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$N = 1$ supersymmetric Proca–Stueckelberg mechanism for extra vector multiplet

Hitoshi Nishino *, Subhash Rajpoot

Department of Physics & Astronomy, California State University, 1250 Bellflower Boulevard, Long Beach, CA 90840, United States

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Abstract

We present a variant formulation of $N = 1$ supersymmetric Proca–Stueckelberg mechanism for an arbitrary non-Abelian group in four dimensions. This formulation resembles our previous variant supersymmetric compensator mechanism in 4D. Our field content consists of the three multiplets: (i) a non-Abelian Yang–Mills multiplet $(A_\mu{}^I, \lambda^I)$, (ii) a tensor multiplet $(B_{\mu\nu}{}^I, \chi^I, \varphi^I)$ and (iii) an extra vector multiplet $(K_\mu{}^I, \rho^I, C_{\mu\nu\rho}{}^I)$ with the index I for the adjoint representation of a non-Abelian gauge group. The $C_{\mu\nu\rho}{}^I$ is originally an auxiliary field dual to the conventional auxiliary field D^I for the extra vector multiplet. The vector $K_\mu{}^I$ and the tensor $C_{\mu\nu\rho}{}^I$ get massive, after absorbing respectively the scalar φ^I and the tensor $B_{\mu\nu}{}^I$. The superpartner fermion ρ^I acquires a Dirac mass shared with χ^I. We fix non-trivial quartic interactions in the total lagrangian, with corresponding cubic interaction terms in field equations.

1. Introduction

Recently, there have been considerable developments [1,2] in the supersymmetrization of the Proca–Stueckelberg compensator mechanism [3]. The supersymmetrization of *non-Abelian* compensator mechanism was first performed in late 1980s [4]. The *Abelian* supersymmetric

* Corresponding author.
E-mail addresses: H.Nishino@csulb.edu (H. Nishino), Subhash.Rajpoot@csulb.edu (S. Rajpoot).

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Table 1
DOF of our field content.

	${A_\mu}^I$	λ^I	${B_{\mu\nu}}^I$	χ^I	φ^I	${K_\mu}^I$	ρ^I	${C_{\mu\nu\rho}}^I$
DOF before absorptions								
Physical	2	2	1	2	1	2	2	0
Unphysical and physical	3	4	3	4	1	3	4	1
DOF after absorptions								
Physical	2	2	0	0	0	3	4	1
Unphysical and physical	3	4	0	0	0	6	8	2

Proca–Stueckelberg mechanism in [5] has a direct application to MSSM [6]. In [1], general representations of *non-Abelian* group are analyzed, and higher-order terms have been also fixed. Even though the original Higgs mechanism [7] has been established experimentally [8], the Proca–Stueckelberg-type compensator mechanism for massive gauge fields [3] is still an important theoretical alternative.

In our recent paper [2], we presented a *variant* supersymmetric compensator mechanism, both in component and superspace [9], with a field content different from [4]. Our formulation in [2] differs also from [1], because the field content in [2] consists of two multiplets: Yang–Mills (YM) vector multiplet (VM) $({A_\mu}^I, \lambda^I, {C_{\mu\nu\rho}}^I)$, and the tensor multiplet (TM) $({B_{\mu\nu}}^I, \chi^I, \varphi^I)$. The ${C_{\mu\nu\rho}}^I$-field is Hodge-dual to the conventional auxiliary field D^I. The 'dilaton' φ^I (or ${B_{\mu\nu}}^I$) is absorbed into the longitudinal component of ${A_\mu}^I$ (or ${C_{\mu\nu\rho}}^I$), making the latter massive [2]. Our compensation mechanism in [2] works even with ${C_{\mu\nu\rho}}^I$ in the adjoint representation.

In this present paper, we demonstrate yet another field content as a supersymmetric compensator system in which an extra vector in the adjoint representation absorbs a scalar. We have three multiplets VM $({A_\mu}^I, \lambda^I)$, TM $({B_{\mu\nu}}^I, \chi^I, \varphi^I)$, and the extra vector multiplet (EVM) $({K_\mu}^I, \rho^I, {C_{\mu\nu\rho}}^I)$. The φ^I and ${B_{\mu\nu}}^I$ in the TM are compensator fields, respectively absorbed into ${K_\mu}^I$ and ${C_{\mu\nu\rho}}^I$-fields in the EVM. *Before* the absorptions, the *physical* degrees of freedom (DOF) count as ${A_\mu}^I(2)$, $\lambda^I(2)$; ${B_{\mu\nu}}^I(1)$, $\chi^I(2)$, $\varphi^I(1)$; ${K_\mu}^I(2)$, $\rho^I(2)$, ${C_{\mu\nu\rho}}^I(0)$. *After* the absorption, the *physical* DOF count as ${A_\mu}^I(2)$, $\lambda^I(2)$; ${K_\mu}^I(3)$, $\rho^I(4)$, ${C_{\mu\nu\rho}}^I(1)$, as summarized in Table 1.

Our new system differs from our recent works [2,10] in terms of the four aspects:

(i) Our present system has three multiplets VM, TM and EVM, while that in [2] has only VM and TM. The new multiplet is EVM $({K_\mu}^I, \rho^I, {C_{\mu\nu\rho}}^I)$, where ${K_\mu}^I$ (or ${C_{\mu\nu\rho}}^I$) absorbs φ^I (or ${B_{\mu\nu}}^I$), getting massive.
(ii) The vector field getting massive is *not* ${A_\mu}^I$, but is the extra vector field ${K_\mu}^I$.
(iii) In [10], the TM $({B_{\mu\nu}}^I, \chi^I, \varphi^I)$ absorbs the EVM $({C_\mu}^I, \rho^I)$, while in our present system the EVM $({K_\mu}^I, \rho^I, {C_{\mu\nu\rho}}^I)$ absorbs the TM $({B_{\mu\nu}}^I, \chi^I, \varphi^I)$. In other words, the roles played by the TM and extra vector multiplets are exchanged.

This paper is organized as follows. In the next section, we give the definitions of field strengths with their Bianchi identities, and tensorial transformations. In Section 3, we present our lagrangian with the $N = 1$ supersymmetry transformation rule. In Section 4, we give the field equations, with a related important lemma. In Section 5, we give the brief sketch of superspace re-formulation. Concluding remarks are given in Section 6.

2. Field strengths and tensorial transformations

The field strengths for our bosonic fields $A_\mu{}^I$, $B_{\mu\nu\rho}{}^I$, $C_{\mu\nu\rho}{}^I$, $K_\mu{}^I$ and φ^I are respectively

$$F_{\mu\nu}{}^I \equiv +2\partial_{[\mu}A_{\nu]}{}^I + mf^{IJK}A_\mu{}^J A_\nu{}^K, \tag{2.1a}$$
$$\mathcal{G}_{\mu\nu\rho}{}^I \equiv +3D_{[\mu}B_{\nu\rho]}{}^I + mC_{\mu\nu\rho}{}^I - 3m^{-1}f^{IJK}F_{[\mu\nu}{}^J\mathcal{D}_{\rho]}\varphi^K, \tag{2.1b}$$
$$H_{\mu\nu\rho\sigma}{}^I \equiv +4D_{[\mu}C_{\nu\rho\sigma]}{}^I + 6f^{IJK}F_{[\mu\nu}{}^J B_{\rho\sigma]}{}^K, \tag{2.1c}$$
$$L_{\mu\nu}{}^I \equiv +2D_{[\mu}K_{\nu]}{}^I + f^{IJK}F_{\mu\nu}{}^J\varphi^K, \tag{2.1d}$$
$$\mathcal{D}_\mu\varphi^I \equiv +D_\mu\varphi^I + mK_\mu{}^I. \tag{2.1e}$$

We use m for the YM coupling constant, while D_μ is the YM-covariant derivative. The $\mathcal{G}$ in (2.1b) instead of G is a reminder that this field strength has an extra term $m^{-1}F \wedge \mathcal{D}\varphi$. Similarly, $\mathcal{D}_\mu$ in (2.1e) is used to be distinguished from D_μ. The mC and mK-terms in the respective field strength $\mathcal{G}$ and $\mathcal{D}\varphi$ are suggestive that these field strengths can be absorbed into the field redefinitions of C and K.

