-dealers of Africa seem to have been ready reckoners, and that, too, for a practical purpose. . . . The shipcaptains are said to have complained that it became more and more difficult to make good bargains with such sharp arithmeticians" (cited in [261, 2]). It would be interesting to explore further this and other possible influences, such as the disappearance or undermining of traditional African mathematical education by the physical elimination or exportation of the bearers of mathematical knowledge.

It might be suggested that it would be interesting to search African literature, including autobiographies (cf. the autobiographical essays [86, 217]), for information on mathematics education in the colonial period and the reaction to it (e.g. attitudes towards mathematics and mathematics teachers). A systematic analysis of the ideas (and prejudices) of missionaries, colonial administrators, and educators about the mathematical capacities of their African 'subjects' (e.g., the comments of Junod on the 'lack of mathematical ability' in Mozambique [164, 151–155, 576–577]) and the implications these attitudes carried might be worthwhile and revealing for present and future generations.

POST-INDEPENDENCE

The impact of the transplantation of curricula on mathematics education and the recent history of mathematics education in Africa has been the subject of diverse publications concerning varied countries. Tanzania [210–212, 251, 265], Nigeria [234, 269], Sierra Leone [310], Sudan [77, 79], Mozambique [40, 70, 71, 95–97, 156], Swaziland [204]. Gerdes [98] reproduces the autobiographies of the winners of the first mathematics olympiads in Mozambique. Navez (University of Burundi) has been researching the evolution of mathematics curricula at the secondary school level in Burundi.

An interesting theme in the recent history of mathematics education in Africa, which seems to deserve study, is that of the emergence and evolution of continental and regional mathematics curriculum development projects, such as the African Mathematics Program (for a short historical overview see [313, 195–199]), School Mathematics Project for East Africa, Joint Mathematics Project, East African Regional Mathematics Program, and West African Regional Mathematics Program. The history of mathematical associations and journals, of mathematical departments and schools, are other possible research topics.

The introduction and spread, in sub-Saharan Africa, of new mathematical research areas and related fields such as statistics, informatics, and computer science in relation to the introduction and spread of new technologies, such as computers, have not been studied as yet, nor have their implications for the countries involved.

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