The Bianchi identities for our field strengths are

$$D_{[\mu}F_{\nu\rho]}{}^I \equiv 0, \tag{2.2a}$$
$$D_{[\mu}\mathcal{G}_{\nu\rho\sigma]}{}^I \equiv +\frac{1}{4}mH_{\mu\nu\rho\sigma}{}^I - \frac{3}{2}f^{IJK}F_{[\mu\nu}{}^J L_{\rho\sigma]}{}^K, \tag{2.2b}$$
$$D_{[\mu}L_{\nu\rho]}{}^I \equiv +f^{IJK}F_{[\mu\nu}{}^J\mathcal{D}_{\rho]}\varphi^K, \tag{2.2c}$$
$$D_{[\mu}\mathcal{D}_{\nu]}\varphi^I \equiv +\frac{1}{2}mL_{\mu\nu}{}^I. \tag{2.2d}$$

There should be proper tensorial transformations [1,2] associated with $B_{\mu\nu}{}^I$, $C_{\mu\nu\rho}{}^I$ and $K_\mu{}^I$ which are symbolized as δ_β, δ_γ and δ_κ. The last δ_κ is for the extra vector $K_\mu{}^I$ which is also a kind of 'tensor' in adjoint representation:

$$\delta_\alpha\big(A_\mu{}^I, B_{\mu\nu}{}^I, C_{\mu\nu\rho}{}^I, K_\mu{}^I, \varphi^I\big)$$
$$= \big(D_\mu\alpha^I, -f^{IJK}\alpha^J B_{\mu\nu}{}^K, -f^{IJK}\alpha^J C_{\mu\nu\rho}{}^K, -f^{IJK}\alpha^J K_\mu{}^K, -f^{IJK}\alpha^J\varphi^K\big), \tag{2.3a}$$
$$\delta_\beta\big(A_\mu{}^I, B_{\mu\nu}{}^I, C_{\mu\nu\rho}{}^I, K_\mu{}^I, \varphi^I\big) = \big(0, +2D_{[\mu}\beta_{\nu]}{}^I, -3f^{IJK}F_{[\mu\nu}{}^J\beta_{\rho]}{}^K, 0, 0\big), \tag{2.3b}$$
$$\delta_\gamma\big(A_\mu{}^I, B_{\mu\nu}{}^I, C_{\mu\nu\rho}{}^I, K_\mu{}^I, \varphi^I\big) = \big(0, -m\gamma_{\mu\nu}, +3D_{[\mu}\gamma_{\nu\rho]}{}^I, 0, 0\big), \tag{2.3c}$$
$$\delta_\kappa\big(A_\mu{}^I, B_{\mu\nu}{}^I, C_{\mu\nu\rho}{}^I, K_\mu{}^I, \varphi^I\big) = \big(0, 0, 0, D_\mu\kappa^I, -m\kappa^I\big), \tag{2.3d}$$

where δ_α is the standard YM gauge transformation. The transformations (2.3c) and (2.3d) indicate that the $C_{\mu\nu\rho}{}^I$ and $K_\mu{}^I$-fields respectively can absorb the compensators $B_{\mu\nu}{}^I$ and φ^I.

Our field strengths are *covariant* under δ_α, while *invariant* under δ_β, δ_γ, δ_γ and δ_κ:

$$\delta_\alpha\big(F_{\mu\nu}{}^I, \mathcal{G}_{\mu\nu\rho}{}^I, H_{\mu\nu\rho}{}^I, L_{\mu\nu}{}^I, D_\mu\varphi^I\big)$$
$$= -f^{IJK}\alpha^J\big(F_{\mu\nu}{}^K, \mathcal{G}_{\mu\nu\rho}{}^K, H_{\mu\nu\rho\sigma}{}^K, L_{\mu\nu}{}^K, D_\mu\varphi^K\big), \tag{2.4a}$$
$$\delta_\beta\big(F_{\mu\nu}{}^I, \mathcal{G}_{\mu\nu\rho}{}^I, H_{\mu\nu\rho}{}^I, L_{\mu\nu}{}^I, D_\mu\varphi^I\big) = (0, 0, 0, 0, 0), \tag{2.4b}$$
$$\delta_\gamma\big(F_{\mu\nu}{}^I, \mathcal{G}_{\mu\nu\rho}{}^I, H_{\mu\nu\rho}{}^I, L_{\mu\nu}{}^I, D_\mu\varphi^I\big) = (0, 0, 0, 0, 0), \tag{2.4c}$$
$$\delta_\kappa\big(F_{\mu\nu}{}^I, \mathcal{G}_{\mu\nu\rho}{}^I, H_{\mu\nu\rho}{}^I, L_{\mu\nu}{}^I, D_\mu\varphi^I\big) = (0, 0, 0, 0, 0). \tag{2.4d}$$

3. Lagrangian and $N = 1$ supersymmetry

Once the invariant field strengths F, $\mathcal{G}$, H, L and $\mathcal{D}\varphi$ have been established, it is straightforward to construct a lagrangian, invariant also under $N = 1$ supersymmetry. Our action $I \equiv \int d^4x\, \mathcal{L}$ has the lagrangian[1]

$$\begin{aligned}
\mathcal{L} = &-\frac{1}{4}\big(F_{\mu\nu}{}^I\big)^2 + \frac{1}{2}\big(\bar{\lambda}^I \partial\!\!\!/ \lambda^I\big) - \frac{1}{12}\big(\mathcal{G}_{\mu\nu\rho}{}^I\big)^2 + \frac{1}{2}\big(\bar{\chi}^I \partial\!\!\!/ \chi^I\big) - \frac{1}{2}(\mathcal{D}_\mu\varphi)^2 - \frac{1}{48}\big(H_{[4]}{}^I\big)^2 \\
&+ \frac{1}{2}\big(\bar{\rho}^I \partial\!\!\!/ \rho^I\big) - \frac{1}{4}\big(L_{\mu\nu}{}^I\big)^2 + m\big(\bar{\chi}^I \rho^I\big) + \frac{1}{48} f^{IJK}\big(\bar{\lambda}^I \gamma^{[4]} \chi^J\big) H_{[4]}{}^K \\
&- \frac{1}{12} f^{IJK}\big(\bar{\lambda}^I \gamma^{[3]} \rho^J\big)\mathcal{G}_{[3]}{}^K - \frac{1}{4} f^{IJK}\big(\bar{\chi}^I \gamma^{\mu\nu} \rho^J\big) F_{\mu\nu}{}^K - \frac{1}{2} f^{IJK}\big(\bar{\lambda}^I \gamma^\mu \rho^J\big)\mathcal{D}_\mu\varphi^K \\
&+ \frac{1}{4} f^{IJK}\big(\bar{\lambda}^I \gamma^{\mu\nu} \chi^J\big) L_{\mu\nu}{}^K + \frac{1}{8} h^{IJ,KL}\big(\bar{\lambda}^I \lambda^K\big)\big[\big(\bar{\rho}^L \rho^J\big) - \big(\bar{\chi}^L \chi^J\big)\big] \\
&+ \frac{1}{8} h^{IJ,KL}\big(\bar{\lambda}^I \gamma_5 \lambda^K\big)\big[\big(\bar{\rho}^L \gamma_5 \rho^J\big) + \big(\bar{\chi}^L \gamma_5 \chi^J\big)\big] \\
&- \frac{1}{8} h^{IJ,KL}\big(\bar{\lambda}^I \gamma_5 \gamma_\mu \lambda^K\big)\big[\big(\bar{\rho}^L \gamma_5 \gamma^\mu \rho^J\big) - \big(\bar{\chi}^L \gamma_5 \gamma^\mu \chi^J\big)\big] \\
&+ \frac{1}{8} h^{IJ,KL}\big(\bar{\chi}^I \gamma_5 \rho^J\big)\big(\bar{\chi}^K \gamma_5 \rho^L\big).
\end{aligned} \tag{3.1}$$

The symbol $h^{IJ,KL}$ is defined by $h^{IJ,KL} \equiv f^{IJM} f^{MKL}$.

The kinetic terms of B and φ, namely, the $(\mathcal{G}_{\mu\nu\rho}{}^I)^2$ and $(\mathcal{D}_\mu\varphi)^2$-terms, which respectively contain m^2C^2 and m^2K^2-terms, play the role of mass terms for the C and K-fields, after the absorptions of DB by C and $\mathcal{D}\varphi$ by K. Because of $N = 1$ supersymmetry, this compensator mechanism between TM and EVM works also for fermionic partners. Namely, the original χ^I-field in TM is mixed with the ρ^I-field in EVM, forming the Dirac mass term $m(\bar{\chi}^I \rho^I)$.

The $N = 1$ supersymmetry transformation rule of our multiplets is

$$\delta_Q A_\mu{}^I = +\big(\bar{\epsilon}\gamma_\mu \lambda^I\big), \tag{3.2a}$$

$$\delta_Q \lambda^I = +\frac{1}{2}\big(\gamma^{\mu\nu}\epsilon\big) F_{\mu\nu}{}^I + \frac{1}{2} f^{IJK}(\gamma_5\epsilon)\big(\bar{\chi}^J \gamma_5 \rho^K\big), \tag{3.2b}$$

$$\delta_Q B_{\mu\nu}{}^I = +\big(\bar{\epsilon}\gamma_{\mu\nu}\chi^I\big) + 2m^{-1} f^{IJK}\big(\bar{\epsilon}\gamma_{[\mu|}\lambda^J\big)\mathcal{D}_{|\nu]}\varphi^K - m^{-1} f^{IJK}\big(\bar{\epsilon}\chi^J\big) F_{\mu\nu}{}^K, \tag{3.2c}$$

$$\begin{aligned}
\delta_Q \chi^I = &+\frac{1}{6}\big(\gamma^{\mu\nu\rho}\epsilon\big)\mathcal{G}_{\mu\nu\rho}{}^I - \big(\gamma^\mu\epsilon\big)\mathcal{D}_\mu\varphi^I + \frac{1}{2} f^{IJK}\big(\gamma^\mu \rho^J\big)\big(\bar{\epsilon}\gamma_\mu \lambda^K\big) \\
&- \frac{1}{2} f^{IJK} \rho^J \big(\bar{\epsilon}\lambda^K\big) + \frac{1}{2} f^{IJK}\big(\gamma_5 \rho^J\big)\big(\bar{\epsilon}\gamma_5 \lambda^K\big),
\end{aligned} \tag{3.2d}$$

$$\delta_Q \varphi^I = +\big(\bar{\epsilon}\chi^I\big), \tag{3.2e}$$

$$\delta_Q K_\mu{}^I = +\big(\bar{\epsilon}\gamma_\mu \rho^I\big) - f^{IJK}\big(\bar{\epsilon}\gamma_\mu \lambda^J\big)\varphi^K, \tag{3.2f}$$

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[1] We also use the symbol $[r]$ for totally antisymmetric indices $\rho_1 \cdots \rho_r$ to save space. Our notation is $(\eta_{\mu\nu}) \equiv \mathrm{diag}(-,+,+,+)$, $\epsilon^{0123} = +1, \epsilon_{\mu_1 \cdots \mu_{4-r}[r]}\epsilon^{[r]\sigma_1 \cdots \sigma_{4-r}} = -(-1)^r (4-r)!(r!)\delta_{[\mu_1}{}^{\sigma_1} \cdots \delta_{\mu_{4-r}]}{}^{\sigma_{4-r}}$, $\gamma_5 \equiv +i\gamma_0\gamma_1\gamma_2\gamma_3$, $\epsilon^{[4-r][r]}\gamma_{[r]} = -i(-1)^{r(r-1)/2}(r!)\gamma_5\gamma^{[4-r]}$.

$$\begin{aligned}\delta_Q \rho^I = {} & +\frac{1}{2}(\gamma^{\mu\nu}\epsilon)L_{\mu\nu}{}^I - \frac{1}{24}(\gamma^{\mu\nu\rho\sigma}\epsilon)H_{\mu\nu\rho\sigma}{}^I + \frac{1}{2}f^{IJK}(\gamma^\mu\chi^J)(\bar\epsilon\gamma_\mu\lambda^K) \\ & + \frac{1}{2}f^{IJK}\chi^J(\bar\epsilon\lambda^K) + \frac{1}{2}f^{IJK}(\gamma_5\chi^J)(\bar\epsilon\gamma_5\lambda^K) \\ & + \frac{1}{4}m^{-1}h^{IJ,KL}\big[(\gamma^\mu\epsilon)(\bar\lambda^J\lambda^K) + (\gamma_5\gamma^\mu\epsilon)(\bar\lambda^J\gamma_5\lambda^K) + (\gamma_5\epsilon)(\bar\lambda^J\gamma_5\gamma^\mu\lambda^K)\big]\mathcal{D}_\mu\varphi^L \\ & + \frac{1}{24}m^{-1}h^{IJ,KL}\big[(\gamma_5\gamma^{\mu\nu\rho}\epsilon)(\bar\lambda^J\gamma_5\lambda^K) + (\gamma^{\mu\nu\rho}\epsilon)(\bar\lambda^J\lambda^K) \\ & - \epsilon(\bar\lambda^J\gamma^{\mu\nu\rho}\lambda^K) + 3(\gamma_5\gamma^{\mu\nu}\epsilon)(\bar\lambda^J\gamma_5\gamma^\rho\lambda^K)\big]\mathcal{G}_{\mu\nu\rho}{}^L, \end{aligned} \tag{3.2g}$$

$$\delta_Q C_{\mu\nu\rho}{}^I = +(\bar\epsilon\gamma_{\mu\nu\rho}\rho^I) - 3f^{IJK}(\bar\epsilon\gamma_{[\mu|}\lambda^J)B_{|\nu\rho]}{}^K. \tag{3.2h}$$

An important corollary is for the arbitrary variations of our field strengths:

$$\delta F_{\mu\nu}{}^I = +2D_{[\mu|}(\delta A_{|\nu]}{}^I), \tag{3.3a}$$

$$\begin{aligned}\delta\mathcal{G}_{\mu\nu\rho}{}^I = {} & +3D_{[\mu|}(\widetilde\delta B_{|\nu\rho]}{}^I) + m(\widetilde\delta C_{\mu\nu\rho}{}^I) \\ & - 3f^{IJK}(\delta A_{[\mu|}{}^J)L_{|\nu\rho]}{}^K - 3f^{IJK}F_{[\mu\nu|}{}^J(\widetilde\delta K_{|\rho]}{}^K),\end{aligned} \tag{3.3b}$$

$$\begin{aligned}\delta H_{\mu\nu\rho\sigma}{}^I = {} & +4D_{[\mu|}(\widetilde\delta C_{|\nu\rho\sigma]}{}^I) + 4f^{IJK}(\delta A_{[\mu|}{}^J)(\mathcal{G}_{|\nu\rho\sigma]}{}^K + 3m^{-1}f^{KLM}L_{|\nu\rho|}{}^L\mathcal{D}_{|\sigma]}\varphi^M) \\ & - 6f^{IJK}(\widetilde\delta B_{[\mu\nu|}{}^J)F_{|\rho\sigma]}{}^K,\end{aligned} \tag{3.3c}$$

$$\delta L_{\mu\nu}{}^I = +2D_{[\mu|}(\widetilde\delta K_{|\nu]}{}^I) + 2f^{IJK}(\delta A_{[\mu|}{}^J)\mathcal{D}_{|\nu]}\varphi^K + f^{IJK}F_{\mu\nu}{}^J(\delta\varphi^K), \tag{3.3d}$$

$$\delta(D_\mu\varphi^I) = +D_\mu(\delta\varphi^I) + m(\widetilde\delta K_\mu{}^I). \tag{3.3e}$$

The modified transformations $\widetilde\delta B$, $\widetilde\delta C$ and $\widetilde\delta K$ are defined by

$$\widetilde\delta B_{\mu\nu}{}^I \equiv +\delta B_{\mu\nu}{}^I - 2m^{-1}f^{IJK}(\delta A_{[\mu|}{}^J)\mathcal{D}_{|\nu]}\varphi^K - m^{-1}f^{IJK}F_{\mu\nu}{}^K(\delta\varphi^K), \tag{3.4a}$$

$$\widetilde\delta C_{\mu\nu\rho}{}^I \equiv +\delta C_{\mu\nu\rho}{}^I + 3f^{IJK}(\delta A_{[\mu|}{}^J)B_{|\nu\rho]}{}^K,$$

$$\widetilde\delta K_\mu{}^I \equiv +\delta K_\mu{}^I + f^{IJK}(\delta A_\mu{}^J)\varphi^K. \tag{3.4b}$$

A special case of (3.3) is the supersymmetry transformation rule,

$$\delta_Q F_{\mu\nu}{}^I = -2(\bar\epsilon\gamma_{[\mu}D_{\nu]}\lambda^I), \tag{3.5a}$$

$$\begin{aligned}\delta_Q\mathcal{G}_{\mu\nu\rho}{}^I = {} & +3(\bar\epsilon\gamma_{[\mu\nu}D_{\rho]}\chi^I) + m(\bar\epsilon\gamma_{\mu\nu\rho}\rho^I) \\ & - 3f^{IJK}(\bar\epsilon\gamma_{[\mu|}\lambda^J)L_{|\nu\rho]}{}^K + 3f^{IJK}(\bar\epsilon\gamma_{[\mu|}\rho^J)F_{|\nu\rho]}{}^K,\end{aligned} \tag{3.5b}$$

$$\begin{aligned}\delta_Q H_{\mu\nu\rho\sigma}{}^I = {} & -4(\bar\epsilon\gamma_{[\mu\nu\rho}D_{\sigma]}\rho^I) + 4f^{IJK}(\bar\epsilon\gamma_{[\mu|}\lambda^J)\mathcal{G}_{|\nu\rho\sigma]}{}^K \\ & - 6f^{IJK}(\bar\epsilon\gamma_{[\mu\nu|}\chi^J)F_{|\rho\sigma]}{}^K,\end{aligned} \tag{3.5c}$$

$$\delta_Q L_{\mu\nu}{}^I = -2(\bar\epsilon\gamma_{[\mu}D_{\nu]}\rho^I) + 2f^{IJK}(\bar\epsilon\gamma_{[\mu|}\lambda^J)\mathcal{D}_{|\nu]}\varphi^K - f^{IJK}(\bar\epsilon\chi^J)F_{\mu\nu}{}^K, \tag{3.5d}$$

$$\delta_Q(\mathcal{D}_\mu\varphi^I) = +(\bar\epsilon D_\mu\chi^I) + m(\bar\epsilon\gamma_\mu\rho^I). \tag{3.5e}$$

In particular, there should be *no* 'bare' potential-field terms, such as 'bare' $B_{\mu\nu}{}^I$ or 'bare' φ^I-term without derivatives in (3.3) or (3.5). The modified transformations (3.4) explain why the terms in $\delta_Q B_{\mu\nu}{}^I$ (3.2c), $\delta_Q C_{\mu\nu\rho}{}^I$ (3.2h) and $\delta_Q K_\mu{}^I$ (3.2f) other than their first linear terms are required. In other words, all the *tilted* transformations $\widetilde{\delta}_Q B_{\mu\nu}{}^I$, $\widetilde{\delta}_Q C_{\mu\nu\rho}{}^I$, $\widetilde{\delta}_Q K_\mu{}^I$ contain only the linear terms in (3.2c), (3.2h) and (3.2f), respectively.

Note the peculiar $m^{-1}F \wedge B$-term in $\mathcal{G}$ in (2.1b). The general variation of this term is

$$\delta\big(-3m^{-1} f^{IJK} F_{[\mu\nu}{}^J \mathcal{D}_{\rho]}\varphi^K\big) = +3D_{[\mu|}\big[-2m^{-1} f^{IJK}\big(\delta A_{|\nu|}{}^J\big)\mathcal{D}_{|\rho]}\varphi^K\big] - 6m^{-1} f^{IJK}\big(\delta A_{[\mu|}{}^J\big) D_{|\nu|}\mathcal{D}_{|\rho]}\varphi^K. \qquad (3.6)$$

The last term is proportional to $(\delta A) \wedge L$ with the original m^{-1} canceled by m in the former resulting in only an m^0-term, interpreted as the third term in (3.3b). The first term of (3.6) with m^{-1} is absorbed into the second term of $\widetilde{\delta} B_{\nu\rho}{}^I$ in (3.4a). This sort of sophisticated Chern–Simons terms at order m^{-1} has not been presented in the past, to our knowledge. This is the result of intricate play between the TM and EVM, where the latter absorbs the former as a compensator multiplet.

The confirmation of the action invariance $\delta_Q I = 0$ is performed as follows. Including the fermionic quartic terms, the confirmation is performed at $\mathcal{O}(\Phi^2)$, $\mathcal{O}(\Phi^3)$ and $\mathcal{O}(\Phi^4)$-terms, where Φ stands for any fundamental field. Depending on the context, we distinguish fermionic fields and bosonic fields by the symbols ψ and ϕ, respectively: $(\Phi) = (\psi, \phi)$. To be more precise, there are four categories of terms to consider: (I) $m^0\Phi^2$, (II) $m\Phi^2$, (III) $m^0\Phi^3$, (IV) $m\Phi^3$, and (V) $m^0\Phi^4$.

The categories (I) and (II) are straightforward quadratic-order confirmations. The category (III) for $m^0\Phi^3$-terms is non-trivial with nine sectors: (i) $\lambda\mathcal{G}H$, (ii) χFH, (iii) $\lambda H\mathcal{D}\varphi$, (iv) $\lambda\mathcal{G}L$, (v) $\rho F\mathcal{G}$, (vi) $\lambda L\mathcal{D}\varphi$, (vii) χFL, (viii) $\rho F\mathcal{D}\varphi$, and (ix) $\lambda\overline{\chi}D\rho$ or $\lambda\overline{\rho}D\chi$. The only subtle sector is (ix), where upon partial integrations, we can get rid of derivatives on λ, such that we are left only with $\lambda\chi D\rho$ or $\lambda\rho D\chi$-terms.[2] After Fierz rearrangements, only the structures $(\bar{\epsilon}\gamma\lambda)(\overline{\chi}\gamma D\rho)$ and $(\bar{\epsilon}\gamma\lambda)(\overline{\rho}\gamma D\chi)$ remain, all of which cancel amongst themselves. The cancellation confirmation of these terms are involved, depending on the number of γ-matrices sandwiched by ϵ and λ. This is carried out by adding the non-trivial $\lambda\rho$-terms in $\delta_Q\chi$, $\lambda\chi$-terms in $\delta_Q\rho$, and $\chi\rho$-terms in $\delta_Q\lambda$.

The category (IV) for $m\Phi^3$-terms has four sectors: (i) $m\lambda\rho^2$, (ii) $m\lambda^3$, (iii) $m\lambda\chi^2$, and (iv) $m\lambda\rho^2$. The confirmation of all of these sectors are relatively easy, consistently with the $\lambda\rho$-terms in $\delta_Q\chi$, $\lambda\chi$-terms in $\delta_Q\rho$, and $\chi\rho$-terms in $\delta_Q\lambda$.

The computation to fix the $\mathcal{O}(\Phi^4)$-terms in the lagrangian is the most involved. All terms in the sector (V) $m^0\Phi^4$ are actually of the type $m^0\psi^3\phi$, *i.e.,* (fermion)^3(boson)-terms. They arise, *e.g.,* from the variations of the $\mathcal{O}(\psi^4)$-terms in the lagrangian. They are categorized into ten sectors: (i) $\lambda^2\rho H$, (ii) $\rho^2\lambda F$, (iii) $\chi^2\rho H$, (iv) $\rho^2\chi\mathcal{G}$, (v) $\lambda^2\chi\mathcal{D}\varphi$, (vi) $\chi^2\lambda F$, (vii) $\rho^2\chi\mathcal{D}\varphi$, (viii) $\lambda^2\chi\mathcal{G}$, (ix) $\chi^2\rho L$, and (x) $\lambda^2\rho L$. The confirmation of these sectors (i) through (x) are the outlined as follows: For (i), there are two sources of terms: $\lambda^2\rho^2$-terms and $\bar{\lambda}\chi H$-term in the lagrangian. After the Fierzing of the latter terms, these contributions simply cancel themselves. For (ii), there are three sources of terms: $\chi\rho F$, $\lambda\rho\mathcal{G}$ and $\lambda^2\rho^2$-terms. The first two group of terms need Fierzing, and they eventually cancel themselves. Similarly for (iii), there are two sources of terms: $\lambda\chi H$ and $\chi^2\rho^2$. For (iv), there are two sources of terms: $\lambda\rho\mathcal{G}$ and $\chi^2\rho^2$. For (v), there are four sources: $\lambda\rho\mathcal{D}\varphi$, $\lambda\chi L$, $m\lambda\rho$ and $\rho^2\chi^2$-terms. In particular, the $m^{-1}\lambda^2\mathcal{D}\varphi$-terms in $\delta_Q\rho$ via $m\lambda\delta_Q\rho$ cancel other terms. For (vi), there are four sources: $\chi\rho F$, $\lambda\chi L$, $\lambda\chi H$ and $\lambda^2\chi^2$-terms. For (vii), there are two sources: $\lambda\rho\mathcal{D}\varphi$ and $\chi^2\rho^2$-terms. For (viii), there are four sources: $\lambda\rho\mathcal{G}$,

[2] Here we do *not* necessary mean the terms of the type $(\bar{\epsilon}\gamma\lambda)(\overline{\chi}\gamma D\rho)$ or $(\bar{\epsilon}\gamma\lambda)(\overline{\rho}\gamma D\chi)$, which are reached *after* Fierz arrangements.

$\lambda\chi H$, $m\chi\rho$ and $\lambda^2\chi^2$-terms. For (ix), there are two sources: $\lambda\chi L$ and $\chi^2\rho^2$-terms. For (x), there are three sources: $\lambda\chi L$, $\lambda\rho\mathcal{G}$ and $\lambda^2\rho^2$-terms. All of these terms cancel themselves, after appropriate Fierz-rearrangements.

4. Field equations

The field equations in our system are highly non-trivial. This is due to the extra Chern–Simon-type terms in various field strengths. Even the simplest field strength $\mathcal{D}_\mu\varphi^I$ has an extra term $mK_\mu{}^I$. The explicit forms of our field equations are

$$\frac{\delta\mathcal{L}}{\delta A_\mu{}^I} \doteq -D_\nu F^{\mu\nu I} + \frac{1}{2}f^{IJK}(\bar{\chi}^J D^\mu\rho^K) + \frac{1}{2}f^{IJK}(\bar{\rho}^J D^\mu\chi^K) - \frac{1}{2}mf^{IJK}(\bar{\lambda}^J\gamma^\mu\lambda^K) + \frac{1}{2}f^{IJK}L_{\nu\rho}{}^J\mathcal{G}^{\mu\nu\rho K} - \frac{1}{6}f^{IJK}\mathcal{G}_{\nu\rho\sigma}{}^J H^{\mu\nu\rho\sigma K} + f^{IJK}L^{\mu\nu J}\mathcal{D}_\nu\varphi^K \doteq 0, \tag{4.1a}$$

$$\frac{\delta\mathcal{L}}{\delta B_{\mu\nu}{}^I} \doteq +\frac{1}{2}D_\rho\mathcal{G}^{\mu\nu\rho I} - \frac{1}{4}mf^{IJK}(\bar{\lambda}^J\gamma^{\mu\nu}\chi^K) + \frac{1}{4}f^{IJK}F_{\rho\sigma}{}^J H^{\mu\nu\rho\sigma K} - \frac{1}{2}f^{IJK}(\bar{\lambda}^J\gamma^{[\mu}D^{\nu]}\rho^K) + \frac{1}{2}f^{IJK}(\bar{\rho}^J\gamma^{[\mu}D^{\nu]}\lambda^K) \doteq 0, \tag{4.1b}$$

$$\frac{\delta\mathcal{L}}{\delta C_{\mu\nu\rho}{}^I} \doteq -\frac{1}{6}D_\sigma H^{\mu\nu\rho\sigma I} - \frac{1}{6}m\mathcal{G}^{\mu\nu\rho I} - \frac{1}{6}mf^{IJK}(\bar{\lambda}^J\gamma^{\mu\nu\rho}\rho^K) - \frac{1}{4}f^{IJK}(\bar{\lambda}^J\gamma^{[\mu\nu}D^{\rho]}\chi^K) + \frac{1}{4}f^{IJK}(\bar{\chi}^J\gamma^{[\mu\nu}D^{\rho]}\lambda^K) \doteq 0, \tag{4.1c}$$

$$\frac{\delta\mathcal{L}}{\delta K_\mu{}^I} \doteq -D_\nu L^{\mu\nu I} - m\mathcal{D}^\mu\varphi^I - \frac{1}{2}f^{IJK}F_{\nu\rho}{}^J\mathcal{G}^{\mu\nu\rho K} - mf^{IJK}(\bar{\lambda}^J\gamma^\mu\rho^K) - \frac{1}{2}f^{IJK}(\bar{\lambda}^J D^\mu\chi^K) - \frac{1}{2}f^{IJK}(\bar{\chi}^J D^\mu\lambda^K) \doteq 0, \tag{4.1d}$$

$$\frac{\delta\mathcal{L}}{\delta\varphi^I} \doteq +D_\mu\mathcal{D}^\mu\varphi^I - \frac{1}{2}mf^{IJK}(\bar{\lambda}^J\chi^K) + \frac{1}{2}f^{IJK}F_{\mu\nu}{}^J L^{\mu\nu K} \doteq 0, \tag{4.1e}$$

$$\frac{\delta\mathcal{L}}{\delta\bar{\lambda}^I} \doteq +\not{D}\lambda^I + \frac{1}{48}f^{IJK}(\gamma^{\mu\nu\rho\sigma}\chi^J)H_{\mu\nu\rho\sigma}{}^K - \frac{1}{12}f^{IJK}(\gamma^{\mu\nu\rho}\rho^J)\mathcal{G}_{\mu\nu\rho}{}^K - \frac{1}{2}(\gamma^\mu\rho^J)\mathcal{D}_\mu\varphi^K + \frac{1}{4}(\gamma^{\mu\nu}\chi^J)L_{\mu\nu}{}^K - \frac{1}{4}h^{IJ,KL}\lambda^K[(\bar{\rho}^L\rho^J) - (\bar{\chi}^L\chi^J)] + \frac{1}{4}h^{IJ,KL}(\gamma_5\lambda^K)[(\bar{\rho}^L\gamma_5\rho^J) + (\bar{\chi}^L\gamma_5\chi^J)] - \frac{1}{4}h^{IJ,KL}(\gamma_5\gamma_\mu\lambda^K)[(\bar{\rho}^L\gamma_5\gamma^\mu\rho^J) - (\bar{\chi}^L\gamma_5\gamma^\mu\chi^J)] \doteq 0, \tag{4.1f}$$

$$\frac{\delta\mathcal{L}}{\delta\bar{\chi}^I} \doteq +\not{D}\chi^I + m\rho^I - \frac{1}{48}f^{IJK}(\gamma^{\mu\nu\rho\sigma}\lambda^J)H_{\mu\nu\rho\sigma}{}^K + \frac{1}{4}f^{IJK}(\gamma^{\mu\nu}\rho^J)F_{\mu\nu}{}^K + \frac{1}{4}f^{IJK}(\gamma^{\mu\nu}\lambda^J)L_{\mu\nu}{}^K + \frac{1}{4}h^{IJ,KL}\chi^K(\bar{\lambda}^L\lambda^J) + \frac{1}{4}h^{IJ,KL}(\gamma_5\chi^K)(\bar{\lambda}^L\gamma_5\lambda^J) + \frac{1}{4}h^{IJ,KL}(\gamma_5\gamma_\mu\chi^K)(\bar{\lambda}^L\gamma_5\gamma^\mu\lambda^J) \doteq 0, \tag{4.1g}$$

$$\begin{aligned}\frac{\delta\mathcal{L}}{\delta\overline{\rho}^I} &\doteq +\not{D}\rho^I + m\chi^I + \frac{1}{12}f^{IJK}(\gamma^{\mu\nu\rho}\lambda^J)\mathcal{G}_{\mu\nu\rho}{}^K - \frac{1}{4}f^{IJK}(\gamma^{\mu\nu}\chi^J)F_{\mu\nu}{}^K \\ &\quad - \frac{1}{4}f^{IJK}(\gamma^\mu\lambda^J)\mathcal{D}_\mu\varphi^K - \frac{1}{4}h^{IJ,KL}\rho^K(\bar{\lambda}^L\lambda^J) + \frac{1}{4}h^{IJ,KL}(\gamma_5\rho^K)(\bar{\lambda}^L\gamma_5\lambda^J) \\ &\quad - \frac{1}{4}h^{IJ,KL}(\gamma_5\gamma_\mu\rho^K)(\bar{\lambda}^L\gamma_5\gamma^\mu\lambda^J) \doteq 0, \end{aligned} \tag{4.1h}$$

where the symbol $\doteq$ is for an equality by the use of field equation(s).

The $m\mathcal{G}$-term in the C-field equation (4.1c) plays the role of the mass term for the C-field after a field-redefinition of C absorbing the $3DB$-term in $\mathcal{G}$. So does the $m\mathcal{D}\varphi$-term in the K-field equation (4.1d).

Our result (4.1) is based on an important lemma about the general variation of our lagrangian up to a total divergence:

$$\begin{aligned}\delta\mathcal{L} &= (\delta A_\mu{}^I)\Bigg[+2D_\nu\left(\frac{\delta\mathcal{L}}{\delta F_{\mu\nu}{}^I}\right) + \left(\frac{\delta\mathcal{L}_{\psi\not{D}\psi}}{\delta A_\mu{}^I}\right) - 3f^{IJK}L_{\nu\rho}{}^J\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^K}\right) \\ &\quad + 4f^{IJK}\mathcal{G}_{\nu\rho\sigma}{}^J\left(\frac{\delta\mathcal{L}}{\delta H_{\mu\nu\rho\sigma}{}^K}\right) + 2f^{IJK}(\mathcal{D}_\nu\varphi^J)\left(\frac{\delta\mathcal{L}}{\delta L_{\mu\nu}{}^K}\right)\Bigg] \\ &\quad + (\delta B_{\mu\nu}{}^I)\left[-3D_\rho\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^I}\right) - 6f^{IJK}F_{\rho\sigma}{}^J\left(\frac{\delta\mathcal{L}}{\delta H_{\mu\nu\rho\sigma}{}^K}\right)\right] \\ &\quad + (\delta C_{\mu\nu\rho}{}^I)\left[+4D_\sigma\left(\frac{\delta\mathcal{L}}{\delta H_{\mu\nu\rho\sigma}{}^I}\right) + m\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^I}\right)\right] \\ &\quad + (\delta K_\mu{}^I)\left[+2D_\nu\left(\frac{\delta\mathcal{L}}{\delta L_{\mu\nu}{}^I}\right) + m\left\{\frac{\delta\mathcal{L}}{\delta(\mathcal{D}_\mu\varphi^I)}\right\} + 3f^{IJK}F_{\nu\rho}{}^J\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^K}\right)\right] \\ &\quad + (\delta\varphi^I)\Bigg[-D_\mu\left\{\frac{\delta\mathcal{L}}{\delta(\mathcal{D}_\mu\varphi^I)}\right\} + 3m^{-1}f^{IJK}F_{\mu\nu}{}^J D_\rho\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^K}\right) \\ &\quad - f^{IJK}F_{\mu\nu}{}^J\left(\frac{\delta\mathcal{L}}{\delta L_{\mu\nu}{}^K}\right)\Bigg] \\ &\quad + (\delta\bar{\lambda}^I)\left(\frac{\delta\mathcal{L}}{\delta\bar{\lambda}^I}\right) + (\delta\overline{\chi}^I)\left(\frac{\delta\mathcal{L}}{\delta\overline{\chi}^I}\right) + (\delta\overline{\rho}^I)\left(\frac{\delta\mathcal{L}}{\delta\overline{\rho}^I}\right). \end{aligned} \tag{4.2}$$

The symbol $(\delta\mathcal{L}_{\psi\not{D}\psi}/\delta A_\mu{}^I)$ in the first line is for the contributions from the minimal couplings in the fermionic kinetic terms of λ, χ and ρ. Use is also made of the general-variation formulae in (3.3) for arranging the whole terms.

In getting the expression (4.2), there are many non-trivial cancellations. For example, the two terms:

$$3f^{IJK}(\delta A_\mu{}^I)B_{\nu\rho}{}^J\left[+4D_\sigma\left(\frac{\delta\mathcal{L}}{\delta H_{\mu\nu\rho\sigma}{}^K}\right) + m\left(\frac{\delta\mathcal{L}}{\delta\mathcal{G}_{\mu\nu\rho}{}^K}\right)\right] \tag{4.3}$$

cancel upon the use of the C-field equation (4.1c). Similarly, the two terms:

$$f^{IJK}(\delta A_\mu{}^I)\varphi^J\left[+2D_\nu\left(\frac{\delta\mathcal{L}}{\delta L_{\mu\nu}{}^K}\right) + m\left\{\frac{\delta\mathcal{L}}{\delta(\mathcal{D}_\mu\varphi^K)}\right\}\right] \tag{4.4}$$

also cancel upon the K-field equation (4.1d).

As an additional confirmation, we can show that the divergence of the A, B, C and K-field equations all vanish. For example, the divergence of the A-field equation is

$$0 \stackrel{?}{=} D_\mu \left(\frac{\delta \mathcal{L}}{\delta A_\mu{}^I} \right) \doteq + m f^{IJK} (\bar{\chi}^J \rho^K) + m f^{IJK} (\bar{\rho}^J \chi^K) - \frac{1}{24} m f^{IJK} H_{\mu\nu\rho\sigma}{}^J H^{\mu\nu\rho\sigma K}$$
$$- \frac{1}{6} f^{IJK} \mathcal{G}_{\nu\rho\sigma}{}^J \mathcal{G}^{\nu\rho\sigma\, K} - \frac{1}{2} m f^{IJK} L_{\mu\nu}{}^J L^{\mu\nu\, K} \doteq 0 \quad \text{(Q.E.D.)}. \tag{4.5}$$

Here we have used other field equations, such as $\not{D}\lambda^I \doteq \cdots$ or $D_\mu H^{\mu\nu\rho\sigma I} \doteq +m\mathcal{G}^{\nu\rho\sigma I} + \cdots$, *etc.* Similarly for the case of C-field equation:

$$0 \stackrel{?}{=} D_\rho \left(\frac{\delta \mathcal{L}}{\delta C_{\mu\nu\rho}{}^I} \right)$$
$$= -\frac{1}{12} m f^{IJK} F_{\rho\sigma}{}^J H^{\mu\nu\rho\sigma K} - \frac{1}{6} m D_\rho \mathcal{G}^{\mu\nu\rho} - \frac{1}{12} m f^{IJK} D_\rho (\bar{\lambda}^J \gamma^{\mu\nu\rho} \rho^K)$$
$$\doteq -\frac{1}{12} f^{IJK} F_{\rho\sigma}{}^J H^{\mu\nu\rho\sigma K} - \frac{1}{6} m \left[-\frac{1}{2} f^{IJK} D_\rho (\bar{\lambda}^J \gamma^{\mu\nu\rho} \rho^K) - \frac{1}{2} f^{IJK} F_{\rho\sigma}{}^J H^{\mu\nu\rho\sigma K} \right]$$
$$- \frac{1}{12} m f^{IJK} D_\rho (\bar{\lambda}^J \gamma^{\mu\nu\rho} \rho^K) \doteq 0, \tag{4.6}$$

where we used the B-field equation for the $D\mathcal{G}$-term.

5. Superspace reformulation

We have so far discussed only component formulation. We can re-formulate our component results in terms of superspace language. In the conventional superspace formalisms for the typical multiplets of VM, chiral multiplets, or *singlet* tensor multiplets are performed in terms of *unconstrained* pre-potential superfields. For example, for a *singlet* (Abelian) tensor multiplet, the *unconstrained* superfield is L [11]. This is possible for the case of *singlet* tensor multiplet, but *not* for our present non-Abelian TM. The obstruction against using the unconstrained superfield L is described with Eq. (4.11) in our previous paper [2]. For this reason, we can *not* rely on the so-called *unconstrained* pre-potential formulation.

Our formulation to be given here is very similar to our previous superspace reformulation for Proca–Stueckelberg mechanism such as in [2]. Our superspace notation has slight difference from our component formulation. We use the indices $A \equiv (a, \alpha)$, $B \equiv (b, \beta)$, ... for superspace coordinates, where $a = 0, 1, \ldots, 3$ (or $\alpha = 1, \ldots, 4$) are for the bosonic (or fermionic) coordinates. Accordingly, our fundamental field content will be VM $(A_a{}^I, \lambda_\alpha{}^I)$, TM $(B_{ab}{}^I, \chi_\alpha{}^I, \varphi^I)$ and EVM $(K_a{}^I, \rho_\alpha{}^I, C_{abc}{}^I)$. The superfield strengths of $A_a{}^I$, $B_{ab}{}^I$ and $C_{abc}{}^I$ are respectively $F_{ab}{}^I$, $G_{abc}{}^I$ and $H_{abc}{}^I$.

These superfields satisfy the superspace Bianchi identities (Bids)[3]

$$+\frac{1}{2} \nabla_{[A} F_{BC)}{}^I - \frac{1}{2} T_{[AB|}{}^D F_{D|C)}{}^I \equiv 0, \tag{5.1a}$$

$$+\frac{1}{6} \nabla_{[A} G_{BCD)}{}^I - \frac{1}{4} T_{[AB|}{}^E G_{E|CD)}{}^I - m H_{ABCD}{}^I$$
$$+ \frac{1}{4} f^{IJK} F_{[AB}{}^J L_{CD)}{}^K - \frac{1}{6} M_{[ABC} \nabla_{D)} \varphi^I \equiv 0, \tag{5.1b}$$

[3] In superspace we use the convention for (anti)symmetrizations of indices, *e.g.*, $[AB) \equiv AB - (-1)^{AB} BA$, so that $[ab] \equiv ab - ba$, and $(\alpha\beta) \equiv \alpha\beta + \beta\alpha$.

$$+\frac{1}{2}\nabla_{[A}\mathcal{D}_{B)}\varphi^I - T_{AB}{}^C\nabla_C\varphi^I \equiv 0, \tag{5.1c}$$

$$+\frac{1}{24}\nabla_{[A}H_{BCDE)}{}^I - \frac{1}{12}T_{[AB|}{}^F H_{F|CDE)}{}^I$$
$$-\frac{1}{12}F_{[AB}{}^J G_{CDE)}{}^K - \frac{1}{12}M_{[ABC}L_{DE)}{}^I \equiv 0, \tag{5.1d}$$

$$+\frac{1}{2}\nabla_{[A}F_{BC)}{}^I - \frac{1}{2}T_{[AB|}{}^D F_{D|C)}{}^I \equiv 0, \tag{5.1e}$$

$$+\frac{1}{6}\nabla_{[A}M_{BCD)} - \frac{1}{4}T_{[AB|}{}^E M_{E|CD)} \equiv 0. \tag{5.1f}$$

The constraints at the engineering dimensions $d = 0,\ 1/2$ and 1 are

$$T_{\alpha\beta}{}^c = +2(\gamma^c)_{\alpha\beta}, \qquad M_{\alpha\beta c} = +2(\gamma_c)_{\alpha\beta}, \tag{5.2a}$$

$$\nabla_\alpha\varphi^I = -\chi_\alpha{}^I, \qquad H_{\alpha bcd}{}^I = -(\gamma_{bcd}\rho^I)_\alpha, \tag{5.2b}$$

$$G_{\alpha bc}{}^I = -(\gamma_{bc}\chi^I)_\alpha - m^{-1}f^{IJK}(\gamma_{[b|}\lambda^J)_\alpha\nabla_{|c]}\varphi^K + m^{-1}f^{IJK}\chi_\alpha{}^J F_{bc}{}^K, \tag{5.2c}$$

$$F_{\alpha b}{}^I = -(\gamma_b\lambda^I)_\alpha, \qquad L_{\alpha b}{}^I = -(\gamma_b\rho^I)_\alpha, \tag{5.2d}$$

$$\nabla_\alpha\lambda_\beta{}^I = +\frac{1}{2}(\gamma^{cd})_{\alpha\beta}F_{cd}{}^I - \frac{1}{2}f^{IJK}(\gamma_5)_{\alpha\beta}(\bar{\chi}^J\gamma_5\rho^K), \tag{5.2e}$$

$$\nabla_\alpha\chi_\beta{}^I = -\frac{1}{6}(\gamma^{cde})_{\alpha\gamma}\mathcal{G}_{cde}{}^I - (\gamma_c)_{\alpha\beta}\nabla_c\varphi^I + \frac{1}{2}f^{IJK}(\gamma_c\lambda^J)_\alpha(\gamma^c\rho^K)_\beta$$
$$-\frac{1}{2}f^{IJK}\lambda_\alpha{}^J\rho_\beta{}^K + \frac{1}{2}f^{IJK}(\gamma_5\lambda^J)_\alpha(\gamma_5\rho^K)_\beta, \tag{5.2f}$$

$$\nabla_\alpha\rho_\beta{}^I = +\frac{1}{2}(\gamma^{cd})_{\alpha\beta}L_{cd}{}^I + \frac{1}{24}(\gamma^{cdef})_{\alpha\beta}H_{cdef}{}^I + \frac{1}{2}f^{IJK}(\gamma_c\lambda^J)_\alpha(\gamma^c\chi^K)_\beta$$
$$+\frac{1}{2}f^{IJK}\lambda_\alpha{}^J\chi_\beta{}^K + \frac{1}{2}f^{IJK}(\gamma_5\lambda^J)_\alpha(\gamma_5\chi^K)_\beta$$
$$+\frac{1}{4}m^{-1}h^{IJ,KL}\big[+(\gamma^d)_{\alpha\beta}(\bar{\lambda}^J\lambda^K) - (\gamma_5\gamma^d)_{\alpha\beta}(\bar{\lambda}^J\gamma_5\lambda^K)$$
$$-(\gamma_5)_{\alpha\beta}(\bar{\lambda}^J\gamma_5\gamma^d\lambda^K) - (\gamma_5\gamma^{cd})_{\alpha\beta}(\bar{\lambda}^J\gamma_5\gamma_c\lambda^K)\big]\mathcal{D}_d\varphi^L$$
$$+\frac{1}{24}m^{-1}h^{IJ,KL}\big[-(\gamma^{cde})_{\alpha\beta}(\bar{\lambda}^J\lambda^K) + (\gamma_5\gamma^{cde})_{\alpha\beta}(\bar{\lambda}^J\gamma_5\lambda^K)$$
$$+C_{\alpha\beta}(\bar{\lambda}^J\gamma^{cde}\lambda^K) + 3(\gamma_5\gamma^{cd})_{\alpha\beta}(\bar{\lambda}^J\gamma_5\gamma^e\lambda^K)\big]\mathcal{G}_{cde}{}^L. \tag{5.2g}$$

Even though *not* explicitly shown, all other independent components are zero, *e.g.*, $F_{\alpha\beta}{}^I = 0$ or $H_{\alpha\beta\gamma\delta}{}^I = 0$, *etc.* As usual in superspace, the Bids at $d = 3/2$ and $d = 2$ give the superfield equations of all of our fundamental fields $A_a{}^I$, $\lambda_\alpha{}^I$, $B_{ab}{}^I$, $\chi_\alpha{}^I$, $K_a{}^I$, $\rho_\alpha{}^I$ and $C_{abc}{}^I$. Since these are consistent with our field equations in (4.1), they are skipped in order to save space.

6. Concluding remarks

In this paper, we have presented a very peculiar supersymmetric system that realizes the Proca–Stueckelberg compensator mechanism [3] for an EVM. Our present model has resemblance to our recent model [2], which had only two multiplets VM and TM.

The peculiar features of our model are summarized as

(i) We have three multiplets VM, TM and EVM, where the EVM will be eventually massive.
(ii) Our peculiar field strength $\mathcal{G} = 3DB + mC - 3m^{-1}F \wedge B$ has the last term with m^{-1}.
(iii) Our model provides yet another mechanism of absorbing the dilaton-type scalar field φ^I into the extra vector $K_\mu{}^I$, different from the conventional YM gauge field $A_\mu{}^I$.
(iv) Even the tensor $C_{\mu\nu\rho}{}^I$ in the EVM gets a mass absorbing $B_{\mu\nu}{}^I$ in the TM.

Even though our system is less economical than [2] with an additional multiplet EVM, it has its own advantage. First, we provide a mechanism for giving a mass to the extra vector $K_\mu{}^I$ in the EVM, which may be not needed as a massless particle at low energy. Second, we have a new compensator mechanism for an extra vector $K_\mu{}^I$ in the adjoint representation, which is *not* the YM gauge field. The derivative $\mathcal{D}_\mu\varphi^I$ is simpler than exponentiations [2].

General formulations for different representations (not necessarily adjoint representations) for supersymmetric compensator mechanism have been given in [1]. However, we emphasize here that the fixing of supersymmetric couplings for our system with a different field content is a highly non-trivial task. Even superspace formulation does not help so much, because of the obstruction described in Section 4 of [2]. The main reason is that the usual unconstrained formalism in terms of the singlet superfield L [11] can *not* describe a tensor multiplet in the adjoint representation.

Our results can be applied to diverse space–time dimensions and also to extended supersymmetric systems.

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